SOUND VISUALIZATION AND MANIPULATION
SOUND VISUALIZATION AND MANIPULATION

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About the Author

Yang-Hann Kim
The research area of Yang-Hann Kim is mainly acoustics, noise/vibration. Experimental approaches and associated digital signal processing are used the most. Research projects include sound field visualization, noise source identification using array microphones, detection and estimation of moving noise source, structural acoustics, duct acoustics, silencer design, diagnostics of machines, and active noise/vibration control. Recently, he has been recognized as a pioneer in the field of sound visualization and manipulation. The latter is to make any sound field or shape in the selected region/regions. Therefore, it can be used for having very focused sound field, private sound zone/zones, or 3D listening field.

Dr Kim joined the Department of Mechanical Engineering as an Associate Professor in 1989. Previously he worked for five years at the Korea Institute of Technology as an Assistant and Associate Professor of the Department of Mechatronics. From 1979 to 1984, he was a research assistant at the Acoustics and Vibration Laboratory of Massachusetts Institute of Technology while pursuing Ph.D. degree in the field of acoustics and vibration, and obtained Ph.D. in February 1985 at M.I.T., Mechanical Engineering (O.E. Program).

He has been on the editorial board of Mechanical Systems and Signal Processing (MSSP), editorial advisor of the Journal of Sound and Vibration (JSV) and Journal of Noise Control Engineering. He also served KSNVE as an editor for three years (1995–97). His research has been recognized in the professional societies and institutes in many respects, including the best paper award by KSNVE (1998), the best research award by ASK (1997), second place award in the sound visualization competition by the Acoustical Society of America (1997), the best international cooperation award from KAIST, and KSNVE, and the best teaching award from KAIST, department of M.E (2010). He is elected as co-chairman of inter-noise 2015, San Francisco, also a director of I-INCE. He is a Fellow of the Acoustical Society of America.

Dr Kim has published more than 100 papers, mostly in the field of sound visualization and manipulation, in the well-known journals, including the Journal of the Acoustical Society of America, Journal of Sound and Vibration, and Journal of Acoustics and Vibration, the Transaction of ASME. He is an author of well-known acoustics text, Sound Propagation: An Impedance Based Approach, published by John Wiley & Sons, Inc. He also wrote the chapter “Acoustic holography” in the Handbook of Acoustics, published by Springer Verlag. He has delivered two plenary lectures in ICA (2004), and Inter-Noise (2012) and one keynote lecture in ICSV (2009). All of these lectures were on acoustic holography, sound visualization, and manipulation.

Jung-Woo Choi
Jung-Woo Choi’s primary research area includes active sound control and array signal processing for loudspeaker/microphone arrays. His research interests also include sound field reproduction, sound focusing, and their application for audio systems. From 1999, he has been working on
sound/noise control over elected regions based on the concept of acoustic contrast, which has been widely adopted for the implementation of personal sound zones. Recently, his research has extended to interactive 3D sound/multi-channel audio systems that can be manipulated in real-time by exploiting the beauty of direct integral formulas.

Dr Choi received his B.Sc., M.Sc., and Ph.D. degrees in Mechanical Engineering from the Korea Institute of Science and Technology (KAIST), Korea, in 1999, 2001, and 2005, respectively. He was a Postdoctoral Research Associate with the Center for Noise and Vibration Control (NOVIC), KAIST, Korea, 2005–06. From 2006 to 2007, he was a Visiting Postdoctoral Researcher at the Institute of Sound and Vibration Research (ISVR), University of Southampton, UK. From 2007 to 2011, he was with Samsung Electronics at the Samsung Advanced Institute of Technology (SAIT) in Korea, working on array-based audio systems as a Research & Development staff member and a Senior Engineer. In 2011, he joined the Department of Mechanical Engineering, KAIST, Korea, and has since been a Research Professor there. He is the author of more than 50 papers/conference articles and 15 patent applications, including five registered patents on loudspeaker array systems.
Preface

If only we could see sound propagation in space with our eyes, and if only the sound could be created in any desired shape! Such a fantastic concept is being realized. New approaches to acoustics and noise engineering have allowed innovative changes in these fields.

So far, extensive efforts have been made using various methods to explain how a medium changes as sound propagates in space or how the shape of the sound propagation changes depending on its frequency and wavelength. There are two main approaches being employed to resolve these questions: theoretical and experimental. The theoretical approach is adopted to develop an understanding of the phenomena of sound propagation in acoustic waves and, through this understanding, to attempt to find a solution. The characteristics of acoustic wave equations in certain cases are interpreted by numerically solving the so-called linear acoustic wave equations. Popular numerical techniques are the finite element method and boundary element method; both have achieved incredible developments owing to continuous evolutions in their background theories and improvements in the arithmetic capacity of computers. The experimental approach has also seen rapid improvements. Developments in semiconductor technologies have reduced the microphone size to eliminate unnecessary scattering induced by them, and the reduction in cost allows for tens and hundreds of microphones to be used at the same time. We can now sample, record, and analyze signals from hundreds of microphones in almost real time. These developments allow us to “visualize” sound using our eyes in the real world, which is something that human beings have long dreamt of. The first half of this book explains various methods to visualize sound.

From a mathematical point of view, sound visualization can be regarded as an exploration of methods to transform measured data into information that is visible to the human eyes. Most information transformation is determined during selection of the desired basis function because information transformation can produce different results depending on its mapping functions, as shown in the figure below. Thus, we need to deal with problems such as selecting a basis function and expressing a sound field as a visible image using the selected basis function. This book explains the planar, cylindrical, and spherical basis functions used in acoustic holography and the functions employed for beamforming methods. Their advantages and disadvantages as well as practical applicability are addressed. The advantage of the acoustic holography method includes visualization of information with great physical significance, such as acoustic pressure, velocity, intensity, and energy. On the other hand, the beamforming method can provide a variety of visualization information depending on the type of basis function used for the beamformer.

The concept that visualization results vary significantly depending on the basis functions can be reasonably expanded to an idea of sound manipulation where arbitrary or desired forms of sound can be created in space. Desired sounds can be produced by selecting basis functions so that the sounds generated from the sound sources arranged in the space are of these types of basis functions, as shown in the figure. Well-known methods include wave field synthesis (WFS) and Ambisonics. WFS is a representative method based on the so-called Kirchhoff–Helmholtz integral
equation, whereas Ambisonics is a technique that expresses sound fields using spherical harmonics and embodies the desired shapes of sound in space using such expressions. From a unifying point of view, the manipulation of a sound field is an issue in obtaining the desired output using the available sound sources; accordingly, we can select the best basis function depending on the definition of the desired function. Based on this idea, the sound focusing problem of concentrating the sound in desired areas or dividing an area into acoustically bright and dark zones and maximizing the ratio of sound energies between the two areas can also be explained.

Both the sound visualization method and sound manipulation method demand considerable theoretical knowledge of mathematics and acoustics as well as knowledge of signal processing to understand their principles and realize their practical applications. To aid potential readers who want to understand the basic concepts or those who will practically apply the methods, the simplest one-dimensional theories are introduced in this book, and their mathematical and theoretical explanations are presented in every chapter. Chapter 1 is intended to aid understanding of the basic physical quantities in acoustics. Part I consists of two chapters. Chapter 1 explains three physical quantities in acoustics using one-dimensional examples: interrelationships among acoustic pressure, particle velocity, and acoustic density. This approach is justified in that the principle of superposition holds for a linear system; hence, most of the concepts explained in one dimension can be extended to multidimensional cases.

Part II introduces the sound visualization methods and explains how their basic principles can be varied depending on certain basis functions. Accordingly, basis functions and approaches for the acoustic holography and the beamforming method are introduced. An appropriate basis function should be used depending on what we want to visualize; depending on this basis function, the information of visualized sound fields can be varied.

In Part III, we deal with sound manipulation techniques. Sound manipulation is carried out using two main methods; both are discussed with respect to how they are embodied in one-dimensional situations. Sound manipulation involves a sound focusing technique that concentrates the sound in specific areas in space and a sound field reproduction method that generates a wave front in the desired forms. For realizing these two methods, unique inputs to generate sound fields in the desired forms need to be determined. Therefore, the sound focusing and reproduction problems are defined as inverse problems corresponding to the beamforming and acoustic holography methods, respectively. Thus, the sections on acoustic holography and sound field reproduction are organized to complement each other. The sections on beamforming and sound focusing address similar issues but explain them from different points of view. The chapter on beamforming focuses on a signal processing technique to extract the parameter determining the locations of sound sources, whereas the chapter on sound focusing explains resolution variations depending on the geometric configuration of arrays and beam pattern variations depending on the basic aperture functions. Thus, Parts II
and III address different and similar issues from complementary points of view; readers interested
in visualization are strongly recommended to read the manipulation part. It would be efficient for
readers of this book to use Part I as a reference when they need to know more about acoustics
while reading Parts II and III.

In conclusion, this book introduces and explains methods for sound visualization and manipula-
tion. The book is organized such that readers can gain a profound understanding of basic concepts
and theoretical approaches from the one-dimensional case. The methods of visualization and manip-
ulation are explained as a unifying approach for creating certain assumed or desired shapes in space
based on the measured or available information using basis functions.

Yang-Hann Kim
Jung-Woo Choi
Acknowledgments

It was around 1990 that the first author had an idea about the basis function described in this book. He visited his old friend, Prof. J. K. Hammond of the Institute of Sound and Vibration Research (ISVR), University of Southampton, who was giving a lecture on nonlinear signal processing for a group of people in the industry at the time. The first page of the handout made for that class included a primitive version of the picture that is published in this book’s preface. In fact, this picture originated and evolved from the image in Science with a Smile (Robert L. Weber, Institute of Physical Publishing, Bristol and Philadelphia, 1992, pp. 111–12). The moment he looked at this picture, it occurred to him that a part of this picture could be used to explain the processing of every signal. Signal processing essentially involves finding desired information using available data. Thus, this picture symbolically shows that the quality of information obtained eventually depends on how well the processing method represents substantial, physical, or mathematical situations. Ultimately, the result is fully dependent on the processing method one has chosen, that is, a basis function. If so, how do we select a basis function? Although it is a very basic question, it is self-evident that if we can suggest the best method to be selected, it would serve as a very innovative and useful method in this discipline.

In fact, since the first author was at the time working on issues such as mechanical noise diagnosis and fault detection using signal processing, he looked at the picture that Prof. Hammond had used in a symbolic manner and gained an idea to view various problems in a unifying manner. He came to realize that both sound visualization techniques – acoustic holography and beamforming – eventually produced different results with regard to visualization owing to differences in the basic functions that were used. If so, in-depth knowledge of whether a basis function used has a mathematical function to perform something well would allow one to have a good understanding of the result of the sound visualization, that is, the picture. Thus, to clearly interpret the visualized information gained through the acoustic holography method and to accurately analyze the desired information, it is necessary to analyze how well the basis function expresses the desired visual information. Similarly, the following questions can be approached from an understanding of the basis function that was used: what is the specific information that can be gained from the spatial distribution of the beamforming power obtained from the beamforming method? Does the maximum value of the beaming power correctly describe the locations of sound or noise sources? What properties of the sources does the spatial distribution of the beaming power represent? In this regard, the visualization described in this book was greatly inspired by the discussions with Prof. Hammond.

The sound manipulation study described in the second half of this book started, it is recalled, around 1999 when the first author thought he had found some improvements in the study of visualization or was getting bored with studying sound visualization. The second author was studying how to focus sounds in an arbitrary space as part of the work for a master’s degree, and based on this, he started a full-scale study on sound manipulation. The first result aimed to practically
implement a system that allows one to hear a desired sound without disturbing others by focusing the sound in a specific space. Fortunately, the experiment was successful, and in 2000, the authors succeeded in focusing sound in a specific space by using six loudspeakers. Later on, this study was expanded to a study of a home/mobile speaker array system by the second author in the industry and to another study of monitor speaker array development by the first author at KAIST. The monitor speaker array system led to the implementation of a personal audio system that focuses sound using nine speakers. Among those who participated in the theoretical development and the experiment are Chan-hee Lee, currently working at Hyundai Heavy Industries Co., Ltd.; Dr Ji-ho Chang, currently at DTU after completing his doctoral degree; and Jin-young Park, currently pursuing his doctoral degree. At that time, the result attracted so much attention that it was broadcast on national TV. Thanks to this, the first author was granted an unexpected research fund, and his team could build a set of experimental equipment consisting of 32 speakers supported by KAIST’s HRHR project. Using this experimental equipment, the research team implemented a method for focusing sounds on specific spots in various ways, and the effects were found to be better than expected. One day, a question was raised about what results would be produced if the focusing point was moved to an arbitrary location. Min-ho Song developed an interface using an iPhone, in which as a finger moved to a location, the location at which sound was focused was also changed, making the sound audible in real time. In fact, from a theoretical viewpoint, they knew that the sound focusing solution had nothing to do with 3D sound. However, the listener could feel the effect of the location of the sound source moving through the sound focusing solution only. As a matter of fact, studies have reported on a focused source using the time-reversed driving function in wave field synthesis, but these did not have sufficient theoretical basis, and no perfect integral equation form was available for the array in the form of surrounding the listener. The theoretical explanation of the experimental results was completed by the second author in 2011, and it was proved that a general solution can be drawn by combining Porter-Bojarski integral with a multipole virtual source. The first work to create sounds using this solution aimed to relocate a mosquito’s sound to a desired space, and it was a great success. This substantial success became a motivation for the book’s third part. The doctoral students, Jeong-min Lee and Dong-su Kang, made substantial contributions to developing a speaker system that implemented a sound ball. In addition, the authors would like to acknowledge Dr Min-Ho Song, who performed great research while completing his doctoral degree at the Graduate School of Cultural Technology and who contributed to developing one particular interface.

In fact, the graduates’ wonderful studies were greatly helpful to the authors in writing the sound visualization part. In particular, studies by Dr Jae-Woong Choi, who has made great achievements in the areas of spherical beamforming and music, and Dr Young-Key Kim, who founded a company and has been disseminating sound visualization technology, were very helpful in writing the beamforming chapter. Dr Hyu-sang Kwon developed moving frame acoustic holography (MFAH), and he is expected to realize great achievements as an expert in this area. Furthermore, Dr Soon-hong Park of the Korea Aerospace Research Institute has made a great contribution to the method by applying MFAH to moving sound sources. The authors also want to acknowledge Dr Sea-Moon Kim of the Korea Institute of Ocean Science and Technology who successfully lead the acoustic holography experiment on the King Seong-deok Bell; Dr Kyung-Uk Nam of Hyundai Motor Company who greatly contributed to the partial field acoustic holography; and Dr Chun-Su Park who developed the time domain acoustic holography technique using the spatio-temporal complex envelope.

Credit for Chapter 4 of this book also belongs to Ku-Hwan Kim, who programmed most of the codes for beamforming simulations. The authors would like to express their appreciation to all the laboratory members – Jung-Min Lee, Dong-Soo Kang, Dae-Hoon Seo, Ki-Won Kim, Myung-Ryun Lee, Seong-Woo Jung – for their enthusiasm in correcting errors and giving advice to improve the content of this book.
The second author also wants to thank his former advisors at KAIST – Yoon-Sik Park, Chong-Won Lee, Jeong-Guon Lee, and Young-Jin Park – for teaching him the fundamentals of sound and vibration. Special thanks must be given to Prof. P. A. Nelson and S. J. Elliott and Dr F. M. Fazi of ISVR for many hours of fruitful discussions with him regarding the sound field reproduction and sound focusing projects. The experiences with his former colleagues at Samsung Electronics – Younghae Kim, Jungho Kim, Sang-Chul Ko, and Seoung-Hun Kim – were greatly helpful in summarizing the techniques discussed in Chapters 4–6.

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Yang-Hann Kim
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Sound is an important part of our lives. Even in the womb, human beings are capable of detecting sounds. We create and enjoy sounds, and we can identify information conveyed by sound. We live with sound and are familiar with the fundamental concepts associated with it. Fundamental concepts of sound visualization and manipulation can also be explained on the basis of the mechanisms by which a sound wave is generated, propagated, and decayed by various internal and external disturbances.

Acoustics is a vast field of study that explains the propagation of waves in different media, and it cannot be completely covered in merely the first two chapters of this book. In this book, however, we limit the scope by focusing on the general idea of acoustics in terms of its essential physical measures. This part of the book discusses essential measures that refer to the primary measures or physical variables used in acoustics, such as acoustic pressure, velocity, intensity, and energy, which can be used to describe sound propagation. Various impedances, radiations, scatterings, surfaces, and so on, are also considered as important measures that affect wave propagation in space. In order to uniquely and conveniently explain the physics of acoustics, this part of the book relies heavily on the concept of impedance as a window to study sound propagation in time and space.

Chapter 1 introduces the essential physical parameters used in acoustics measurements. The significance of physical parameters other than impedance, such as sound pressure, speed, energy, power, and intensity, are explained (Figure I.1). It is emphasized that these parameters form the fundamental concepts required for understanding the propagation of sound waves. The aforementioned parameters are explained by using a one-dimensional approach. The Euler equation is used to describe the relation between the sound pressure and the particle velocity in a given medium. The state equation, on the other hand, is used to evaluate the relation between the acoustic density and the fluctuating pressure, which is the acoustic pressure that causes the sound propagation. The third equation used is the law of conservation of mass for the compressible fluid, which defines how the fluctuating density and the fluid particle velocity are associated with each other. Therefore, the three essential variables: sound pressure, particle velocity, and fluctuating density, follow these three equations. This enables the derivation of the acoustic wave equation that governs all the parameters associated with acoustic wave propagation. Chapter 1 discusses the two different approaches that can be used to solve this acoustic wave equation. The first one is based on the eigenfunction analysis, in which the solution is determined as the superposition of eigenmodes. Another approach uses Green’s function, which describes how a sound field is constructed when the field has a monopole source at an arbitrary position in space. This approach leads to the Kirchhoff–Helmholtz equation.
Equation of state

\[
\frac{p'}{\rho'} = c^2
\]

Linear Euler’s equation

\[
-\frac{\partial p'}{\partial x} = \rho_0 \frac{\partial u}{\partial t}
\]

**Figure I.1** Pictorial relation between three variables that govern acoustic wave propagation (\(p_0\) and \(\rho_0\) express the mean pressure and static density, respectively; \(p'\) and \(\rho'\) denote acoustic pressure and fluctuating density, respectively; \(c\) denotes the speed of propagation, and \(u\) is the velocity of the fluctuating medium)

**Figure I.2** Reflection and transmission phenomena using the principle of superposition

Chapter 2 takes a rather ambitious route to describe how sound wave reacts under impedance mismatch in space and time using the concept of radiation, scattering, and diffraction. It is believed that the scattering and the diffraction of sound can be explained by acoustic radiation. For instance, a scattered sound field is a result of radiation scattering (Figure I.2), whereas diffraction is a result of the radiation from an object that has spatial impedance mismatch.

An understanding of the first two chapters is expected to help in analyzing and explaining the results obtained by the sound visualization described in Chapters 3 and 4, and the manipulation described in Chapters 5 and 6.
Chapter 1

Acoustic Wave Equation and Its Basic Physical Measures

1.1 Introduction

The waves along a string propagate along its length, but the string itself moves perpendicular to the propagation direction. It therefore forms a transverse wave. If the particle of a medium moves in the direction of propagation, we refer to it as a longitudinal wave. The waves in air, water, or any compressible medium are longitudinal waves, which are often referred to as acoustic waves. This chapter explores the underlying physics and sensible physical measures related to acoustic waves, including pressure, velocity, intensity, and energy. Impedance plays a central role with regard to its effect on these measures.

In the area of sound visualization, our objectives are to determine a rational means to convert essential acoustic variables such as pressure, velocity, and density, or other physically sensible acoustic measures such as intensity or energy, into visible representations. One very straightforward way to accomplish these objectives is to express acoustic pressure by using a color code. Notably, there are many ways to visualize a sound field, depending on a mapping or general basis function, which relates acoustic variables to visual expressions. Therefore, this chapter starts with a discussion on the visualization of a one-dimensional acoustic wave.

1.2 One-Dimensional Acoustic Wave Equation

The simplest case is illustrated in Figure 1.1. The end of a pipe or duct which is filled with a homogeneous compressible fluid (air, water, etc.) is excited with a radian frequency ($\omega = 2\pi f$, $f$: frequency in Hz). If the pipe is semi-infinitely long, then the pressure in the pipe ($p(x, t)$) can be mathematically written as

$$p(x, t) = P_0 \cos(kx - \omega t + \phi)$$  \hspace{1cm} (1.1)

where $P_0$ is the pressure magnitude and $\phi$ is an initial phase. Here, $k$ represents the spatial frequency ($k = 2\pi / \lambda$, $\lambda$: wavelength in m) of the pressure field, which is often called wavenumber.

If the pipe is of finite length $L$, then the possible acoustic pressure in the pipe can be written as

$$p(x, t) = P_0 \cos k(L - x) \cos \omega t.$$  \hspace{1cm} (1.2)

1 Sections of Chapter 1 have been re-used with permission from Ref. [1].
This depicts the waves that can be generated when we excite one end of the pipe, harmonically.

Equations (1.1) and (1.2) are different simply because of the boundary conditions: the former has no boundary condition prescribed at $x = L$, but the latter has a rigid-wall condition (velocity is zero).

To understand what is happening in the pipe, we have to understand how pressures and velocities of the fluid particles behave and are associated with each other. This motivates us to look at an infinitesimal element of the volume of the fluid in the pipe; specifically, we will investigate the relation between force and motion.

As illustrated in Figure 1.1, the forces acting on the fluid between $x$ and $x + \Delta x$ and its motion will follow the conservation of momentum principle. That is,

$$\text{Sum of the forces acting on the fluid} = \text{momentum change} \tag{1.3}$$

We can mathematically express this equality as

$$(pS)_x - (pS)_{x+\Delta x} = \rho S \frac{du}{dt} \Delta x \tag{1.4}$$

where it has already been assumed that the viscous force, which likely exists in the fluid, is small enough (relative to the force induced by pressure) to be neglected.

The rate of change of velocity ($du/dt$) can be expressed by

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \tag{1.5}$$

where $u$ is a function of position ($x$) and time ($t$) and velocity is the time rate change of the displacement. Therefore, we can rewrite Equation (1.5) as

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x}. \tag{1.6}$$
If the cross-section between $x$ and $x + \Delta x$ is maintained constant and $\Delta x$ becomes small ($\Delta x \to 0$), then Equation (1.4) can be expressed as

$$\frac{\partial p}{\partial x} = \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) = \rho \left( \frac{\partial }{\partial t} + u \frac{\partial }{\partial x} \right) \frac{Du}{Dt}$$

(1.7)

where

$$p = p_0 + p'$$

(1.8)

$$\rho = \rho_0 + \rho'$$

(1.9)

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial }{\partial x}$$

(1.10)

Note that the pressure ($p$) is composed of the static pressure ($p_0$) and the acoustic pressure ($p'$), which is induced by the small fluctuation of fluid particles. The density also has two components: the static density ($\rho_0$) and the small fluctuating density ($\rho'$).

Equation (1.10) is the total derivative, and is often called the material derivative. The first term expresses the rate of change with respect to time, and the second term can be obtained by examining the change with respect to space as we move with the velocity $u$. As can be anticipated, the second term is generally smaller than the first.

If the static pressure ($p_0$) and density ($\rho_0$) do not vary significantly in space and time, then Equation (1.7) becomes

$$- \frac{\partial p'}{\partial x} = \rho_0 \frac{\partial u}{\partial t}$$

(1.11)

where $p'$ is acoustic pressure and is directly related to acoustic wave propagation. As already implied in Equation (1.8), acoustic pressure is considerably smaller than static pressure. Equation (1.11) essentially means that a small pressure change across a small distance ($\Delta x$) causes the fluid of mass/unit volume $\rho_0$ to move with the acceleration of $\partial u/\partial t$. This equation is generally referred to as a linearized Euler equation. Equation (1.7), on the other hand, is an Euler equation.

Equations (1.7) and (1.11) describe three physical parameters, pressure, fluid density, and fluid particle velocity. In other words, they express the relations between these three basic variables. In order to completely characterize the relations, two more equations are needed.

The relation between density and fluid particle velocity can be obtained by using the conservation of mass. Figure 1.2 shows how much fluid enters the cross-section at $x$ and how much exits through the surface at $x + \Delta x$. If we apply the principle of conservation of mass law to the fluid volume between $x$ and $x + \Delta x$, the following equality can be written.

\[
\frac{\partial}{\partial t} (\rho S \Delta x) = (\rho u S)_x - (\rho u S)_{x+\Delta x}
\]

(1.12)

Note that we used $\partial x/\partial t = u$ in Equation (1.6). This is the Lagrangian description, which describes the motion of a mass of fluid at $\Delta x$. The other method to describe the momentum change through fixed infinitesimal control volume is by using the Euler description (Section 8.1.2). Note also that a more precise momentum balance can be expressed as $-\frac{\partial p}{\partial x} = \frac{\partial p u}{\partial x}$.

We assume that the effect of mass transport is negligible.

We also refer to the acoustic pressure as “access pressure” or “sound pressure.”
Acoustic Wave Equation and Its Basic Physical Measures

\[
\frac{\partial}{\partial t} (\rho S \Delta x) = (\rho u S)_x + \Delta x
\]

\(S\) : cross section area (m²)
\(u\) : fluid particle velocity in \(x\) direction (m/s)
\(\rho\) : density of fluid (kg/m³)

Figure 1.2 Conservation of mass in an infinitesimal element of fluid (increasing mass of the infinitesimal volume results from a net decrease of the mass through the surfaces of the volume)

as illustrated in Figure 1.2. As assumed before, if the area of the cross-section \((S)\) remains constant, then Equation (1.12) can be rewritten as

\[
\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x} (\rho u). \tag{1.13}
\]

We can linearize this equation by substituting Equation (1.9) into Equation (1.13). Equation (1.13) then becomes

\[
\frac{\partial \rho'}{\partial t} = -\rho_0 \frac{\partial u}{\partial x}. \tag{1.14}
\]

Equations (1.11) and (1.14) express the relation between sound pressure and fluid particle velocity, as well as the relation with fluctuating density and fluid particle velocity, respectively. One more equation is therefore needed to completely describe the relations of the three acoustic variables: acoustic pressure, fluctuating density, and fluid particle velocity. The other equation must describe how acoustic pressure is related to fluctuating density. Recall that a pressure change will induce a change in density as well as other thermodynamic variables, such as entropy. This leads us to postulate that acoustic pressure is a function of density and entropy, that is,

\[
p = p(\rho, s) \tag{1.15}
\]

where \(s\) denotes entropy. We can then write the change of pressure, or fluctuating pressure, \(dp\) or \(p'\), by modifying Equation (1.15) as follows:

\[
dp = \left. \frac{\partial p}{\partial \rho} \right|_s d\rho + \left. \frac{\partial p}{\partial s} \right|_\rho ds. \tag{1.16}
\]

This equation simply states that a pressure change causes a density change \((d\rho)\) and entropy variation \((ds)\). It is noticeable that the fluid obeys the law of isentropic processes when it oscillates within the range of the audible frequency: 20 Hz to 20 kHz. The second term on the right-hand side of Equation (1.16) is therefore negligible. This implies that the small change of sound pressure with regard to the infinitesimal change of density can be assumed to have certain proportionality. (An alternative way to deduce the same relation can be found in Appendix B, Section B.1.3.) Note that the second relation of Equation (1.16) is mostly found experimentally. This reduces Equation (1.16) to the form

\[
\frac{p'}{\rho'} = \frac{B}{\rho_0} = c^2 \tag{1.17}
\]

\(^5\) This is possible if the period of oscillation by the fluid particle is much smaller than the time required to dissipate or transfer the heat energy within the wavelength of interest.
where $B$ is the bulk modulus that expresses the pressure required for a unit volume change and $c$ is the speed of sound. We may obtain Equation (1.17) by introducing a gas dynamics model. This equation is an equation of state. Tables 1.1 and 1.2 summarizes the speed of sound in accordance with the state of gas [2]. An alternative method of deducing Equations (1.16) and (1.17) can be found in Appendix B, Section B.1.3.

Table 1.1 The dependency of the speed of sound on temperature

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<tr>
<th>Temperature (°C)</th>
<th>Speed of sound (m/s)</th>
<th>Temperature (°C)</th>
<th>Speed of sound (m/s)</th>
<th>Temperature (°C)</th>
<th>Speed of sound (m/s)</th>
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Table 1.2 The dependency of the speed of sound on relative humidity and on frequency

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<th>Relative humidity/ frequency</th>
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<th>30% Decay rate (%)</th>
<th>Speed of sound (m/s)</th>
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<th>100% Decay rate (%)</th>
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</table>

Note that Equation (1.17) expresses how the access pressure or acoustic pressure communicates with the fluctuating density. Equations (1.11) and (1.14) completely express the laws that govern the waves in which we are interested. Therefore, we can summarize the relations as

$$\frac{-\partial p'}{\partial x} = \rho_0 \frac{\partial u}{\partial t} \quad (1.11)$$

$$\frac{\partial \rho'}{\partial t} = -\rho_0 \frac{\partial u}{\partial x} \quad (1.14)$$

$$\frac{p'}{\rho'} = c^2. \quad (1.17)$$

Figure 1.3 demonstrates how these equations and physical variables are related. If we eliminate $\rho'$ and $u$ from Equations (1.11), (1.14), and (1.17), then we obtain

$$\frac{\partial^2 p'}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 p'}{\partial t^2} \quad (1.18)$$

This is a linearized acoustic wave equation.\(^6\)

Equation (1.18) is essentially a general one-dimensional acoustic wave, that is, the waves in compressible fluid. A similar relation can be found from the propagation of a string wave. The only difference between the waves along a string and acoustic waves lies in whether the directions of wave propagation and velocity fluctuation of medium are collinear or perpendicular. Note that the propagation direction of the waves along a string is perpendicular to that of the motion of the string. Conversely, the acoustic wave propagates in the direction of the fluid particle’s velocity.\(^7\)

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\(^6\) If we eliminate $p'$ and $u$ or change to $p'$ and $\rho'$, then we can obtain the equation for $\rho'$ and $u$, respectively.

\(^7\) The linearized Euler equation (Equation (1.11)) essentially states that the pressure difference induces time rate of velocity $u$ in the $x$ direction.