DESIGN OF ROTATING ELECTRICAL MACHINES
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Second Edition

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Preface

Electrical machines are almost entirely used in producing electricity, and there are very few electricity-producing processes where rotating machines are not used. In such processes, at least auxiliary motors are usually needed. In distributed energy systems, new machine types play a considerable role: for instance, the era of permanent magnet machines has commenced.

About half of all electricity produced globally is used in electric motors, and the share of accurately controlled motor drives applications is increasing. Electrical drives provide probably the best control properties for a wide variety of processes. The torque of an electric motor may be controlled accurately, and the efficiencies of the power electronic and electromechanical conversion processes are high. What is most important is that a controlled electric motor drive may save considerable amounts of energy. In the future, electric drives will probably play an important role also in the traction of cars and working machines. Because of the large energy flows, electric drives have a significant impact on the environment. If drives are poorly designed or used inefficiently, we burden our environment in vain. Environmental threats give electrical engineers a good reason for designing new and efficient electric drives.

Finland has a strong tradition in electric motors and drives. Lappeenranta University of Technology and Aalto University have found it necessary to maintain and expand the instruction given in electric machines. The objective of this book is to provide students in electrical engineering with an adequate basic knowledge of rotating electric machines, for understanding of the operating principles of these machines as well as developing elementary skills in machine design. Although, due to the limitations of this material, it is not possible to include all the information required in electric machine design in a single book, this material will serve as a manual for a machine designer in the early stages of his or her career. The bibliographies at the end of chapters are intended as sources of references and recommended background reading. The Finnish tradition of electrical machine design is emphasized in this monograph through the important contributions of Professor Tapani Jokinen, who has spent decades in developing the Finnish machine design profession. Equally important is the view of electrical machine design provided by Professor Valéria Hrabovcová from Slovak Republic, which also has a strong industrial tradition.

In the second edition, some parts of the first edition have been rewritten to make the text proceed more logically and many printing errors have been corrected. Especially, permanent magnet machine and synchronous reluctance machine chapters are now much more comprehensive including new research results. Also the Eco-design principles and economical considerations in machine design are shortly introduced.
The authors are thankful for Dr. Hanna Niemelä for translating the original Finnish material for the first edition.

We express our gratitude to the following persons, who have kindly provided material for this book: Professor Antero Arkkio (Aalto University), Dr Jorma Haataja, Dr Tanja Hedberg (ITT Water and Wastewater AB), Mr Jari Jäppinen (ABB), Dr Hanne Jussila (LUT), Dr Panu Kurronen (The Switch Oy), Dr Janne Nerg (LUT), Dr Markku Niemelä (ABB), Dr Asko Parviainen (AXCO Motors), Dr Sami Ruoho (Teollisuuden Voima), Dr Marko Rilla (Visedo), Dr Pia Salminen (LUT), Dr Ville Sihvo (MAN Turbo), Mr Pavel Ponomarev, Mr Juho Montonen, Ms Julia Alexandrova, Dr. Henry Hämäläinen and numerous other colleagues. Dr Hanna Niemelä’s contribution to the first edition and the publication process of the original manuscript is particularly acknowledged.

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Abbreviations and Symbols

- $A$: linear current density, [A/m]
- $A$: magnetic vector potential, [Vs/m]
- $A$: magnetic vector potential scalar value, [Vs/m]
- $A$: temperature class 105 °C
- $AC$: alternating current
- $AM$: asynchronous machine
- $A1-A2$: armature winding of a DC machine
- $A_{1n}, A_{2n}, A_{3n}$: factors for defining permanent magnet flux density
- $a$: number of parallel paths in windings without commutator: per phase, in windings with a commutator: per half armature, diffusivity
- $B$: magnetic flux density, vector [Vs/m²], [T]
- $B$: magnetic flux density scalar value, [Vs/m²]
- $B_r$: remanent flux density, [T]
- $B_{sat}$: saturation flux density, [T]
- $B$: temperature class 130 °C
- $B1-B2$: commutating pole winding of a DC machine
- $b$: width, [m]
- $b_{0c}$: conductor width [m]
- $b_c$: conductor width [m]
- $b_d$: tooth width, [m]
- $b_{dr}$: rotor tooth width, [m]
- $b_{ds}$: stator tooth width, [m]
- $b_r$: rotor slot width, [m]
- $b_s$: stator slot width, [m]
- $b_0$: slot opening, [m]
- $b_v$: width of ventilation duct, [m]
- $C$: capacitance, [F], machine constant, integration constant, fabrication cost, [€]
- $C$: temperature class >180 °C
- $C1-C2$: compensating winding of a DC machine
- $C_f$: friction coefficient
- $C_M$: torque coefficient
- $C_s$: saving cost per year, [€/a]
Abbreviations and Symbols

- **c**: specific heat capacity, [J/kgK], capacitance per unit of length, factor, divider, constant
- **C_{diff}**: increase of the purchase cost, \([\text{€}]\)
- **c_e**: energy cost, \([\text{€/kWh}]\)
- **c_p**: specific heat capacity of air in constant pressure
- **C_{pw}**: cost per one kilowatt of loss over the life of motor, \([\text{€/kW}]\)
- **c_{th}**: heat capacity
- **CTI**: Comparative Tracking Index
- **c_v**: specific volumetric heat, \([\text{kJ/Km}^3]\)
- **D**: direct current
- **DOL**: direct- on-line
- **D_s**: inner diameter of the stator, \([\text{m}]\)
- **D_{sc}**: outer diameter of the stator, \([\text{m}]\)
- **D_t**: outer diameter of the rotor, \([\text{m}]\)
- **D_{ri}**: inner diameter of the rotor, \([\text{m}]\)
- **D1-D2**: series magnetizing winding of a DC machine
- **d**: thickness, \([\text{m}]\)
- **d_l**: thickness of the fringe of a pole shoe, \([\text{m}]\)
- **E**: electromotive force (emf), \([\text{V}]\), RMS, electric field strength, \([\text{V/m}]\), scalar, elastic modulus, Young’s modulus, \([\text{Pa}]\), bearing load
- **E_a**: activation energy, \([\text{J}]\)
- **E**: electric field strength, vector, \([\text{V/m}]\)
- **E**: electric field strength scalar value, \([\text{V/m}]\)
- **E**: temperature class 120 °C
- **E**: irradiation intensity \([\text{W/m}^2]\)
- **E1-E2**: shunt winding of a DC machine
- **e**: electromotive force \([\text{V}]\), instantaneous value \(e(t)\)
- **e**: Napier’s constant
- **emf**: electromotive force, \([\text{V}]\)
- **F**: force, \([\text{N}]\), scalar
- **F**: force, \([\text{N}]\), vector
- **F**: temperature class 155 °C
- **FEA**: Finite Element Analysis
- **F_g**: geometrical factor
- **F_{mm}**: magnetomotive force \(\oint \mathbf{H} \cdot d\mathbf{l}\), \([\text{A}]\), (mmf)
- **F1-F2**: separate magnetizing winding of a DC machine or a synchronous machine
- **f**: frequency, \([\text{Hz}]\), Moody friction factor
- **f_{Br}**: factor for defining permanent magnet radial flux density
- **f_{Bth}**: factor for defining permanent magnet tangential flux density
- **g**: coefficient, constant, thermal conductance per unit length
- **G**: electrical conductance
- **G_{th}**: thermal conductance
- **H**: magnetic field strength, \([\text{A/m}]\)
- **H**: magnetic field strength scalar value, \([\text{A/m}]\)
- **H_c, H_{cB}**: coercivity related to flux density, \([\text{A/m}]\)
Abbreviations and Symbols

$H_{cJ}$ coercivity related to magnetization, [A/m]

H temperature class 180 °C

$H_n$ number of partial discharges

$h$ height, [m]

$h_{0c}$ conductor height [m]

$h_c$ conductor height [m]

$h_d$ tooth height [m]

$h_p$ height of a subconductor, [m]

$h_{p2}$ height of pole body, [m]

$h_{ys}$ height of stator yoke, [m]

$h_{yr}$ height of rotor yoke, [m]

$h_s$ stator slot height, [m]

$I$ electric current, [A], RMS, brush current, second moment of inertia of an area, [m^4]

IM induction motor

$I_{ns}$ counter-rotating current (negative sequence component), [A]

$I_o$ current of the upper bar, [A]

$I_n$ current of the lower bar, slot current, slot current amount, [A]

$I_s$ conductor current

IC classes of electrical machines

IEC International Electrotechnical Commission

Im imaginary part

$i$ current, [A], instantaneous value $i(t)$, per unit value of current, [pu], annual rate of interest

$J$ moment of inertia, [kgm^2], current density [A/m^2], magnetic polarization

$J_{0PM}$ current density on the PM surface, [A/m^2]

$J_{PM}$ eddy current density, [A/m^2]

$\mathbf{J}$ Jacobian matrix

$J_{\text{ext}}$ moment of inertia of load, [kgm^2]

$J_M$ moment of inertia of the motor, [kgm^2]

$J_{\text{sat}}$ saturation of polarization, [Vs/m^2]

$\mathbf{J}_s$ surface current, vector, [A/m]

$J_s$ surface current vector scalar value, [A/m]

$j$ difference of the numbers of slots per pole and phase in different layers

$j$ imaginary unit

$K$ transformation ratio, constant, number of commutator segments

$K_{Br}$ factor for defining permanent magnet radial flux density

$K_{B\theta}$ factor for defining permanent magnet tangential flux density

$K_L$ inductance ratio

$k$ connecting factor (coupling factor), correction coefficient, safety factor, ordinal of layers, roughness coefficient

$k_E$ machine-related constant

$k_C$ Carter factor

$k_{Cu}, k_{Fe}$ space factor for copper, space factor for iron

$k_d$ distribution factor, correction factor, saliency factor in d- axis

$k_q$ saliency factor in q- axis
$k_{dsat}$  saliency factor taking into account saturation in d-axis

$k_{qpar}$  saliency factor taking into account parallel magnetic lines in q-axis

$k_{Fe+n}$  correction factor

$k_k$  short circuit ratio

$k_{L}$  skin effect factor for the inductance

$k_p$  pitch factor

$k_{pw}$  pitch factor due to coil side shift, present worth factor of an equal payment series

$k_{R}$  skin effect factor for the resistance

$k_{sat}$  saturation factor

$k_{sq}$  skewing factor

$k_{th}$  coefficient of heat transfer, [W/m²K]

$k_v$  pitch factor of the coil side shift in a slot

$k_w$  winding factor

$k_{\sigma}$  safety factor in the yield

$L$  self inductance [H]

$L$  characteristic length, characteristic surface length, tube length [m]

$LC$  inductor-capacitor

$L_{d}$  tooth tip leakage inductance, synchronous inductance in d-axis [H]

$L_{q}$  synchronous inductance in q-axis [H]

$L_{d}/L_{q}$  inductance ratio

$L_{k}$  short-circuit inductance, [H]

$L_{m}$  magnetizing inductance, [H]

$L_{md}$  magnetizing inductance of an m-phase synchronous machine, in d-axis,[H]

$L_{mq}$  magnetizing inductance of an m-phase synchronous machine, in q-axis, [H]

$L_{mn}$  mutual inductance, [H]

$L_{mp}$  magnetizing inductance of single-phase winding, [H]

$L_{pd}$  main inductance of a single phase, [H]

$L_{sq}$  skew leakage inductance, [H]

$L_{u}$  slot inductance, [H]

$L_{w}$  end winding leakage inductance, [H]

$L_{\delta}$  air-gap leakage inductance, [H]

$L_{m\delta}$  magnetizing inductance of synchronous machines with non-salient poles, [H]

$L'$  transient inductance, [H]

$L''$  subtransient inductance, [H]

$L_1, L_2, L_3$  network phases

$l$  length [m], closed line, distance, inductance per unit of length, relative inductance (inductance per unit value), gap spacing between the electrodes

$l$  unit vector collinear to the integration path

$l'$  effective core length, [m]

$l_{ew}$  average conductor length of winding overhang, [m]

$l_p$  wetted perimeter of tube, [m]

$l_{pu}$  inductance as a per unit value

$l_w$  length of coil ends, [m]

$l_{sub}$  length of one sub-stack, [m]
Abbreviations and Symbols

$M$ \hspace{1cm} mutual inductance [H], magnetization [A/m]

$M_{\text{sat}}$ \hspace{1cm} saturation magnetization, [A/m]

$m$ \hspace{1cm} number of phases, mass, [kg]

$m_c$ \hspace{1cm} mutual coupling factor

$m_0$ \hspace{1cm} constant

$\text{mmf}$ \hspace{1cm} magnetomotive force, [A]

$N$ \hspace{1cm} number of turns in a winding, number of turns in series

$N_{f1}$ \hspace{1cm} number of coil turns in series in a single pole

$Nu$ \hspace{1cm} Nusselt number

$N_{ul}$ \hspace{1cm} number of bars of a coil side in the slot

$N_p$ \hspace{1cm} number of turns of one pole pair

$N_k$ \hspace{1cm} number of turns of compensating winding

$N_v$ \hspace{1cm} number of conductors in each side

$N$ \hspace{1cm} Non-drive end

$\mathbb{N}$ \hspace{1cm} set of integers

$N_{\text{even}}$ \hspace{1cm} set of even integers

$N_{\text{odd}}$ \hspace{1cm} set of odd integers

$n$ \hspace{1cm} normal unit vector of the surface

$n$ \hspace{1cm} rotation speed (rotation frequency), [1/s], ordinal of the harmonic (sub), ordinal of the critical rotation speed, integer, exponent, years of saving (motor life time)

$n_v$ \hspace{1cm} number of ventilation ducts

$n_{U}$ \hspace{1cm} number of section of flux tube in sequence

$n_{\Phi}$ \hspace{1cm} number of flux tube

$P$ \hspace{1cm} power, losses [W]

$P_{\text{in}}$ \hspace{1cm} input power, [W]

$\text{PAM}$ \hspace{1cm} Pole-Amplitude-Modulation

$\text{PM}$ \hspace{1cm} permanent magnet

$\text{PMSM}$ \hspace{1cm} permanent magnet synchronous motor (or machine)

$\text{PWM}$ \hspace{1cm} Pulse Width Modulation

$P_1, P_{\text{ad}}, P_{\text{LL}}$ \hspace{1cm} additional loss, [W]

$P_{\text{ew}}$ \hspace{1cm} end winding losses, [W]

$Pr$ \hspace{1cm} Prandtl number

$P_{\rho}$ \hspace{1cm} friction loss, [W]

$P_{\text{diff}}$ \hspace{1cm} reduction of the purchase cost, [$\text{€}$]

$P_{\text{PM}}$ \hspace{1cm} eddy current loss in permanent magnet, [W]

$p$ \hspace{1cm} number of pole pairs, ordinal, losses per core length, resistive losses per core length, [W/m], pressure, [Pa]

$P_{\text{Al}}$ \hspace{1cm} aluminium content

$p^*$ \hspace{1cm} number of pole pairs of a base winding

$pd$ \hspace{1cm} partial discharge

$Q$ \hspace{1cm} electric charge, [C], number of slots, reactive power, [VA]

$Q_{av}$ \hspace{1cm} average number of slots of a coil group

$Q_p$ \hspace{1cm} number of slots per pole

$Q_o$ \hspace{1cm} number of free slots

$Q'$ \hspace{1cm} number of radii in a voltage phasor graph
Abbreviations and Symbols

$Q^*$ number of slots of a base winding
$Q_{th}$ quantity of heat
$q$ number of slots per pole and phase, instantaneous charge, $q(t)$, [C]
$q_k$ number of slots in a single zone
$q_m$ mass flow, [kg/s]
$q_{th}$ density of the heat flow, [W/m$^2$]
$R$ resistance, [$\Omega$], gas constant, 8.314472 [J/K·mol], thermal resistance, reactive parts
$R_{bar}$ bar resistance, [$\Omega$]
RM reluctance machine
RMS root mean square
$R_m$ reluctance, [$A/Vs = 1/H$]
$R_{th}$ thermal resistance, [K/W]
Re real part
$Re$ Reynolds number
$Re_{crit}$ critical Reynolds number
RR Resin Rich impregnation method
$r$ radius, [m], thermal resistance per unit length, per unit resistance [pu], coefficient of radiation
$r$ radius unit vector
S1-S8 duty types
$S$ apparent power, [VA], cross-sectional area
SM synchronous motor
SR switched reluctance
SyRM synchronous reluctance machine
$S_c$ cross-sectional area of conductor, [m$^2$]
$S_p$ pole surface area, [m$^2$]
$S_r$ rotor surface area facing the air gap, [m$^2$]
$S$ Poynting’s vector, [W/m$^2$], unit vector of the surface
$s$ slip, skewing measured as an arc length
$s_b$ slip at maximum torque
$s_{sp}$ skewing expressed as a number of slot pitches
$T$ torque, [Nm], absolute temperature, [K], period, operating time of the motor per year, [h/a]
$Ta$ Taylor number
$T_{a_m}$ modified Taylor number
$T_b$ pull-out torque, peak (maximum) torque [Nm]
$t_c$ commutation period, [s]
TEFC totally enclosed fan-cooled
$T_j$ mechanical time constant, [s]
$T_{mec}$ mechanical torque, [Nm]
$T_{pb}$ payback time
$T_s$ temperature of the plane
$T_u$ pull-up torque, [Nm]
$T_v$ counter torque, [Nm]
$T_l$ locked rotor torque, [Nm]
Abbreviations and Symbols

TC  tooth coil

\( t \)  time, [s], number of phasors of a single radius, largest common divider, lifetime of insulation

\( t \)  tangential unit vector

\( t_c \)  commutation period, [s]

\( t_r \)  rise time, [s]

\( t^* \)  number of layers in a voltage vector graph for a base winding

\( U \)  voltage, [V], RMS

\( U_{\text{contact}} \)  depiction of a phase

\( U_{\text{m}} \)  contact voltage drop, [V]

\( U_{\text{r}} \)  resistive voltage, [V]

\( U_{\text{sj}} \)  peak value of the impulse voltage, [V]

\( U_v \)  coil voltage, [V]

\( U_1 \)  terminal of the head of the U-phase of the machine

\( U_2 \)  terminal of the end of the U-phase of the machine

\( u \)  voltage, instantaneous value \( u(t) \), [V], number of coil sides in a layer, per unit value of voltage, [pu]

\( u_{\text{b1}} \)  blocking voltage of the oxide layer, [V]

\( u_{\text{c}} \)  commutation voltage, [V]

\( u_m \)  mean fluid velocity in tube, [m/s]

\( V \)  volume, [m³], electric potential

\( V \)  depiction of a phase

\( V_{\text{m}} \)  scalar magnetic potential, [A]

VPI  Vacuum Pressure Impregnation

\( V_1 \)  terminal of the head of the V-phase of the machine

\( V_2 \)  terminal of the end of the V-phase of the machine

\( v \)  speed, velocity, [m/s]

\( v \)  vector

\( W \)  energy, [J], coil span (width), average coil span [m]

\( W \)  depiction of a phase

\( W_{\text{fc}} \)  energy stored in the magnetic field in SR machines

\( W_d \)  energy returned through the diode to the voltage source in SR drives

\( W_{\text{mt}} \)  energy converted into mechanical work when the transistor is conducting in SR drives

\( W_{\text{md}} \)  energy converted to mechanical work while de-energizing the phase in SR drives

\( W_R \)  energy returning to the voltage source in SR drives

\( W' \)  co-energy, [J]

\( W_1 \)  terminal of the head of the W-phase of the machine

\( W_2 \)  terminal of the end of the W-phase of the machine

\( W_\Phi \)  magnetic energy, [J]

\( w \)  length, [m], energy per volume unit

\( w_{\text{PM}} \)  permanent magnet width, [m]

X  reactance, [Ω]

\( x \)  coordinate, length, ordinal number, coil span decrease [m]
Abbreviations and Symbols

\nx_m\n\nrelative value of reactance

Y
\nadmittance, [S]

Y
\ntemperature class 90°C

y
\ncoordinate, length, step of winding

y_m
\nwinding step in an AC commutator winding

y_n
\ncoil span in slot pitches

y_Q
\ncoil span of full-pitch winding in slot pitches, pole pitch expressed in number
\nof slots per pole

y_v
\ncoil span decrease in slot pitches

y_1
\nstep of span in slot pitches, back end connector pitch

y_2
\nstep of connection in slot pitches, front end connector pitch

y_C
\ncommutator pitch in number of commutator segments

Z
\nimpedance, [Ω], number of bars, number of positive and negative phasors of
\nthe phase

Z_M
\ncharacteristic impedance of the motor, [Ω]

Z_s
\nsurface impedance, [Ω]

Z_0
\ncharacteristic impedance, [Ω]

z
\ncoordinate, length, integer, total number of conductors in the armature
\nwinding

z_a
\nnumber of adjacent conductors

z_b
\nnumber of brushes

z_c
\nnumber of coils

z_cs
\nnumber of conductors in half slot

z_p
\nnumber of parallel-connected conductors

z_Q
\nnumber of conductors in a slot

z_t
\nnumber of conductors on top each other

α
\angle, [rad], [°], coefficient, temperature coefficient, relative pole width
\nof the pole shoe, convection heat transfer coefficient, [W/K], skew angle,
\n[rad], [°]

\n1/α
\ndepth of penetration

α_{DC}\n\nrelative pole width coefficient for DC machines

α_i
\ratio of the arithmetical average of the flux density to its peak value

α_m\n\nmass transfer coefficient, [(mol/sm²)/(mol/m³)] = m/s

α_{PM}\n\nrelative permanent magnet width

α_{SM}\n\nrelative pole width coefficient for synchronous machines

α_r\n\nheat transfer coefficient of radiation

α_{str}\n\angle between the phase winding

α_{th}\n\nheat transfer coefficient [W/(m²K)]

α_{ph}\n\angle between the phase winding

α_u\n\slot angle, [rad], [°]

α_z\n\phasor angle, zone angle, [rad], [°]

α_ρ\n\angle of single phasor, [rad], [°]

β\n\angle [rad], [°]

β\n\nabsorptivity

Γ\n\energy ratio, integration route

Γ_c\n\interface between iron and air
Abbreviations and Symbols

γ  angle, [rad], [°], coefficient

γ′c  commutation angle, [rad], [°]

γD  switch conducting angle, [rad], [°]

δ  air gap (length), penetration depth [m], dielectric loss angle, [rad], [°], dissipation angle, [rad], [°], load angle, [rad], [°]

δc  the thickness of concentration boundary layer, [m]

δg(x)  air gap profile function in d- axis, [m]

δq(x)  air gap profile function in q- axis, [m]

δef  effective air gap (influence of iron and slotting taken into account)

δPM  depth of penetration in PM, [m]

δv  velocity boundary layer, [m]

δT  temperature (thermal) boundary layer, [m]

δ′  load angle, [rad], [°], corrected air gap, [m]

δ0  minimum air gap, [m]

δ0e  air gap in the middle of the pole corrected with Carter factor, [m]

δde  equivalent d- axis air gap, [m]

δqe  equivalent q- axis air gap, [m]

Δ2  damping factor

ε  permittivity [F/m], position angle of the brushes, [rad], [°], stroke angle, [rad], [°], amount of short pitching

εsp  amount of short pitching expressed as slot pitches

εth  emissitivity

εthr  relative emissitivity

ε0  permittivity of vacuum 8.854·10^{-12} [F/m]

ζ  phase angle, [rad], [°], harmonic factor, saliency ratio, phase angle of the rotor impedance

ζd  harmonic factor in d-axis

ζq  harmonic factor in q-axis

η  efficiency, empirical constant, experimental pre-exponential constant

ηr  reflectivity, thermal conductivity

Θ  current linkage, [A], temperature rise (difference) [K]

Θk  compensating current linkage, [A]

ΘΣ  total current linkage [A]

θ  angle, position, [rad], [°]

θ′  angle, [rad], [°]

κ  angle, [rad], [°], factor for reduction of slot opening

κr  transmissivity

Λ  permeance, [Vs/A], [H]

Λ′  specific permeance, [Vs/A/m^2]

Λ0′  average of specific permeance, [Vs/A/m^2]

λ  thermal conductivity [W/m-K], permeance factor, proportionality factor, inductance factor, inductance ratio

μ  permeability [Vs/Am, H/m], number of pole pairs operating simultaneously per phase, friction coefficient

μr  relative permeability
Abbreviations and Symbols

\( \mu \)  
\( \mu_0 \)  
\( \nu \)  
\( \nu \)  
\( \xi \)  
\( \rho \)  
\( \rho_A \)  
\( \rho_E \)  
\( \rho_\nu \)  
\( \sigma \)  
\( \sigma_\delta \)  
\( \sigma_F \)  
\( \sigma_{F_n} \)  
\( \sigma_{F_{tan}} \)  
\( \sigma_{mec} \)  
\( \sigma_{SB} \)  
\( \tau \)  
\( \tau_p \)  
\( \tau_{q2} \)  
\( \tau_r \)  
\( \tau_s \)  
\( \tau_u \)  
\( \tau_v \)  
\( \tau_d' \)  
\( \tau_d'' \)  
\( \tau_d'\theta \)  
\( \tau_d''\theta \)  
\( \tau_q' \)  
\( \tau_q'' \)  
\( \tau_q'\theta \)  
\( \tau_q''\theta \)  
\( \upsilon \)  
\( \Phi \)  
\( \Phi_{th} \)  
\( \Phi_\delta \)  
\( \phi \)  
\( \varphi \)  
\( \varphi' \)  
\( \psi \)  
\( \psi_\theta \)  
\( \psi_{m} \)  
\( \psi_{mp} \)  

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>dynamic viscosity, ([\text{Pa} \cdot \text{s}, \text{kg/(s} \cdot \text{m)}])</td>
</tr>
<tr>
<td>( \mu_0 )</td>
<td>permeability of vacuum, (4 \pi \cdot 10^{-7} \ [\text{Vs/Am, H/m}])</td>
</tr>
<tr>
<td>( \nu )</td>
<td>ordinal of harmonic, Poisson's ratio, reluctivity, ([\text{Am/Vs, m/H}]), pole pair</td>
</tr>
<tr>
<td>( \nu )</td>
<td>number of harmonics, kinematic viscosity of the coolant</td>
</tr>
<tr>
<td>( \xi )</td>
<td>pulse velocity</td>
</tr>
<tr>
<td>( \rho )</td>
<td>reduced conductor height</td>
</tr>
<tr>
<td>( \rho )</td>
<td>resistivity, ([\Omega \cdot \text{m}]), electric charge density, ([\text{C}/\text{m}^2]), density, ([\text{kg}/\text{m}^3]), reflection factor, ordinal number of a single phasor</td>
</tr>
<tr>
<td>( \rho_A )</td>
<td>absolute overlap ratio</td>
</tr>
<tr>
<td>( \rho_E )</td>
<td>effective overlap ratio</td>
</tr>
<tr>
<td>( \rho_\nu )</td>
<td>transformation ratio for IM impedance, resistance, inductance</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>specific conductivity, electric conductivity ([\text{S}/\text{m}]), leakage factor, ratio of the leakage flux to the main flux</td>
</tr>
<tr>
<td>( \sigma_\delta )</td>
<td>air gap harmonic leakage factor</td>
</tr>
<tr>
<td>( \sigma_F )</td>
<td>tension, ([\text{Pa}])</td>
</tr>
<tr>
<td>( \sigma_{F_n} )</td>
<td>normal tension, ([\text{Pa}])</td>
</tr>
<tr>
<td>( \sigma_{F_{tan}} )</td>
<td>tangential tension, ([\text{Pa}])</td>
</tr>
<tr>
<td>( \sigma_{mec} )</td>
<td>mechanical stress, ([\text{Pa}])</td>
</tr>
<tr>
<td>( \sigma_{SB} )</td>
<td>Stefan-Boltzmann constant, (5.670400 \times 10^{-8} \ \text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-4})</td>
</tr>
<tr>
<td>( \tau )</td>
<td>relative time, span of the lamination thickness on one pole pitch</td>
</tr>
<tr>
<td>( \tau_p )</td>
<td>pole pitch, ([\text{m}])</td>
</tr>
<tr>
<td>( \tau_{q2} )</td>
<td>pole pitch on the pole surface, ([\text{m}])</td>
</tr>
<tr>
<td>( \tau_r )</td>
<td>rotor slot pitch, ([\text{m}])</td>
</tr>
<tr>
<td>( \tau_s )</td>
<td>stator slot pitch, ([\text{m}])</td>
</tr>
<tr>
<td>( \tau_u )</td>
<td>slot pitch, ([\text{m}])</td>
</tr>
<tr>
<td>( \tau_v )</td>
<td>zone distribution</td>
</tr>
<tr>
<td>( \tau_d' )</td>
<td>direct transient short-circuit time constant, ([\text{s}])</td>
</tr>
<tr>
<td>( \tau_d'' )</td>
<td>direct transient open-circuit time constant, ([\text{s}])</td>
</tr>
<tr>
<td>( \tau_d'\theta )</td>
<td>direct subtransient short-circuit time constant, ([\text{s}])</td>
</tr>
<tr>
<td>( \tau_d''\theta )</td>
<td>direct subtransient open-circuit time constant, ([\text{s}])</td>
</tr>
<tr>
<td>( \tau_q' )</td>
<td>quadrature subtransient short-circuit time constant, ([\text{s}])</td>
</tr>
<tr>
<td>( \tau_q'' )</td>
<td>quadrature subtransient open-circuit time constant, ([\text{s}])</td>
</tr>
<tr>
<td>( \upsilon )</td>
<td>factor, kinematic viscosity, (\mu/\rho), ([\text{Pa} \cdot \text{s}/(\text{kg}/\text{m}^3)])</td>
</tr>
<tr>
<td>( \Phi )</td>
<td>magnetic flux, ([\text{Vs}], [\text{Wb}])</td>
</tr>
<tr>
<td>( \Phi_{th} )</td>
<td>thermal power flow, heat flow rate ([\text{W}])</td>
</tr>
<tr>
<td>( \Phi_\delta )</td>
<td>air gap flux, ([\text{Vs}], [\text{Wb}])</td>
</tr>
<tr>
<td>( \phi )</td>
<td>magnetic flux, instantaneous value (\phi(t)), ([\text{Vs}], [\text{Wb}]), electric potential ([\text{V}])</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>phase shift angle, ([\text{rad}], [\text{°}])</td>
</tr>
<tr>
<td>( \varphi' )</td>
<td>function for skin effect calculation</td>
</tr>
<tr>
<td>( \psi )</td>
<td>magnetic flux linkage, ([\text{Vs}])</td>
</tr>
<tr>
<td>( \psi_\theta )</td>
<td>electric flux, ([\text{C}])</td>
</tr>
<tr>
<td>( \psi_{m} )</td>
<td>air gap flux linkage ([\text{Vs}])</td>
</tr>
<tr>
<td>( \psi_{mp} )</td>
<td>magnetic flux linkage of phase winding ([\text{Vs}])</td>
</tr>
<tr>
<td>( \psi )</td>
<td>function for skin effect calculation</td>
</tr>
</tbody>
</table>
Abbreviations and Symbols

χ     length/diameter ratio, shift of a single pole pair
Ω     mechanical angular speed [rad/s]
ω     electric angular velocity [rad/s], angular frequency [rad/s]
Δ     difference, drop
ΔT    temperature rise (difference) [K], [°C]
∇T    temperature gradient [K/m], [°C/m]
Δp    pressure drop [Pa]

Subscripts

0     section
1     primary, fundamental component, beginning of a phase, locked rotor torque
2     secondary, end of a phase
Al    aluminum
a     armature, shaft
ad    additional (loss)
av    average
B     brush
b     base value, peak value of torque, blocking, damper bar
bar   bar
bearing bearing (losses)
C     capacitor
Cu    copper
Cuw   End winding conductor
conv  convection
c     conductor, commutation
cf    centrifugal
cp    commutating poles
contact brush contact
cr, crit critical
DC    direct current
D     direct, damper
d     tooth, direct, tooth tip leakage flux
diff  difference
E     emf
e     equivalent
ef    effective
electric
em    electromagnetic
ew    end winding
ext   external
F     force
Fe    iron
f     field
Ft    eddy current
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>Hy</td>
<td>hysteresis</td>
</tr>
<tr>
<td>i</td>
<td>internal, insulation, ordinal</td>
</tr>
<tr>
<td>in</td>
<td>input</td>
</tr>
<tr>
<td>k</td>
<td>compensating, short circuit, ordinal</td>
</tr>
<tr>
<td>lam</td>
<td>laminations</td>
</tr>
<tr>
<td>LL</td>
<td>additional load losses</td>
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<tr>
<td>M</td>
<td>motor</td>
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<tr>
<td>max</td>
<td>maximum</td>
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<tr>
<td>m</td>
<td>mutual, main, magnetizing</td>
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<tr>
<td>mag</td>
<td>magnetizing, magnetic</td>
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<td>mec</td>
<td>mechanical</td>
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<td>min</td>
<td>minimum</td>
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<td>mut</td>
<td>mutual</td>
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<td>mp</td>
<td>single-phase magnetizing</td>
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<td>N</td>
<td>rated</td>
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<td>n</td>
<td>nominal, normal</td>
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<tr>
<td>ns</td>
<td>negative-sequence component</td>
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<tr>
<td>o</td>
<td>starting, upper, over</td>
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<tr>
<td>opt</td>
<td>optimal</td>
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<td>out</td>
<td>output</td>
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<tr>
<td>PM</td>
<td>permanent magnet</td>
</tr>
<tr>
<td>p</td>
<td>pole, primary, subconductor, pole leakage flux, operational harmonic</td>
</tr>
<tr>
<td>p1</td>
<td>pole shoe</td>
</tr>
<tr>
<td>p2</td>
<td>pole body</td>
</tr>
<tr>
<td>ph</td>
<td>phasor, phase</td>
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<tr>
<td>ps</td>
<td>positive-sequence component</td>
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<tr>
<td>pu</td>
<td>per unit</td>
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<tr>
<td>q</td>
<td>quadrature, zone</td>
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<tr>
<td>r</td>
<td>rotor, remanence, relative, damper ring short circuit</td>
</tr>
<tr>
<td>res</td>
<td>resultant</td>
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<tr>
<td>S</td>
<td>surface</td>
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<td>s</td>
<td>stator</td>
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<td>sj</td>
<td>impulse wave</td>
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<td>saturation</td>
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<td>phase section</td>
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<td>th</td>
<td>thermal</td>
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<tr>
<td>tot</td>
<td>total</td>
</tr>
<tr>
<td>u</td>
<td>slot, lower, under, bottom, slot leakage flux, pull-up torque</td>
</tr>
<tr>
<td>v</td>
<td>zone, coil side shift in a slot, coil</td>
</tr>
<tr>
<td>x</td>
<td>x-direction</td>
</tr>
<tr>
<td>y</td>
<td>y-direction, yoke</td>
</tr>
<tr>
<td>ya</td>
<td>armature yoke</td>
</tr>
</tbody>
</table>
Abbreviations and Symbols

yr  rotor yoke
ys  stator yoke
w   end winding
z   z-direction, phasor of voltage phasor graph
ρ   ordinal number of single phasor
ρ   friction loss
ρw  windage (loss)
δ   air gap
Φ   flux
ν   harmonic
σ   flux leakage
γ   ordinal of a subconductor
μ   harmonic ordinal

Superscripts

^  peak/maximum value, amplitude
'  imaginary, apparent, reduced, virtual, referred to the stator
*  base winding, complex conjugate

Boldface symbols are used for vectors with components parallel to the unit vectors $i, j,$ and $k$.

$A$  vector potential, $A = iA_x + jA_k + kA_z$
$B$  flux density, $B = iB_x + jB_k + kB_z$
$I$  complex phasor of the current
$\overline{I}$  bar above the symbol denotes average value