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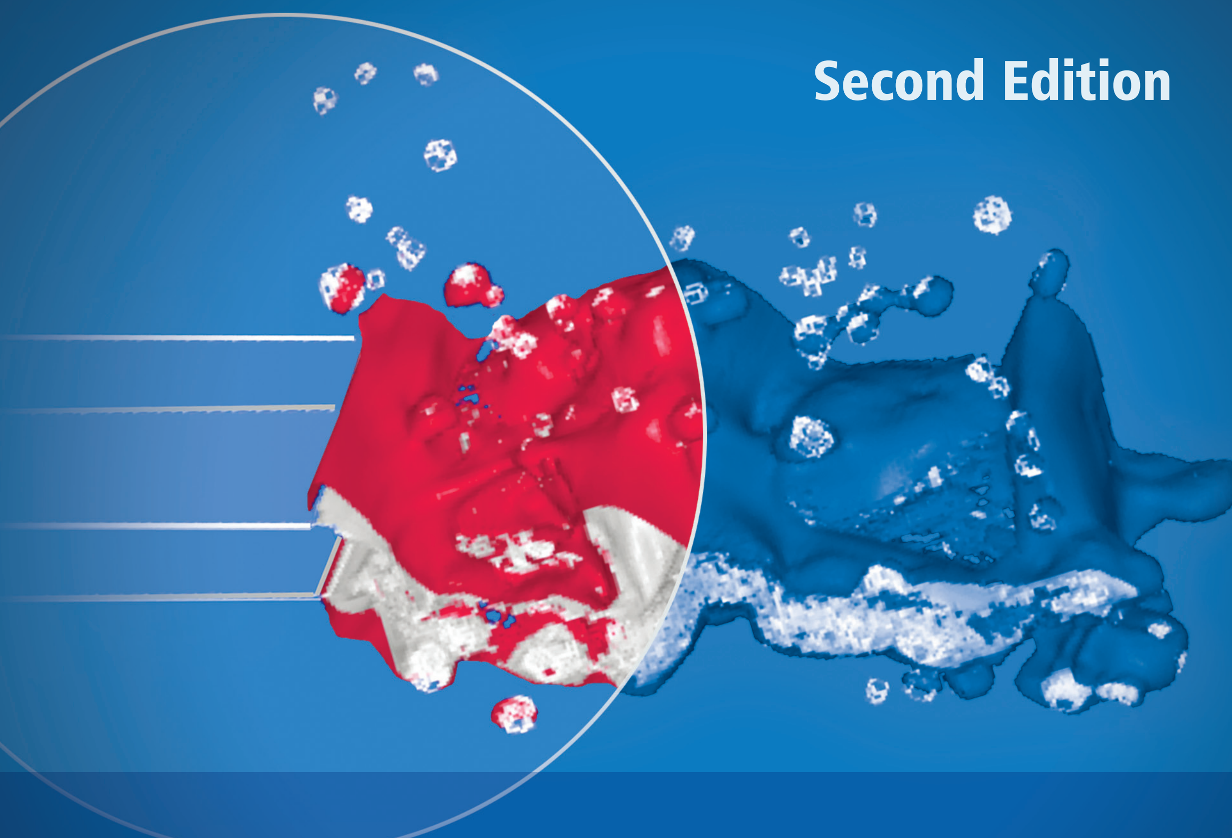
Nonlinear Finite Elements for Continua and Structures

Second Edition



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NONLINEAR FINITE ELEMENTS FOR CONTINUA AND STRUCTURES

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On the cover: Fracture-process profile of a Ti-modified 4330 steel, shown in the unloaded configuration. An initial microvoid volume fraction of 0.04% is simulated using the multiresolution continuum theory described in chapter 12 with about 270 million 8-node elements and approximately 3 billion degrees of freedom.

To our families

Contents

Foreword

This book provides a comprehensive introduction to the theory of nonlinear finite element analysis and its various implementation strategies. It is intended for beginning graduate students studying the areas of mechanical engineering, civil engineering, applied mathematics, engineering mechanics and materials science.

The authors provide a wide selection of material to suit the needs and preferences of many instructors. This second edition provides a thorough coverage of the key topics for the benefit of students, practitioners and developers of the nonlinear finite element software. Since 2001, I have taught the first edition of this book as a textbook to students at Columbia University, RPI and engineers at the Knolls Nuclear Laboratory. Students absolutely loved it due to the fact that difficult concepts are explained in a way that engineers and graduate students can easily comprehend.

The second edition of the book comes with a solutions manual for the benefit of instructors and students. The solutions are accompanied by MATLAB® and FORTRAN codes for select computer problems, to facilitate and accelerate the implementation of the more advanced concepts and algorithms covered in the book.

The second edition also includes three new chapters, which offer a concise introduction to some of the cutting-edge methods that have evolved in recent years in the field of nonlinear finite elements, namely: the eXtended Finite Element Method (XFEM), Multiresolution Continuum Theory (MCT) for multiscale microstructures, and dislocation-density-based crystalline plasticity. With these timely additions, the book will now reach a broader community at the forefront of research.

In summary, the book has proven over the years to be an excellent reference on nonlinear finite elements for instructors, students, engineers and researchers. It can be easily adopted as a graduate textbook or used by students for self-study. With the addition of a solutions manual, along with the relevant computer codes, the learning curve for some of the most advanced concepts in nonlinear finite analysis could be greatly shortened. Finally, the additional three chapters add the new dimension of cutting edge research.

Jacob Fish
Columbia University, USA

Preface

The objective of this book is to provide a comprehensive introduction to the methods and theory of nonlinear finite element analysis. We have focused on the formulation and solution of the discrete equations for various classes of problems that are of principal interest in applications of the finite element method to solid mechanics, the mechanics of materials, and structural mechanics. The core topics are presented first, which include: the discretization by finite elements of continua in one dimension and in multi-dimensions; the formulation of constitutive equations for nonlinear materials and large deformations; and procedures for the solution of the discrete equations, including considerations of both numerical and physical instabilities. More specialized applications are then presented. These include: the treatment of structural and contact-impact problems; representation of weak and strong discontinuities that evolve in failing solids; and mechanism-based modeling of material nonlinearities, illustrating advanced treatments of their multiscale aspects and microstructural origins. These are the topics which are of relevance to industrial and research applications and which are essential to those in the practice, research, and teaching of nonlinear finite elements.

The book has a mechanics style rather than a mathematical style. Although it includes analyses of the stability of numerical methods and the relevant partial differential equations, the objective is to teach methods of finite element analysis and the properties of the solutions and the methods. Topics such as proofs of convergence and the mathematical properties of solutions are not considered.

In the formulation of the discrete equations, we start with the governing equations based on the mechanics of the system, develop a weak form, and use this to derive the discrete equations. Weak forms and the discrete equations are developed for Lagrangian, arbitrary Lagrangian, and Eulerian meshes, for in the simulation of industrial processes and research, problems with large deformations that cannot be treated by Lagrangian meshes are becoming more common. Both the updated Lagrangian and the total Lagrangian approaches are thoroughly described.

Since a fundamental understanding of the equations requires substantial familiarity with continuum mechanics, Chapter 3 summarizes the continuum mechanics which is pertinent to the topics in this book. The chapter begins with a basic description of motion with an emphasis on rotation. Strain and stress measures are described along with transformations between them, which are later generalized as push-forward and pullback operations. The basic conservation laws are described in both so-called Eulerian and Lagrangian descriptions. Objectivity, often known as frame invariance, is introduced.

Chapter 4 describes the formulation of the discrete equations for Lagrangian meshes. We start with the development of the weak forms of momentum balance and use these to develop the discrete equations. Both the total Lagrangian and the updated Lagrangian formulations are thoroughly described, and methods and approaches for transforming between these formulations are discussed. Examples are given of the development of various elements in two and three dimensions.

Chapter 5 treats constitutive equations, with particular emphasis on the aspects of material models that are relevant to the treatment of material nonlinearities and large deformations.

Solution procedures and analyses of stability are described in Chapter 6. Both explicit and implicit integration procedures are described for transient processes and solutions; continuation procedures for equilibrium problems are considered. Newton methods and the linearization procedures required for the construction of the Newton equations are developed. In the solution of nonlinear problems, the stability of the numerical procedures and of the physical processes is crucial. Therefore, the theory of stability is summarized and applied to the determination of the stability of solutions and numerical procedures. Both geometric and material stability are considered.

Chapter 7 deals with arbitrary Lagrangian Eulerian methods. This chapter also provides the tools for Eulerian analysis. Numerical techniques needed for this class of meshes, such as upwinding and the SUPG formulation, are described.

Chapter 8 deals with element technology, the special techniques which are needed for the successful design of elements in constrained media problems. Emphasis is placed on the problem of incompressible materials but the techniques are described in a general context. One-point quadrature elements and hourglass control are also described.

Chapter 9 is devoted to structural elements, particularly shells and beams; plates are not treated separately because they are special cases of shells. We emphasize continuum-based structural formulations because they are more easily learned and more widely used for nonlinear analysis. The various assumptions are carefully studied and continuum-based formulations for beams and shells are developed. Much of this chapter rests heavily on the preceding chapters, since continuum-based elements can be developed from continuum elements with minor modifications. Therefore, topics such as linearization and material models are treated only briefly.

Contact-impact is described in Chapter 10. Contact-impact is viewed as a variational inequality, so that the appropriate contact inequalities are met in the discrete equations. Both displacement-based and velocity-based formulations are described. Attention is focused on the nonsmooth character of contact-impact and its effect on solution procedures and simulations.

Chapter 11 covers the modeling of strong and weak discontinuities. An overview of methods in classical finite elements is provided as a historical introduction. The chapter focuses on using the extended finite element method (XFEM) to model discontinuities with non-conforming meshes. For strong discontinuities the emphasis is on modeling fracture, with extensions to other problems. For weak discontinuities emphasis is on material interfaces, but the developments presented are easily extendable to other weak discontinuities. The discussion begins with the 1D formulation and then builds to multiple dimensions. Discussions are included for both implementation and integration of XFEM as well as a brief overview of the level set method, which is often coupled with XFEM. The chapter concludes with an example.

The role of material microstructure in defining material nonlinearities is introduced in Chapter 12. Emphasis is made on the *multiresolution continuum theory*, a multiscale mechanics theory for the large deformation of heterogeneous materials. Its aim is to link the mechanics of solids to materials science. The theory is developed from variational principles and discretized for finite element implementation. Representative volume elements (RVEs) and their role in developing mechanism-based multiscale constitutive formulations are then discussed and integrated in the multiresolution framework.

RVE modeling of single crystals by finite elements is discussed in Chapter 13, as an example of mechanism-based modeling of non-linear materials. From materials science, the crystallographic description of cubic and non-cubic crystals and the theory of dislocation densities are linked to a non-linear constitutive algorithm that governs inhomogeneous deformation in crystalline materials at the continuum level.

This book is intended for beginning graduate students in programs in mechanical engineering, civil engineering, applied mathematics, and engineering mechanics. The book assumes some familiarity with the finite element method, such as a one-semester course or a four- to five-week section in a larger course. The student should be familiar with shape functions, stiffness, and force assembly; it is also helpful to have some background in variational or energy methods. In addition, students should have had some exposure to strength of materials and continuum mechanics. Familiarity with indicial notation and matrix notation is essential.

Most instructors will choose not to cover this entire book. To do so would require a one-year course. Our aim has been to include a wide selection of material to suit the needs and preferences of many instructors. Moreover, the additional material provides the interested student with a source of background reading before embarking into the literature.

Shorter courses, such as a 10-week quarter or a 16-week semester, require a judicious selection of material which reflects the aims and taste of the instructor. The book presents most material in both the total and the updated Lagrangian format. Thus, an introductory course can focus on the updated Lagrangian viewpoint from Chapter 2 to Chapter 4, with selected topics from Chapters 5 and 6 to familiarize the student with material models and solution procedures. Some instructors may opt to skip the one-dimensional treatment in Chapter 2, leaving it as perhaps required reading. The total Lagrangian formulation can then be introduced by simply showing the transformation in Chapter 4. A similar course can be designed with an emphasis on the total Lagrangian formulation.

We have endeavored to use a unified style and notation throughout this book. This is important because, for students, drastic changes in notation and formalism often impede learning. This, at times, causes divergence from notation customary in the literature of a particular area, but we hope that the consistency of presentation will help the student.

For the second edition of this book a solution manual is available, which includes solutions to all exercises in the book, including MATLAB® and/or FORTRAN codes for the prescribed computer problems.

We would like to thank our many friends, colleagues, and former students who read preliminary versions of this book and who provided numerous suggestions, feedback and corrections, especially

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List of Boxes