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Hugo E. Hernández-Figueroa
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Preface

Diffraction and dispersion effects are well known from centuries and are recognized to be limiting factors for many industrial and technological applications based, for example, on electromagnetic (EM) beams and pulses.

Diffraction is always present, affecting any waves that propagate in two-dimensional or three-dimensional (2D or 3D) media. Pulses and beams are constituted by waves traveling along different directions, which produces a gradual spatial broadening. This effect is a limiting factor whenever a pulse is needed, which maintains its transverse localization, for example, in free space communications, image forming, optical lithography, and EM tweezers.

Dispersion acts on pulses propagating in material media, causing mainly a temporal broadening; an effect due to the variation of the refraction index with the frequency, so that each spectral component of the pulse possesses a different phase-velocity. This entails a gradual temporal widening, which constitutes a limiting factor when a pulse is requested to maintain its time width, for example, in communication systems.

Consequently, the development of techniques capable of alleviating the signal degradation effects caused by these two effects is of crucial importance. Non-diffracting waves (NDWs), known also as localized waves, are indeed able to resist diffraction for a long distance. Today, NDWs are well established both theoretically and experimentally, and are having innovative applications not only in vacuum but in material (linear or nonlinear) media too, showing to be able to resist also dispersion. Moreover, the NDWs allow compensating even for effects such as attenuation. Indeed, in dispersing homogeneous media, it is possible for instance to construct pulses that simultaneously resist the effects of diffraction and dispersion; and, in absorbing homogeneous media, it is possible to construct beams that resist the simultaneous effects of diffraction and attenuation.

As expounded in this book, their potential applications are being intensively explored, always with surprising results, in fields such as microwaves, optics, acoustics, and are promising also in mechanics, geophysics, elementary particle physics, and gravitational waves. To confine ourselves to electromagnetism, let us mention the work on EM tweezers, optical (or acoustic) scalpsels, optical guiding of atoms or (charged or neutral) corpuscles, optical lithography, optical images, communications in free space, remote optical alignment, optical acceleration of
charged corpuscles, and so on. The NDWs are suitable superpositions of Bessel beams. (Let us recall in particular that a peculiar superposition of Bessel beams can be used to obtain “static” NDW fields, with high transverse localization, and whose longitudinal intensity pattern can assume any shape within a chosen interval of the propagation axis. Such beams, called “frozen waves” (FWs), have been experimentally produced in recent times in the case of optics as reported also in this book; they too promise to have very important applications, even in the field of medicine, for example, for tumor curing.)

Considering the significant amount of exciting and impressive results published in the recent 5 years or so, we decided to edit this book on this topic, after the first one appeared in 2008, published by John Wiley & Sons, Inc. These books are the first ones of their kind in the literature. The present volume is composed of 22 chapters authored by many of the worldwide most productive researchers in the field, with a balanced presentation between theory and experiments.

Chapter 1, by Recami et al., presents a thorough review of NDWs, emphasizing their theoretical foundations (in terms of exact solutions to the wave equations) along with historical aspects and the interconnections of this subject with other technological and scientific areas. It shows in particular how to eliminate any backward-traveling components (also known as noncausal components) in the case of both ideal and realistic finite-energy NDW pulses; and a method is then presented for an analytical description of truncated beams. The interesting case of the subluminal NDWs, or bullets, is moreover investigated, which leads to a new analytic description of FWs (endowed with a static envelope) in terms of continuous Bessel beam superpositions. The production of FWs is studied for absorbing media too. The role of special relativity and of Lorentz transformations, for the physical comprehension of the whole issue of NDWs, is stressed. Further topics are the use of higher order Bessel beams; an application to biomedical optics (by recourse to the generalized Lorenz–Mie theory); and, last but not least, the important fact that “soliton-like” solutions can be found also in the rather different case of the ordinary, linear Schrödinger equation within standard quantum mechanics.

Chapter 2 is authored by Ziolkowski, who coined the term “localized waves” and was involved with the topic at its inception. He reviews the initial years of focus wave modes, EM bullets, EM missiles, acoustic directed energy pulse trains (ADEPTs), electromagnetic directed energy pulse trains (EDEPTs), Bessel beams, complex beams, etc., until around the mid-1990s.

Chapter 3, by Mazilu and Dholakia, reviews theory, generation, properties, and applications of various nondiffracting beams, particularly the Bessel beam and Airy beams, and describes some emergent applications including imaging, micromanipulation, and cell transfection.

Chapter 4, by Saari, faces the circumstance that NDWs naturally became attractive to representatives of various fields, so that some misunderstandings showed up among the newcomers, for example, about nature, propagation velocity, and other properties of the NDWs. The first part of this chapter attempts to clarify issues such as the superluminal group velocity of X-type waves as opposed to their energy
transport and signal velocity. It introduces the concept of superluminal accelerating and decelerating quasi-Bessel-X pulses, which are locally propagation-invariant. The second part of this chapter overviews experimental studies, where such a concept has been applied in time-domain treatment of diffraction of ultrashort light pulses on various apertures and optical elements.

Chapter 5, by J.-Y. Lu, deals with applications of limited-diffraction beams, such as X-waves, in high frame rate medical imaging. Various techniques related to such imaging method are introduced, including improvements of image quality and development of techniques for commercial realization of the method.

Chapter 6 is by Besieris and Shaarawi, who carefully discuss all the salient properties of spatiotemporally localized null EM waves (including their vortex structure, the Bateman constraint satisfied by them, and total energy and total angular momentum they carry). They show the Whittaker–Bateman potential theory to be a unifying approach for constructing wide classes of novel spatiotemporally localized luminal, superluminal, and hybrid null EM waves.

Chapter 7 is authored by Besieris, Shaarawi, and Ziolkowski. It aims at finding out a large class of nonsingular, localized, traveling wave solutions to the linear 3D Schrödinger equation, based on two interesting ansatzs. The second part of this chapter provides an account of a broad class of finite-energy accelerating localized wave solutions to the 3D Schrödinger equation, based on generalization of previous work on one-dimensional (1D) infinite-energy nonspreading wavepackets by Berry and Balazs.

In Chapter 8, Dubietis, Faccio, and Valiulis deal with the spontaneous formation of nonlinear X-waves, which is a known feature of intense ultrashort pulse propagation in transparent dielectrics (closely related to white-light continuum generation and femtosecond filamentation phenomena), and study the statistical aspects of the nonlinear X wave formation in presence of intensity, energy, and phase noise; meeting signatures of extreme events, that is, heavy-tailed statistical distributions. Such X-waves are interpreted as spatiotemporal optical rogue waves.

In Chapter 9, Conti shows how X-waves can be a basis for the second quantization of the optical field and how this approach enables to investigate nonlinear optical processes when employing highly nonmonochromatic beams. Implications on quantum entanglement and quantum information are discussed.

In Chapter 10, Hillion starts by recalling that the Helmholtz equation satisfied by the transverse electric (TE) and transverse magnetic (TM) fields (which is not the scalar Helmholtz equation) has elementary solutions in terms of Bessel and Hankel functions, and proves the existence of two different classes of solutions. He looks for solutions of the Helmholtz equation in the absence, or presence, of nonlinearities, showing, for example, that they are not of the Bessel type.

Chapter 11, by Bock and Grunwald, regards reflective axicons as modified, rotationally symmetric versions of the double slit setup, generating Bessel-like localized waves. Their experimental method works nondiffractively even for ultrashort pulses with large spectral bandwidths. They show how to reconstruct not only the spatial but also the temporal pulse information from quantum interference patterns formed by single photons; and introduce a spatiotemporal characterization
of pulsed nondiffracting beams even at the quantum level ("quantum nondiffracting pulses"). Their results allow to exclude, incidentally, the interpretations of the double-slit experiment in which an interaction is assumed between separated slits (e.g., via surface plasmons). They finally propose a new method for a complete pulse reconstruction based on the nonlinear conversion of single photons from localized wavepackets.

Chapter 12 is authored by Bock et al. It shows that the arrays of free-space localized wavepackets allow an improved diagnostics of ultrashort pulses. In particular, the "needle pulses" can be used for analyzing the temporal properties of wavepackets with spatial resolution. The concept of needle beams is then extended toward more complex nondiffracting patterns. With reflective liquid-crystal-on-silicon-type spatial light modulators (SLMs), shaping and characterization of wavepackets at pulse durations down to 6 fs are demonstrated.

In Chapter 13, Utkin observes that the well-known frequency domain methods, for describing wave generation and propagation, disseminated the belief that a wave is something having a phase and an amplitude, rather than a solution of the wave equations. Although being complex and less universal, the space–time domain methods may be more adequate for solving wave propagation problems; for example, when the source term has complicated spatiotemporal structure. This chapter introduces a new space–time domain ansatz, calling into the play an inhomogeneous partial differential equation (PDE) of the hyperbolic type. In many practically important cases, the canonical PDE has a known Riemann function, which makes possible to construct the unique solution to the above initial value problem harnessing the Riemann–Volterra formula. The applicability of the method for causal description of launching localized waves by physically admissible sources is demonstrated for two practically important cases: generation of a finite-support focus wave mode by a luminal-speed pulse with the Gaussian transverse variation, and launching of a droplet-shaped wave by a line source traveling with a superluminal speed.

Chapter 14, by Saastamoinen, Friberg, and Turunen, overviews a wide class of optical fields, which possess the same spectral density distribution across every plane perpendicular to the nominal propagation direction. Examples are given for both stationary and nonstationary fields with different spatial and temporal coherence properties. The simplest special cases include sharply peaked Bessel fields, Bessel-correlated fields, and localized wave packets, such as X-waves and focus wave modes.

Chapter 15, by Mechler and Kukhlevsky, shows—using the scalar diffraction theory and the image method—how any arbitrary scalar field, confined by a 2D or 3D optical waveguide, can be generated in free space by the appropriate light source. The correspondence between the guided and free-space waves is illustrated using several particular fields, such as the diffraction-free, self-imaging, ultra-short, soliton-like, partially coherent waves, and laser fractals.

Chapter 16, by Ramos, Castellanos, and Callás, starts referring to the experimental production of X-shaped acoustic waves by Lu et al. in 1992, which was based on the sequential excitations of annular array rings and subsequent synthesis by
software composition. Its extension for a strict real-time regime would need an expensive, fast instrumentation with parallel electronic channels; however, fast beam synthesis is actually required in ultrasonic applications for medical diagnosis by imaging or noninvasive inspection in quality control. In this chapter, principles and implementation details are described for achieving real-time radiation of localized ultrasonic beams in pulsed regime, in such a way to achieve efficient implementations of fast X-beam collimations in the megahertz range at a low cost, for the multichannel electronics involved. A real-time ultrasonic emitting and beam-forming experiment in laboratory is described in detail. Optimization results for the annuli emissions and acoustic beam-forming patterns, from a specially designed piezoelectric annular array, are also compared with those obtained using the ideal zero-order X-wave solutions proposed in the classical approach by Lu et al.; and a very acceptable approximation level is obtained.

Chapter 17 is authored by Sheppard. It starts from pulsed beams generated by coherent superposition of Bessel beams. They can be compared among themselves using 3D (generalized) pupils. Applications in microscopy and tomography are discussed.

In Chapter 18, Porras reviews the properties of lossy light bullets; an alternative form of light wave localization in nonlinear media with dissipation (which is between a soliton and a conical wave). Unlike well-known dissipative light bullets, lossy light bullets do not require a continuous gain to sustain stationary propagation. Lossy light bullets are self-healing and are stabilized by losses, which make them attract the self-focusing dynamics in dissipative media.

Chapter 19, by Ranfagni and Mugnai, recalls how in 1952 Toraldo di Francia proposed an intriguing method to increase the optical resolving power. His theory might seem to be in contradiction with Heisenberg’s uncertainty correlations. In this chapter, the authors report the results of microwave investigations, which demonstrate the correctness of that theoretical prediction and its interpretation in relation to the uncertainty principle. Experimental measurements, employing special composed pupils, have been made in order to verify the possibility of obtaining a considerable reduction in the beam width and a field concentration along the axial axis, as compared to a simple pupil. Further experiments have been devoted to evaluate the pulse delay in the propagation, in the presence of this kind of pupils.

Chapter 20, by Vieira, Gesualdi, and Zamboni-Rached, presents, for the first time, the experimental generation of FWs in optics, obtained using a setup for the optical reconstruction of computer-generated holograms (CGHs), based on a 4-F Fourier filtering system and a nematic liquid crystal spatial light modulator (LC-SLM). The CGHs have been implemented computationally and, subsequently, electronically in the LC-SLM for optical reconstruction. The results agree with the corresponding theoretical (analytic) solutions and bear excellent perspectives for scientific and technological applications.

Chapter 21 is authored by Nóbrega, Dartora, and Zamboni-Rached. It presents an analytic method for the description of Airy-type beams when truncated by finite apertures: a method based on suitable superposition of exponentially decaying
Airy beams. The results can be quickly evaluated via the simple analytic solution proposed in this chapter, and agree with those obtained in the literature through numerical methods. Three different truncated beams are analyzed: ideal Airy, Airy-Gauss, and Airy-exponential beams.

Chapter 22, by Kutz, deals with sources of ultrashort light pulses, which enable direct observation of some of the fastest processes in nature, along with studies of matter under extreme conditions (leading to the first studies of the hitherto unexplored field of attosecond physics). It envisions that even single-electron transition events can now be captured. The theoretical models, however, have lagged behind because of their adherence to standard center-frequency expansion techniques for modeling the electric field envelope in Maxwell’s equations; whereas below a few femtosecond regimes such theories begin to breakdown and new approaches must be developed. In this chapter, a mode-locking theory is developed, which is valid in the ultrashort pulse regime, the starting point being Maxwell’s equations. When pushed to the extreme of a few femtoseconds or attosecond pulses, even the so-called nonlinear Schrödinger equation (NLS) description becomes suspect. Thus, a simplified approach is taken in this chapter, where the description of the pulse is derived directly from Maxwell’s equations. Specifically, a mode-locking in a laser cavity is considered, taking advantage of the robust and stable mode-locking that results in the short-pulse limit.

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1

Non-Diffracting Waves: An Introduction

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1.1 A General Introduction

1.1.1 A Prologue

In this chapter, which essentially deals with exact solutions to the wave equations, we begin by introducing the topic of non-diffracting waves (NDW), including some brief historical remarks, and by a simple definition of NDWs; afterward we present some recollections – besides of ordinary waves (Gaussian beams, Gaussian pulses) – of the simplest NDWs (Bessel beams, X-shaped pulses, etc.). More details can be found in the first two (introductory) chapters of the volume on Localized Waves published [1] in 2008. In section 1.2 we go on to show how to eliminate any backward-traveling components (also known as non-causal components), first in the case of ideal NDW pulses, and then, in section 1.3, for realistic, finite-energy NDW pulses. In particular, in section 1.3.1 we forward a general functional expression for any totally-forward non-diffracting pulses. Then, in section 1.4 an efficient method is set forth for the analytic description of truncated beams, a byproduct of its being the elimination of any need of lengthy numerical calculations. In section 1.5 we explore the not-less-interesting question of the subluminal NDWs, or bullets, in terms of two different methods, the second one being introduced as it allows the analytic description of NDWs with $v = 0$ that is of NDWs with a static envelope (“frozen waves” (FW)) in terms of continuous Bessel beam superpositions. The production of such FWs (which, indeed, have been generated experimentally in recent time for optics) is developed theoretically in section 1.6 also for the case of absorbing media. Section 1.7 discusses the role of special relativity and of Lorentz transformations (LTs), which is relevant for the physical comprehension of the whole issue of NDWs. In section 1.8 we present further analytic solutions to the wave equations, with use of higher-order components.

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Bessel beams (namely, non-axially symmetric solutions). Next, section 1.9 deals in detail with an application of NDWs to biomedical optics by having recourse to the generalized Lorenz–Mie theory (GLMT). In section 1.10 we exploit the important fact that “soliton-like” solutions can be found also in the rather different case of the ordinary, linear Schrödinger equation – which is not a properly said wave equation – within standard quantum mechanics; by also constructing, for instance, a general exact non-diffracting solution for such equation. These “localized” solutions to the Schrödinger equation may a priori be of help for a better understanding, say, of de Broglie’s approach and of the particle-wave duality. Some complementary issues are mentioned in the last section.

Let us now start by recalling that diffraction and dispersion are long-known phenomena limiting the applications of beams or pulses.

Diffraction is always present, affecting any waves that propagate in two or three-dimensional (3D) media. Pulses and beams are constituted by waves traveling along different directions, which produces a gradual spatial broadening. This effect is a limiting factor whenever a pulse is needed, which maintains its transverse localization, like, for example, in free space communications, image forming, optical lithography, and electromagnetic tweezers, etc.

Dispersion acts on pulses propagating in material media causing mainly a temporal broadening, an effect due to the variation of the refraction index with the frequency, so that each spectral component of the pulse possesses a different phase velocity. This entails a gradual temporal widening, which constitutes a limiting factor when a pulse is needed that maintains its time width, like, for example, in communication systems.

It has been important, therefore, to develop techniques able to reduce those phenomena. NDW, known also as localized waves, are, indeed, able to resist diffraction for a long distance. Today, NDW are well-established both theoretically and experimentally, and have innovative applications not only in vacuum, but also in material (linear or nonlinear) media, also showing resistance to dispersion. As mentioned, their potential applications are being explored intensively, always with surprising results, in fields like acoustics, microwaves, and optics, and are also promising in mechanics, geophysics [2], and even elementary particle physics [3] and gravitational waves. One interesting acoustic application has been already obtained in high-resolution ultra-sound scanning of moving organs in the human body. We shall see that NDWs are suitable superpositions of Bessel beams. And worth noticing is that peculiar superposition of Bessel beams can be used to obtain “static” NDW fields, with high transverse localization, and whose longitudinal intensity pattern can assume any desired shape within a chosen interval \(0 \leq z \leq L\) of the propagation axis; such waves with a static envelope [1, 4–7], that we called FW, have been produced experimentally in recent times in the case of optics, as reported elsewhere also in this book. These FWs promise to have very important applications (even in the field of medicine and of tumor curing [8]).

To confine ourselves to electromagnetism, let us recall again the present-day studies on electromagnetic tweezers, optical (or acoustic) scalpels, optical