Introduction to

Modern Power Electronics

Andrzej M. Trzynadlowski

THIRD EDITION

with website

WILEY

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For Dorota, Bart, Nicole, Genie, Gary, and Guy

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PREFACE

This text is primarily intended for a one-semester introductory course in power electronics at the undergraduate level. However, containing a comprehensive overview of modern tools and techniques of electric power conditioning, the book can also be used in more advanced classes. Practicing engineers wishing to refresh their knowledge of power electronics, or interested in branching into that area, are also envisioned as potential readers. Students are assumed to have working knowledge of the electric circuit analysis and basic electronics.

During the five years since the second edition of the book was published, power electronics has enjoyed robust progress. Novel converter topologies, applications, and control techniques have been developed. Utilizing advanced semiconductor switches, power converters reach ratings of several kilovolts and kiloamperes. The threat of unchecked global warming, various geopolitical and environmental issues, and the monetary and ecological costs of fossil fuels represent serious energy challenges, which set off intensive interest in sources of clean power. As a result, power electronic systems become increasingly important and ubiquitous. Changes made to this third edition reflect the dominant trends of modern power electronics. They encompass the growing practical significance of PWM rectifiers, the Z-source dc link, matrix converters, and multilevel inverters, and their application in renewable energy systems and powertrains of electric and hybrid vehicles.

In contrast with most books, which begin with a general introduction devoid of detailed information, Chapter 1 constitutes an important part of the teaching process. Employing a hypothetical generic power converter, basic principles and methods of power electronics are explained. Therefore, whatever content sequence an instructor wants to adopt, Chapter 1 should be covered first.

Chapters 2 and 3 provide description of semiconductor power switches and supplementary components and systems of power electronic converters. The reader should be aware of the existence and function of those auxiliary but important parts although the book is mostly focused on power circuits, operating characteristics, control, and applications of the converters.

The four fundamental types of electrical power conversion—ac to dc, ac to ac, dc to dc, and dc to ac—are covered in Chapters 4 through 7, respectively. Chapters 4 and 7, on rectifiers and inverters, are the longest chapters, reflecting the great importance of those converters in modern power electronics. Chapter 8 is devoted to switching dc power supplies, and Chapter 9 covers applications of power electronics in clean energy systems.

Each chapter begins with an abstract and includes a brief summary that follows the main body. Numerical examples, homework problems, and computer assignments complement most chapters. Several relevant and easily available references are provided after each of them. Three appendices conclude the book.

The textbook is accompanied by a series of forty-six PSpice circuit files constituting a virtual power electronics laboratory, and available at http://www.wiley.com/go/modernpowerelectronics3e. The files contain computer models of most power electronic converters covered in the book. The models are a valuable teaching tool, giving the reader an opportunity to tinker with the converters and visualize their operation. Another teaching tool, a PowerPoint presentation, which contains all figures, tables, and most important formulas, is also available, at http://www.wiley.com/go/modernpowerelectronics3e. It will ease the instructor from drawing the often complex circuit diagrams and waveforms on the classroom board.

Against most of the contemporary engineering textbooks, the book is quite concise. Still, covering the whole material in a single-semester course requires from the students a substantial homework effort. The suggested teaching approach would consist in presenting the basic issues in class and letting the students to broaden their knowledge by reading assigned materials, solving problems, and performing PSpice simulations.

I want to express my gratitude to the reviewers of the book proposal, whose valuable comments and suggestions have been greatly appreciated. My students at the University of Nevada, Reno, who used the first and second editions for so many years, provided very constructive critiques as well. Finally, my wife Dorota and children Bart and Nicole receive apologies for my long preoccupation, and many thanks for their unwavering support.

ANDRZEJ M. TRZYNADLOWSKI

ABOUT THE COMPANION WEBSITE

This book is accompanied by a companion website:

www.wiley.com/go/modernpowerelectronics3e

The website includes:

- PSpice circuit files
- Power Point Presentation
- Solutions Manual available for instructors

1 Principles of Electric Power Conversion

In this introductory chapter, fundamentals of power electronics are outlined, including the scope, tools, and applications of this area of electrical engineering. The concept of generic power converter is introduced to illustrate the operating principles of power electronic converters and the types of power conversion performed. Components of voltage and current waveforms, and the related figures of merit, are defined. Two basic methods of magnitude control, that is, phase control and pulse width modulation (PWM), are presented. Calculation of current waveforms is explained. The single-phase diode rectifier is described as the simplest power electronic converter.

1.1 WHAT IS POWER ELECTRONICS?

Modern society with its conveniences strongly relies on the ubiquitous availability of electric energy. The electricity performs most of the physical labor, provides the heating and lighting, activates electrochemical processes, and facilitates information collecting, processing, storage, and exchange.

Power electronics can be defined as a branch of electrical engineering devoted to conversion and control of electric power, using electronic converters based on semiconductor power switches. The power grid delivers an ac voltage of fixed frequency and magnitude. Typically, homes, offices, stores, and other small facilities are supplied from single-phase, low-voltage power lines, while three-phase supply systems with various voltage levels are available in industrial plants and other large commercial enterprises. The 60-Hz (50-Hz in most other parts of the world) fixed-voltage electric power can be thought of as raw power, which for many applications must be conditioned. The power conditioning involves conversion, from ac to dc or vice-versa, and control of the magnitude and/or frequency of voltages and currents. Using the electric lighting as a simple example, an incandescent bulb can directly be supplied with the raw power. However, a fluorescent lamp requires electronic ballast that starts and stabilizes the electric arc. The ballast is thus a power conditioner, necessary for proper operation of the lamp. If used in a movie theater, the incandescent bulb mentioned before is supplied from an ac voltage controller that allows dimming

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of the light just before the movie begins. Again, this controller constitutes an example of power conditioner, or power converter.

Raw dc power is usually supplied from batteries and, increasingly, from photovoltaic sources and fuel cells. Photovoltaic energy systems are usually connected to the grid, and the necessary power conditioning involves dc-to-ac voltage conversion and control of the ac voltage. If a dc source feeds an electric motor, as in a golf cart or an electric wheelchair, a power electronic converter between the battery and the motor performs voltage control and facilitates reverse power flow during braking or downhill ride.

The birth of power electronics can be traced back to the dawn of twentieth century when the first mercury arc rectifiers were invented. However, for conversion and control of electric power, *rotating electro-machine converters* were mostly used in the past. An electro-machine converter was an electric generator driven by an electric motor. If, for instance, adjustable dc voltage was to be obtained from fixed ac voltage, an ac motor operated a dc generator with controlled output voltage. Conversely, if ac voltage was required and the supply energy came from a battery pack, a speed-controlled dc motor and an ac synchronous generator were employed. Clearly, the convenience, efficiency, and reliability of such systems were inferior in comparison with today's *static power electronic converters* performing motionless energy conversion and control.

Today's power electronics has begun with the development of the *silicon controlled rectifier* (*SCR*), also called a *thyristor*, by the General Electric Company in 1958. The SCR is a unidirectional semiconductor power switch that can be turned on ("closed") by a low-power electric pulse applied to its controlling electrode, the gate. The available voltage and current ratings of SCRs are very high, but the SCR is inconvenient for use in dc-input power electronic converters. It is a *semi-controlled* switch, which when conducting current cannot be turned off ("opened") by a gate signal. Within the last few decades, several kinds of *fully controlled* semiconductor power switches that can be turned on and off have been introduced to the market.

Widespread introduction of power electronic converters to most areas of distribution and usage of electric energy is common for all developed countries. The converters condition the electric power for a variety of applications, such as electric motor drives, uninterruptable power supplies, heating and lighting, electrochemical and electro-thermal processes, electric arc welding, high-voltage dc transmission lines, active power filters and reactive power compensators in power systems, and high-quality supply sources for computers and other electronic equipment.

It is estimated that at least half of the electric power generated in the USA flows through power electronic converters, and an increase of this share to close to 100% in the next few decades is expected. In particular, a thorough revamping of the existing US national power grid is envisioned. Introduction of power electronic converters to all stages of the power generation, transmission, and distribution, coupled with extensive information exchange ("smart grid"), allows a dramatic increase of the grid's capabilities without investing in new power plants and transmission lines. The important role of power electronics in renewable energy systems and electric and hybrid vehicles is also worth stressing. It is safe to say that practically every electrical engineer encounters some power electronic converters in his/her professional career.

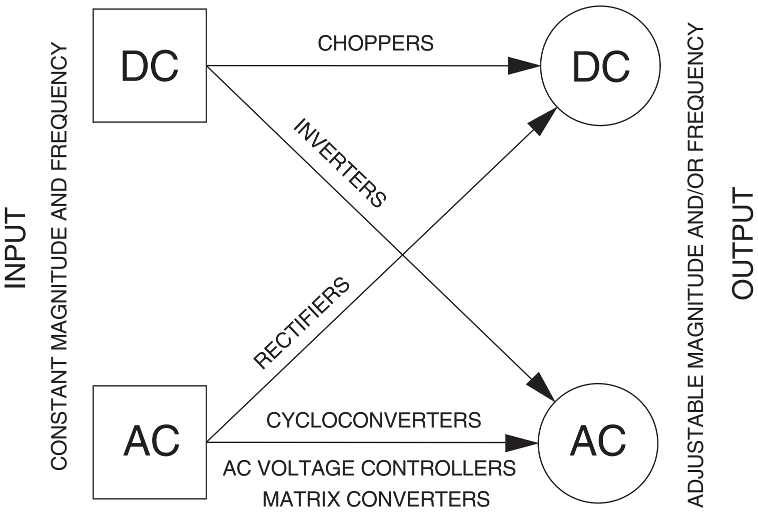

Figure 1.1 Types of electric power conversion and the corresponding power electronic converters.

Types of electric power conversion and the corresponding converters are presented in Figure 1.1. For instance, the ac-to-dc conversion is accomplished using rectifiers, which are supplied from an ac source and whose output voltage contains a fixed or adjustable dc component. Individual kinds of power electronic converters are described and analyzed in Chapters 4 through 8. Basic principles of power conversion and control are explained in the following sections of this chapter.

1.2 GENERIC POWER CONVERTER

Though not a practical apparatus, the hypothetical *generic power converter* shown in Figure 1.2 is a useful tool for illustration of the principles of electric power conversion and control. It is a two-port network of five switches. Switches S1 and

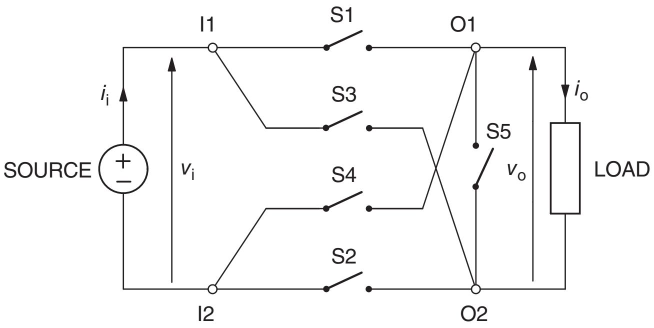

Figure 1.2 Generic power converter.

S2 provide *direct connection* between the input (supply) terminals, I1 and I2, and the output (load) terminals, O1 and O2, respectively, while switches S3 and S4 allow *cross connection* between these pairs of terminals. A voltage source, either dc or ac, supplies the electric power to a load through the converter. Practical loads often contain a significant inductive component, so a resistive–inductive (RL) load is assumed in the subsequent considerations. To ensure a closed path for the load current under any operating conditions, a fifth switch, S5, is connected between the output terminals of the converter and closed when switches S1 through S4 are open. It is assumed that the switches open or close instantaneously.

The supply source is an ideal voltage source and as such it may not be shorted. Also, the load current may not be interrupted. As the voltage across inductance is proportional to the rate of change of current, a rapid drop of that current would cause a high and potentially damaging overvoltage. Therefore, the generic converter can only assume the following three states:

State 0: Switches S1 through S4 are open and switch S5 is closed, shorting the output terminals and closing a path for the lingering load current, if any. The output voltage is zero. The input terminals are cut off from the output terminals so that the input current is also zero.
State 1: Switches S1 and S2 are closed, and the remaining ones are open. The output voltage equals the input voltage and the output current equals the input current.
State 2: Switches S3 and S4 are closed, and the remaining ones are open. Now, the output voltage and current are reversed with respect to their input counterparts.

Let us assume that the generic converter is to perform the ac-to-dc conversion. The sinusoidal input voltage, v_i, whose waveform is shown in Figure 1.3, is given by

$$v_\mathrm{i} = V_\mathrm{i,p} \sin(\omega t), \tag{1.1}$$

where $V_\mathrm{i,p}$ denotes the peak value of that voltage and ω is the input radian frequency. The output voltage, v_o, of the converter should contain a possibly large dc component.

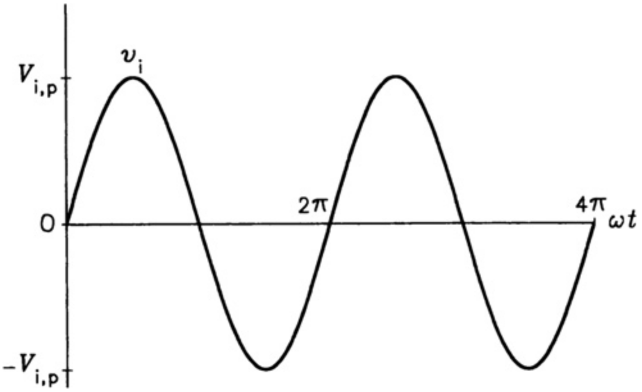

Figure 1.3 Input ac voltage waveform.

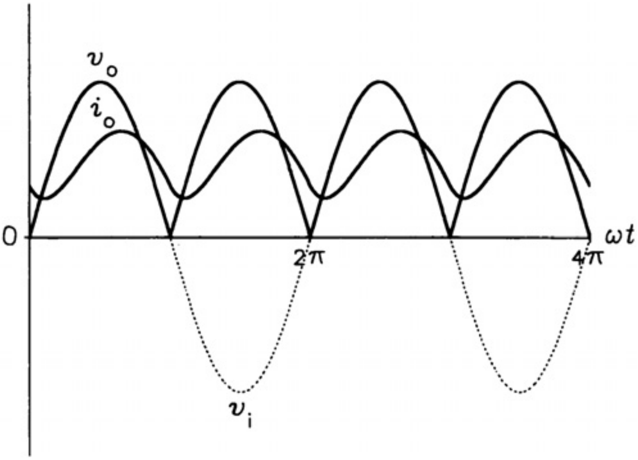

Figure 1.4 Output voltage and current waveforms in the generic rectifier.

Note that the output voltage is not expected to be of ideal dc quality, since such voltage and current waveforms are not feasible in the generic converter, as well as in practical power electronic converters. The same applies to the ideally sinusoidal output voltage and current in ac output converters. If within the first half-cycle of the input voltage, the converter is in state 1, and within the second half-cycle in state 2, the output voltage waveform will be such as depicted in Figure 1.4, that is,

$$v_o = |v_i| = V_{i,p}|\sin(\omega t)|. \tag{1.2}$$

The dc component is the average value of the voltage. Power electronic converters performing the ac-to-dc conversion are called *rectifiers*.

The output current waveform, i_o, can be obtained as a numerical solution of the load equation:

$$L\frac{di_o}{dt} + Ri_o = v_o. \tag{1.3}$$

Techniques for analytical and numerical computation of voltage and current waveforms in power electronic circuits are described at the end of this chapter. Here, only general features of the waveforms are outlined. The output current waveform of the considered generic rectifier is also shown in Figure 1.4. It can be seen that this waveform is closer to an ideal dc waveform than is the output voltage waveform because of the frequency-dependent load impedance. The kth harmonic, $v_{o,k}$, of the output voltage produces the corresponding harmonic, $i_{o,k}$, of the output current such that

$$I_{o,k} = \frac{V_{o,k}}{\sqrt{R^2 + (k\omega_o L)^2}}, \tag{1.4}$$

where $I_{o,k}$ and $V_{o,k}$ denote root mean square (rms) values of the current and voltage harmonics in question, respectively. In the considered rectifier, the fundamental radian frequency, ω_o, of the output voltage is twice as high as the input frequency, ω. The load impedance (represented by the denominator at the right-hand side of Eq. 1.4) for individual current harmonics increases with the harmonic number, k. Clearly, the dc component ($k = 0$) of the output current encounters the lowest impedance, equal to the load resistance only, while the load inductance attenuates only the ac component. In other words, the RL load acts as a low-pass filter.

Interestingly, if an ac output voltage is to be produced and the generic converter is supplied from a dc source, so that the input voltage is $v_i = V_i = \text{const.}$, the switches are operated in the same manner as in the previous case. Specifically, for every half period of the desired output frequency, states 1 and 2 are interchanged. In this way, the input terminals are alternately connected and cross-connected with the output terminals, and the output voltage acquires the ac (although not sinusoidal) waveform shown in Figure 1.5. The output current is composed of growth-function and decay-function segments, typical for transient conditions of an RL circuit subjected to dc excitation. Again, thanks to the attenuating effects of the load inductance, the current waveform is closer to the desired sinusoid than is the voltage waveform. In practice, the dc-to-ac power conversion is performed by power electronic *inverters*. In the case described, the generic inverter is said to operate in the *square-wave mode*.

If the input or output voltage is to be a three-phase ac voltage, the topology of the generic power converter portrayed here would have to be expanded, but it still would be a network of switches. Real power electronic converters are *networks of semiconductor power switches*, too. For various purposes, other elements, such as inductors, capacitors, fuses, and auxiliary circuits, are employed besides the switches in power circuits of practical power electronic converters. Yet, in most of these converters, the fundamental operating principle is the same as in the generic converter, that is, the

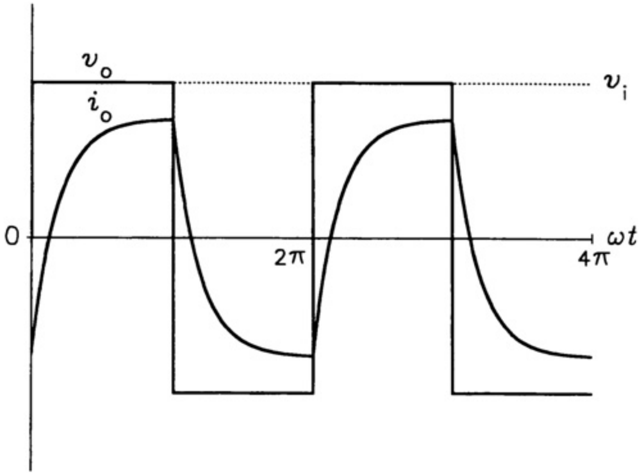

Figure 1.5 Output voltage and current waveforms in the generic inverter.

input and output terminals are being connected, cross-connected, and disconnected in a specific manner and sequence required for the given type of power conversion. Typically, as in the generic rectifier and inverter presented, the load inductance inhibits the switching-related undesirable high-frequency components of the output current.

Although a *voltage source* has been assumed for the generic power converter, some power electronic converters are supplied from *current sources*. In such converters, a large inductor is connected in series with the input terminals to prevent rapid changes of the input current. Analogously, voltage-source converters usually have a large capacitor connected across the input terminals to stabilize the input voltage. Inductors or capacitors are also used at the output of some converters to smooth the output current or voltage, respectively.

According to one of the tenets of circuit theory, two unequal ideal current sources may not be connected in series and two unequal ideal voltage sources may not be connected in parallel. Consequently, the load of a current-source converter may not appear as a current source while that of a voltage-source converter as a voltage source. As illustrated in Figure 1.6, it means that in a current-source power electronic converter a capacitor should be placed in parallel with the load. In addition to smoothing the output voltage, the capacitor prevents the potential hazards of connecting the input inductance conducting certain current with a load inductance conducting different current. In contrast, in voltage-source converters, no capacitor may be connected across the output terminals and it is the load inductance, or an extra inductor between the converter and the load, that is smoothing the output current.

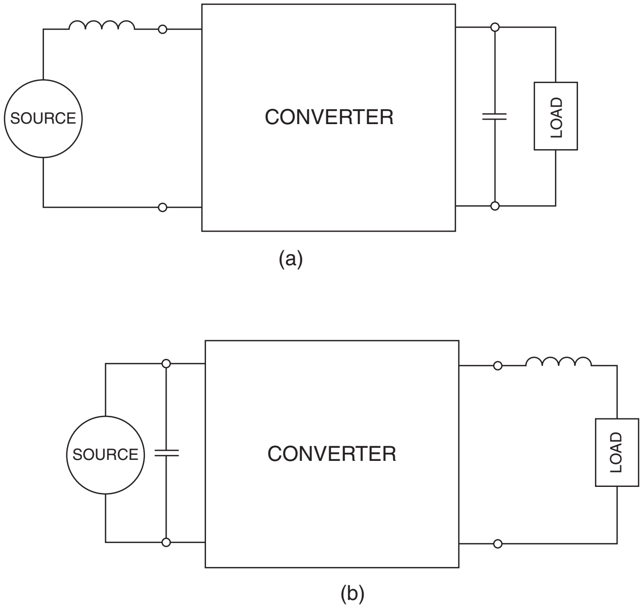

Figure 1.6 Basic configurations of power electronic converters: (a) current source, (b) voltage source.

1.3 WAVEFORM COMPONENTS AND FIGURES OF MERIT

Terms such as the “dc component,” “ac component,” and “harmonics” mentioned in the preceding section deserve closer examination. Knowledge of the basic components of voltage and current waveforms allows evaluation of performance of a converter. Certain relations of these components are commonly used as performance indicators, or *figures of merit*.

A time function, $\psi(t)$, here a waveform of voltage or current, is said to be *periodic* with a period T if

$$\psi(t) = \psi(t + T), \tag{1.5}$$

that is, if the pattern (shape) of the waveform is repeated every T seconds. In the realm of power electronics, it is often convenient to analyze voltages and currents in the *angle domain* instead of the *time domain*. The so-called *fundamental frequency*, f_1, in Hz, is defined as

$$f_1 = \frac{1}{T}, \tag{1.6}$$

and the corresponding *fundamental radian frequency*, ω, in rad/s, as

$$\omega = 2\pi f_1 = \frac{2\pi}{T}. \tag{1.7}$$

Now, a periodic function $\psi(\omega t)$ can be defined as such that

$$\psi(\omega t) = \psi(\omega t + 2\pi). \tag{1.8}$$

The *rms value*, Ψ, of waveform $\psi(\omega t)$ is defined as

$$\Psi \equiv \sqrt{\frac{1}{2\pi} \int_0^{2\pi} \psi^2(\omega t) d\omega t}, \tag{1.9}$$

and the *average value*, or *dc component*, Ψ_{dc}, of the waveform as

$$\Psi_{\text{dc}} \equiv \frac{1}{2\pi} \int_0^{2\pi} \psi(\omega t) d\omega t. \tag{1.10}$$

When the dc component is subtracted from the waveform, the remaining waveform, $\psi_{\text{ac}}(\omega t)$, is called the *ac component*, or *ripple*, that is,

$$\psi_{\text{ac}}(\omega t) = \psi(\omega t) - \Psi_{\text{dc}}. \tag{1.11}$$

The ac component has an average value of zero and the fundamental frequency of f_1.

The rms value, Ψ_{ac}, of $\psi_{ac}(\omega t)$ is defined as

$$\Psi_{ac} \equiv \sqrt{\frac{1}{2\pi} \int_0^{2\pi} \psi_{ac}^2(\omega t) d\omega t}, \tag{1.12}$$

and it can be shown that

$$\Psi^2 = \Psi_{dc}^2 + \Psi_{ac}^2. \tag{1.13}$$

For waveforms of the desirable ideal dc quality, such as the load current of a rectifier, a figure of merit called a *ripple factor*, *RF*, is defined as

$$\mathrm{RF} = \frac{\Psi_{ac}}{\Psi_{dc}}. \tag{1.14}$$

A low value of the ripple factor indicates high quality of a waveform.

Before proceeding to other waveform components and figures of merit, the terms and formulas introduced so far will be illustrated using the waveform of output voltage, v_o, of the generic rectifier, shown in Figure 1.4. The waveform pattern is repeating itself every π radians and, within the 0 to π interval, $v_o = v_i$. Therefore, the average value, $V_{o,dc}$, of the output voltage can most conveniently be determined by calculating the area under the waveform from $\omega t = 0$ to $\omega t = \pi$ and dividing it by the length, π, of the considered interval. Thus,

$$V_{o,dc} = \frac{1}{\pi} \int_0^{\pi} V_{i,p} \sin(\omega t) d\omega t = \frac{2}{\pi} V_{i,p} = 0.64 V_{i,p}. \tag{1.15}$$

Note that the formula above differs from Eq. (1.10). Since $\omega_1 = \omega_o = 2\omega$, the integration is performed in the 0 to π interval of ωt instead of the 0 to 2π interval of $\omega_1 t$.

Similarly, the rms value, V_o, of the output voltage can be calculated as

$$V_o = \sqrt{\frac{1}{\pi} \int_0^{\pi} [V_{i,p} \sin(\omega t)]^2 d\omega t} = \frac{V_{i,p}}{\sqrt{2}} = 0.71 V_{i,p}. \tag{1.16}$$

The result in Eq. (1.16) agrees with the well-known relation for a sine wave as $v_o{}^2 = v_i{}^2$.

Based on Eqs. (1.13) and (1.14), the rms value, $V_{\mathrm{o,ac}}$, of ac component of the voltage in question can be calculated as

$$V_{\mathrm{o,ac}} = \sqrt{V_{\mathrm{o}}^2 - V_{\mathrm{o,dc}}^2} = \sqrt{\left(\frac{V_{\mathrm{i,p}}}{\sqrt{2}}\right)^2 - \left(\frac{2}{\pi}V_{\mathrm{i,p}}\right)^2} = 0.31V_{\mathrm{i,p}}, \tag{1.17}$$

and the ripple factor, $\mathrm{RF_V}$, of the voltage as

$$\mathrm{RF_V} = \frac{V_{\mathrm{o,ac}}}{V_{\mathrm{o,dc}}} = \frac{0.31V_{\mathrm{i,p}}}{0.64V_{\mathrm{i,p}}} = 0.48. \tag{1.18}$$

Decomposition of the analyzed waveform into the dc and ac components is shown in Figure 1.7.

To analytically determine the ripple factor, $\mathrm{RF_I}$, of the output current, the output current waveform, $i_{\mathrm{o}}(\omega t)$, would have to be expressed in a closed form. Instead, numerical computations were performed on the waveform in Figure 1.4, and $\mathrm{RF_I}$ was found to be 0.31. This value is 36% lower than that of the output voltage. This is an example only, but output currents in power electronic converters routinely have higher quality than the output voltages. It is worth mentioning that the obtained value of $\mathrm{RF_I}$ is poor. Practical high-quality dc current waveforms have the ripple factor in the order of few percentage points, and below the 5% level the current is considered as ideal. The current ripple factor depends on the type of converter, and it decreases with an increase in the inductive component of the load. Components of the current waveform evaluated are shown in Figure 1.8.

The ripple factor is of no use for quality evaluation of ac waveforms, such as the output current of an inverter, which ideally should be pure sinusoids. However, as

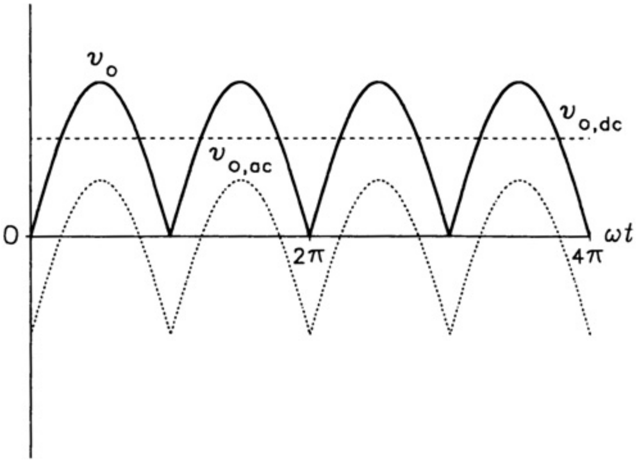

Figure 1.7 Decomposition of the output voltage waveform in the generic rectifier.

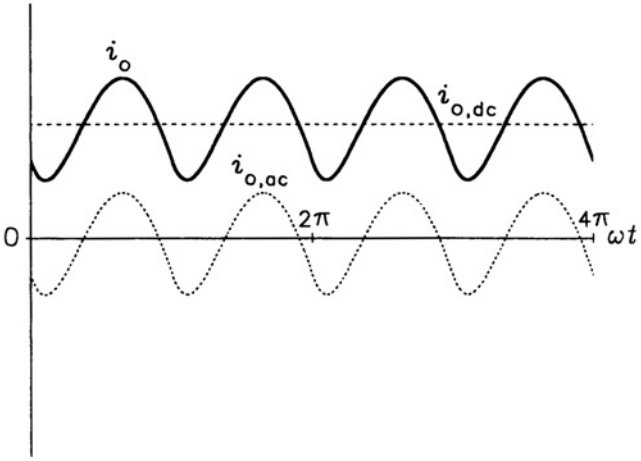

Figure 1.8 Decomposition of the output current waveform in the generic rectifier.

already mentioned and exemplified by the waveforms in Figure 1.5, purely sinusoidal voltages and currents cannot be produced by switching power converters. Therefore, an appropriate figure of merit must be defined as a measure of deviation of a practical ac waveform from its ideal counterpart.

Following the theory of Fourier series (see Appendix B), the ac component, $\psi_{ac}(t)$, of a periodic function, $\psi(t)$, can be expressed as an infinite sum of harmonics, that is, sine waves whose frequencies are multiples of the fundamental frequency, f_1, of $\psi(t)$. In the angle domain,

$$\psi_{ac}(\omega t) = \sum_{k=1}^{\infty} \psi_k(k\omega t) = \sum_{k=1}^{\infty} \Psi_{k,p} \cos(k\omega t + \varphi_k), \tag{1.19}$$

where k is the *harmonic number*, and $\Psi_{k,p}$ and φ_k denote the peak value and phase angle of the kth harmonic, respectively. The first harmonic, $\psi_1(\omega t)$, is called a *fundamental*. Terms "fundamental voltage" and "fundamental current" are used throughout the book to denote the fundamental of a given voltage or current.

The peak value, $\Psi_{1,p}$, of fundamental of a periodic function, $\psi(\omega t)$, is calculated as

$$\Psi_{1,p} = \sqrt{\Psi_{1,c}^2 + \Psi_{1,s}^2}, \tag{1.20}$$

where

$$\Psi_{1,c} = \frac{1}{\pi} \int_0^{2\pi} \psi(\omega t) \cos(\omega t) d\omega t, \tag{1.21}$$

$$\Psi_{1,s} = \frac{1}{\pi} \int_0^{2\pi} \psi(\omega t) \sin(\omega t) d\omega t, \tag{1.22}$$

and the rms value, Ψ_1, of the fundamental is

$$\Psi_1 = \frac{\Psi_{1,\mathrm{p}}}{\sqrt{2}}. \tag{1.23}$$

Since the fundamental of a function does not depend on the dc component of the function, the ac component, $\psi_{\mathrm{ac}}(\omega t)$, can be used in Eqs. (1.21) and (1.22) in place of $\psi(\omega t)$.

When the fundamental is subtracted from the ac component, the so-called *harmonic component*, $\psi_{\mathrm{h}}(\omega t)$, is obtained as

$$\psi_{\mathrm{h}}(\omega t) = \psi_{\mathrm{ac}}(\omega t) - \psi_1(\omega t). \tag{1.24}$$

The rms value, Ψ_{h}, of $\psi_{\mathrm{h}}(\omega t)$, called a *harmonic content* of function $\psi(\omega t)$, can be calculated as

$$\Psi_{\mathrm{h}} = \sqrt{\Psi_{\mathrm{ac}}^2 - \Psi_1^2} = \sqrt{\Psi^2 - \Psi_{\mathrm{dc}}^2 - \Psi_1^2} \tag{1.25}$$

and used for calculation of the so-called *total harmonic distortion*, *THD*, defined as

$$\mathrm{THD} \equiv \frac{\Psi_{\mathrm{h}}}{\Psi_1}. \tag{1.26}$$

The concept of total harmonic distortion is also widely employed outside power electronics as, for instance, in the characterization of quality of audio equipment. Conceptually, the total harmonic distortion constitutes an ac counterpart of the ripple factor.

In the generic inverter, whose output waveforms have been shown in Figure 1.6, the rms value, V_{o}, of output voltage equals the dc input voltage, V_{i}. Since v_{o} is either V_{i} or $-V_{\mathrm{i}}$, $v_{\mathrm{o}}{}^2 = V_{\mathrm{i}}{}^2$. The peak value, $V_{\mathrm{o,1,p}}$, of fundamental output voltage is

$$V_{\mathrm{o,1,p}} = V_{\mathrm{o,1,s}}, \tag{1.27}$$

because the waveform in question has the odd symmetry (see Appendix B). Consequently,

$$V_{\mathrm{o,1,p}} = \frac{2}{\pi}\int_0^{\pi} V_{\mathrm{i}} \sin(\omega t) d\omega t = \frac{4}{\pi} V_{\mathrm{i}} = 1.27 V_{\mathrm{i}}. \tag{1.28}$$

Now, the fundamental output voltage, $v_{\mathrm{o,1}}(\omega t)$, can be expressed as

$$v_{\mathrm{o,1}}(\omega t) = V_{\mathrm{o,1,p}} \sin(\omega t) = \frac{4}{\pi} V_{\mathrm{i}} \sin(\omega t). \tag{1.29}$$