PASSIVE
MACROMODELING
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KAI CHANG, Editor
Texas A&M University

A complete list of the titles in this series appears at the end of this volume.
PASSIVE MACROMODELING

Theory and Applications

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To the memory of my parents S. G.-T.
To my family B. G.
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Appendix A Notation

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Numerical modeling and simulation is essential for understanding the behavior
of structures and systems in practically every branch of science and engineering.
Unfortunately, the enormous complexity of physical systems often prevents direct
approaches where the whole system is characterized using first-principle models
(typically systems of partial differential equations (PDEs)) and then simulated on a
suitable computer. As an example, assessing the quality of signal and power distribution
on a printed circuit board requires computing the transient electromagnetic fields over
a large domain, including metal and dielectric structures with many small geometrical
details, with localized nonlinear devices acting as termination networks. This task is
prohibitive using a direct approach.

Divide and conquer techniques have been demonstrated as excellent alternatives to
reduce this complexity and to perform efficient numerical simulations with reasonable
runtimes. The overall system is first partitioned into well-defined substructures (e.g.,
connectors, via fields, coupled interconnect segments), which are characterized
separately through approximate reduced-complexity behavioral models, so-called
“macromodels.” Macromodels can be identified from terminal responses either in time
or frequency domain, leading to compact state-space representations of the broadband
dynamics of the structure. Alternatively, macromodels may be obtained by applying
model order reduction (MOR) techniques to the large systems of ordinary differential
equations (ODEs) arising from a spatial discretization of Maxwell’s equations in the
domain of interest. Once available, these models are connected to build an approximate
yet accurate representation of the whole system, which can then be solved with limited
computing resources.

This approach has become a standard for the analysis of electrical interconnects
at chip, package, or board level, microwave components, and high-voltage power
grids including their many components such as transmission lines and transformers.
The main enabling factor is linearity of the constitutive equations, which simplifies
the model extraction algorithms. The obtained macromodels can be cast as simplified electric networks made of elementary components, which can be easily simulated using off-the-shelf circuit solvers.

Although the main idea of building reduced-complexity macromodels has been pursued for several decades, an enormous advancement in algorithm and tool development took place starting from the late 1990s with the introduction of the so-called vector fitting (VF) algorithm. Despite its simplicity, this algorithm outperforms other techniques in terms of accuracy, robustness, and scalability. Thus, VF quickly became very popular and is now ubiquitous in many application areas of electrical and electronic engineering.

Additional momentum to the diffusion of macromodeling techniques came few years later with the publication of several technical papers on passivity enforcement. Only passive macromodels can be shown to preserve numerical stability in system-level transient simulations. The availability of reliable algorithms for checking and enforcing model passivity was thus the last step toward the routine application of reduced-order macromodeling. Nowadays, most electronic design automation (EDA) tools used by designers of electrical and electronic systems make extensive use of these techniques.

The rapid diffusion of passive macromodeling techniques was fostered in the early 2000s by a few software tools that were made available on the web, including the original VF code and associated software package from B.G., and the IdEM software package by S.G.-T. and his group. The importance of this subject for both academic and professional communities was further confirmed by the several dedicated tutorials, keynotes, and special sessions in many technical conferences in the fields of microwave, packaging, computer-aided design and by the fact that several leading universities worldwide started offering graduate-level courses on various aspects of macromodeling.

ABOUT THIS BOOK

The purpose of this book is to offer a complete and well-organized presentation of state-of-the-art passive macromodeling techniques, with emphasis on black-box approaches. This book can be used as a reference text for senior or graduate-level courses in electrical engineering programs, as well as a reference for both scholars and engineers active in the field of numerical modeling, simulation, design, and optimization of electrical/electronic systems.

Since the target readers may have a diverse background and experience, we included in the book some chapters with standard background material (e.g., linear time-invariant circuits and systems, basic discretization of field equations, state-space systems), as well as few appendices collecting basic facts from linear algebra (Appendix C), optimization templates (Appendix D), and signals and transforms (Appendix E). These parts may be skipped by the most experienced readers but will be valuable to more practical readers who need a self-contained book for quick reference. Furthermore, the book includes some more technical or advanced topics, which may be omitted at first reading. These topics are flagged with an asterisk (*).

In preparing this text, we have put our best effort to achieve a good compromise between readability and mathematical rigor. In fact, many excellent books and technical papers on system and control theory, identification, approximation, and reduction are
available (and duly cited in this work). Unfortunately, most of these books are hard to read for the nonspecialist. Design engineers and even graduate students in applied disciplines may fail in grasping the main concepts if the presentation flow is too formal. However, the subject of this book requires a minimum of derivation and justification of the main results in order to have a clear understanding of the advantages and disadvantages of the developed algorithms, including applicability limits. With this in mind, we tried to elaborate all main concepts and results in a mathematically precise way, but without excessive formalism. Many proofs are omitted and only sketched or left as exercises for the readers. Most crucial concepts are illustrated through dedicated examples. Each chapter is further complemented by a selected set of problems.

The first part of this book provides some basic definitions and background material on passive linear systems and circuits, setting notation for later chapters (lists of symbols and acronyms are available in Appendices A and B). Chapter 1 provides an overview of the book content and defines the context where macromodeling can or should be applied. Chapter 2 discusses linear time-invariant (LTI) systems based on their “external” or input–output characterization, without assuming any knowledge on their internal structure. Chapter 3 focuses instead on lumped LTI systems described by state-space or descriptor forms. These two chapters discuss in detail the concepts of stability, causality, and passivity – crucial properties that black-box macromodels must adhere to.

Macromodeling aims at representing complex electromagnetic systems as low-complexity equivalent circuits, or more generally, at approximating systems of PDEs via small systems of ODEs. Chapter 4 provides an overview of the necessary steps that must be undertaken to enable such approximations, based on simple 1D and 2D distributed-parameter examples. A short review of the leading methods for the numerical solution of general electromagnetic problems is also included, but here the presentation aims at convincing the reader of high potential for complexity reduction and at further justifying and motivating the need for macromodeling.

Although the main focus of this book is on black-box macromodeling via system identification from external responses, complementary approaches are possible. In particular, Chapter 5 provides an overview of the prominent MOR techniques. These methods start with a large system of ODEs, for example, arising from a spatial discretization of field equations. Compact models are obtained through a projection or truncation process.

The subject of black-box macromodeling via data fitting is introduced in Chapter 6. After a general formulation, we review the various techniques that were widely used before the advent of the VF algorithm. Although the latter outperforms such preexisting techniques, we include a summary and some application examples, so that the advantages of VF are best understood, also from a historical perspective.

Chapter 7 is one of the core chapters of the book. A comprehensive presentation of the VF algorithm is included in frequency and time domains, applied to scalar single-input single-output systems. In addition to a formal development of the main scheme, several enhancements and implementation schemes are presented and discussed, with the help of various examples. Chapter 8 discusses instead the advanced features that make VF the method of choice for macromodel identification. These advanced features include several approaches for ensuring a good scalability with dynamic order and number of input/output ports.
Verification and enforcement of model passivity is fundamental to ensure the numerical stability of subsequent macromodel-based time-domain simulations. These two aspects are thoroughly discussed in Chapters 9 and 10. The former presents several passivity characterizations that are available for impedance, admittance, and scattering descriptions, based on either frequency-dependent or algebraic conditions. Building on these conditions and casting them as constraints, Chapter 10 presents various approaches for “correcting” a given macromodel by perturbing its parameters so that the final result is passive.

Once compact, accurate, and passive macromodels are available, they can be used in system-level transient simulations. Chapter 11 describes various approaches for including such macromodels within system-level simulation flows, either using out-of-the-box commercial solvers such as SPICE or EMTP-type tools or via direct numerical implementation.

Chapter 12 provides an overview of state-of-the-art macromodeling techniques for structures characterized by nonnegligible signal propagation delays between interface ports. Examples can be electrically long multiconductor transmission lines or even more complex interconnects including transmission line segments and discontinuities. Various approaches for extracting and including propagation delays in the macromodels are discussed and compared.

Finally, Chapter 13 presents a number of applications from different areas of engineering. Emphasis is on modeling and simulation of electromagnetic systems, ranging from high-frequency applications in microwave engineering, electromagnetic compatibility, and signal/power integrity, to low-frequency applications in power systems. The frequency responses of all such systems have quite similar features, even if geometry and frequency scales are different. These applications confirm the suitability of macromodels for efficient system simulation, both under normal operation conditions and during unexpected transients caused, for example, by unexpected disturbances or failures. We also include a few examples from nonelectrical disciplines, such as mechanical and hydraulic systems, demonstrating the interdisciplinary character of the techniques that are presented in the book.

SOME REMARKS ON NOTATION

Although a detailed list of symbols is provided in Appendix A, and each symbol is properly defined at its first occurrence, a few preliminary remarks are in order. Throughout this book, we denote scalar, vector, and matrix variables with $x$, $\mathbf{x}$, and $\mathbf{X}$, respectively. We denote with $\mathbf{X}^*$, $\mathbf{X}^\mathsf{T}$, and $\mathbf{X}^\mathsf{H}$ the complex conjugate, transpose, and conjugate (Hermitian) transpose, respectively, of a given matrix $\mathbf{X}$. The imaginary unit $j = \sqrt{-1}$ is denoted with an upright font and is not to be confused with the generic index $j$. The all-zero and identity matrices are denoted with $\mathbf{0}$ and $\mathbf{I}$, respectively. We will often need to construct matrices by stacking matrix blocks horizontally or vertically: the compact Matlab notation is used at times, with $(\mathbf{X}_1; \mathbf{X}_2)$ indicating a block-column matrix (i.e., $\mathbf{X}_2$ is placed below $\mathbf{X}_1$), and $(\mathbf{X}_1, \mathbf{X}_2)$ indicating a block-row matrix (i.e., $\mathbf{X}_2$ is placed at the right of $\mathbf{X}_1$). The set of all eigenvalues and singular values of $\mathbf{X}$ are denoted with $\lambda(\mathbf{X})$ and $\sigma(\mathbf{X})$, respectively.
The main focus of this book is on continuous-time, linear, time-invariant systems with a real-valued impulse response $h(t)$. For these systems, we will generally use or derive state-space descriptions, with associated state-space matrices $\{A, B, C, D\}$. Most results are stated assuming that these state-space matrices are real-valued; the generalization to the complex-valued case is straightforward and not discussed in the book. Lowercase fonts are generally used to denote time-domain signals $x(t)$, where $t$ is time, whereas uppercase fonts describe their associated frequency-domain representation $X(s)$, where $s$ is the Laplace variable. Special signals (distributions) are the unit impulse or Dirac’s delta $\delta(t)$, and the Heaviside unit step $\theta(t)$. A generic transfer matrix is denoted as $H(s)$, and the corresponding impulse response matrix as $h(t)$. Whenever the system under investigation is natively discrete-time, the corresponding signals will be denoted as $x[k]$ or $x_k$, where $k$ is the discrete time index.

The reader will soon realize that, in some cases, the same symbol or letter is used to denote more than one physical, geometrical, or mathematical entity. As an example, $C$ can be used both as the per-unit-length capacitance matrix of a transmission line and as one of the state-space matrices of a linear dynamical system. We have struggled to resolve most of these ambiguities, but a few still remain. This is mainly because this book collects results from different disciplines (such as circuit and electromagnetic theory, system and control theory, system identification, and numerical analysis and optimization). Each of these disciplines has a quite consolidated “library” of standard symbols that are of common use, and conflicts are unavoidable. We believe, however, that this should not cause any confusion, since each symbol is properly defined and used with a unique meaning within the scope of a book unit (section or chapter).

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INTRODUCTION

This introductory chapter provides a quick preview of the book contents, with the main purpose of defining what a macromodel is, motivating why macromodels are important, and overviewing the most important requirements that macromodels should meet in order to be of practical use. There are no derivations in this chapter, and any results that are stated here are not proved but only illustrated through simple examples. Details will be discussed in the following chapters of this book.

1.1 WHY MACROMODELING?

Many real-world problems are too complex to be modeled in full detail starting from first principles. In fact, the processing time and the memory requirements for a direct simulation of a fully detailed system, such as a chip-package-board electronic structure or a high-voltage power system, are prohibitive on any computer. For this reason, common engineering flows are based on divide and conquer approaches. Different devices and subblocks, which comprise the system, are characterized and modeled independently, with a level of accuracy that meets the application requirements. These individual models are then suitably interconnected for system-level analyses, usually in the form of time-domain simulations, allowing a full system study with an acceptable computational effort. The extraction of the models, that is, the macromodeling task, is of course an essential step in the overall procedure.

We understand by the term macromodel a reduced-complexity behavioral description of a device or a collection of devices. Macromodels are inherently approximate, since
their construction deliberately neglects some aspects that are deemed unimportant for the system behavior. Nonetheless, macromodels must be accurate enough to allow appropriate decisions by designers based on the results of subsequent numerical simulations. Depending on the application, macromodels may have to fulfill additional properties. Among these, the most common is passivity, which arises as a fundamental constraint when representing structures that are unable to generate energy, such as electrical interconnects; hence, the title of this book, “Passive Macromodeling”.

Many different approaches to macromodeling are available. Two popular classes of techniques, sometimes denoted as white-box and gray-box, assume a model structure that reproduces or mimics the physical topology of the system that the macromodel intends to represent. For instance, a set of physical conductors may be represented by a network of resistances (representing ohmic losses in each conductor), capacitances (representing charge accumulation at conductors in close proximity), and possibly coupled inductances (representing magnetic coupling between conductors). Either an automated extraction software or an expert designer is required for the construction of such models. This task may, however, become very difficult when the topology of the underlying physical structure becomes very complex. Even worse, the internal structure of the device to be modeled may be only partially known to the engineer who is in charge of building the macromodel. In such cases, exploiting an incomplete knowledge of the system in the model development is very difficult and likely to fail.

This book develops a complementary black-box approach. We seek for models that reproduce the behavior of a physical structure with respect to its input and output characteristics, observed from well-defined external terminals. No information on the internal structure of the actual system is exploited in the construction of the macromodel. This implies that there is usually no direct link between the internal topological and dynamical structure of the model and that of the physical system. A typical application scenario involves the availability of a limited number of time- or frequency-domain responses, obtained by a direct measurement or through a commercial field solver. Macromodels are constructed by fitting the parameters of a suitable model class to this data. The following example illustrates this point on a simple two-port circuit block.

Let us consider the two-port circuit element depicted in Figure 1.1(a), assumed to be the reference (true) system. We evaluate the corresponding \( 2 \times 2 \) admittance matrix

\[
\begin{bmatrix}
1 \Omega & 1 H \\
1 H & 1 \Omega \\
1 H & 1 H \\
3 \Omega & 7 \Omega
\end{bmatrix}
\]

(a)

\[
\begin{bmatrix}
-14 \Omega & -6 H \\
2 \Omega & 2 H \\
3 H & 3 H \\
7 \Omega & 7 \Omega
\end{bmatrix}
\]

(b)

Figure 1.1 Original two-port circuit (a) and its synthesized model (b).
$Y(s)$ defined at ports (1) and (2). The elements of this matrix are rational functions of the complex frequency $s$, which can be written as

$$
Y_{11}(s) = Y_{22}(s) = \frac{2s + 4}{3s^2 + 10s + 7}, \quad Y_{12}(s) = Y_{21}(s) = \frac{-s - 3}{3s^2 + 10s + 7}.
$$

Starting from (1.1) and applying one of the (black-box) circuit synthesis methods, which will be discussed later in Chapter 11, leads to the equivalent circuit depicted in Figure 1.1(b). Since no information on the original topology was used in the synthesis, this equivalent circuit looks very different from the original. Some circuit elements are even negative. Yet, the behavior of the two circuit realizations as observed from ports (1) and (2) is identical, as a frequency sweep of the admittance matrix entries depicted in Figure 1.2 confirms.

One may ask the question whether one circuit realization is preferable to the other. As a general guideline, whenever some information is available on the original system, this should be exploited in the construction of the model. Referring to the aforementioned example, if we know that the original system is comprised of three inductors connected in a “wye” configuration, then it is clear that the circuit topology in Figure 1.1(a) is preferable. Even if the value of the circuit elements is not known, it can be easily

![Admittance responses](image)

**Figure 1.2** Admittance responses of the two circuit realizations shown in Figure 1.1.
estimated from measured port responses through a fitting process. However, when the number of connected elements in the original system is very large, as in the case of parasitic networks, with possibly many internal nodes that are not available externally, guessing the original topology and estimating the component values from external responses is not feasible. In this case, one should abandon the hope for a topology-aware model and resort to a pure black-box approach. If relevant for the application at hand, one may further seek for circuit realizations of the macromodel that do not include negative elements. This is indeed possible, as discussed later in Chapter 11.

1.2 SCOPE

In this book, we restrict ourselves to the treatment of systems and devices that are linear in their input–output behavior. This may at first sound restrictive, as nonlinear devices are found in many real-world systems. However, for most electronic and high-voltage power applications, most of the overall complexity is due to electromagnetic coupling or interaction between various system parts. Sometimes this interaction is unwanted and labeled as parasitic, while sometimes it is required and designed for proper system functioning. Such effects are inherently linear in the vast majority of applications. Therefore, linear macromodeling can be viewed as a process that replaces a high-complexity network of parasitics, or more generally a complex electromagnetic structure, with a lower complexity model at a well-defined set of interface ports. Additional devices with a nonlinear behavior can be modeled separately and included later in the same time-domain simulation, by suitably interconnecting all individual blocks at their interface ports. This modular approach is straightforward with computer simulation programs such as Simulation Program with Integrated Circuit Emphasis (SPICE) or Electromagnetic Transients Program (EMTP). Modeling of nonlinear devices requires techniques that are fundamentally different from those of linear theory, and as such it falls outside the scope of this book.

We further restrict ourselves to univariate modeling, by assuming only frequency or time as the free (independent) variable, although the topic of multivariate macromodeling is emerging as a more and more important field of research and application. Multivariate macromodeling involves introducing in the model expressions one or more additional variables, such as material or device design parameters. Preservation of this dependence in behavioral macromodels is extremely valuable for designers, who need to determine the best parameter configuration that allows suitable system performance metrics to be met, possibly after an optimization loop. Multivariate macromodeling is still a somehow immature field, with only partial results and sometimes inefficient or inaccurate algorithms available. For this reason, we have decided to not discuss multivariate macromodeling except for one introductory section in Chapter 14.

There are various other reasons why macromodeling approaches are important or prove useful in applications, some of which are listed as follows.

Inclusion of frequency-dependent effects. Many devices are characterized by strong and possibly complicated frequency-dependent effects. The series impedance of a transmission line is a simple example, where the frequency