MECHANICAL SCIENCE

THIRD EDITION

W. Bolton

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Mechanical Science

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Preface to third edition

This book aims to give a comprehensive coverage of mechanical science, covering:

- principles of statics
- mechanics of materials
- principles of dynamics
- dynamics of machines

A background of engineering science from a National Certificate/Diploma or A-level Physics or Engineering Science has been assumed, although all relevant basic principles are revised. A basic knowledge of algebra and calculus has also been assumed, although, to assist the reader, a review of the basic principles of differentiation and integration is given in the Appendix.

This book aims to give a comprehensive coverage of mechanical science which is suitable for:

- HNC/HND students taking Mechanical Engineering courses, including all the topics likely to be covered in both years of such courses
- first year undergraduate courses in Mechanical Engineering

In the third edition the coverage of power transmission systems has been extended to give a better introduction to the principles.

Whilst it is recognised that analysis of mechanical science problems of any complexity is generally carried out by means of a computer, it was felt that within a book devoted to the establishment of principles, the exercise of working things 'by hand' was more appropriate in this context and would enable the reader to appreciate more readily the principles involved. Almost 500 problems are included, answers being given to all. In addition each chapter includes fully worked examples, of which there are almost 200.

W. Bolton

Chapter 1
Forces and equilibrium

1.1 Scalars and vectors

There are two types of quantities in mechanics: those which have magnitude but no directional properties and are called *scalar quantities* and those, called *vector quantities* which are associated with a direction as well as having magnitude.

Scalar quantities, e.g. mass and energy, can be added or subtracted by the ordinary mathematical rules for addition and subtraction. The convention that is used in books is that scalar quantities are represented by letters in italic type, e.g. mass m.

Vector quantities, such as acceleration and force, cannot be added or subtracted by the ordinary mathematical rules of addition and subtraction. Their directions have to be taken into account. Vector quantities can be represented by arrow-headed straight lines, the length of the line representing the magnitude of the quantity and the direction of the arrow the direction of the quantity. The convention that is often adopted in books is that vector quantities are represented by letters in bold type, e.g. force $\mathbf{F}$. When we are referring to just the magnitude of a vector quantity then it is represented by just the letter in italic type, e.g. the magnitude of a force F, or the letter in italic type between vertical lines, e.g. $|F|$. Often, when the direction of a vector is implied, or specified by some diagram, the vector is just referred to by its magnitude.

The term *coplanar vectors* is used for vectors which lie in the same plane and *concurrent vectors* for those which have lines of action which all pass through the same point.

1.1.1 Vector addition and subtraction

In general, two concurrent vectors are added together by means of the *parallelogram law*. The term *resultant* is used for the resulting vector. The parallelogram law can be stated as: the resultant vector $\mathbf{V}$ obtained by adding two vectors $\mathbf{V_1}$ and $\mathbf{V_2}$ is the diagonal of the parallelogram in which $\mathbf{V_1}$ and $\mathbf{V_2}$ are represented by arrow-headed lines as adjacent sides. Figure 1.1 shows the parallelogram.

If a vector $\mathbf{V_2}$ is drawn with its line of action unchanged but the arrow pointing in the opposite direction then we have the vector $-\mathbf{V_2}$. Thus the difference between two vectors $\mathbf{V_1}$ and $\mathbf{V_2}$ can be obtained by using the

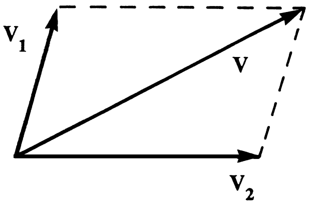


Fig. 1.1 Parallelogram law.

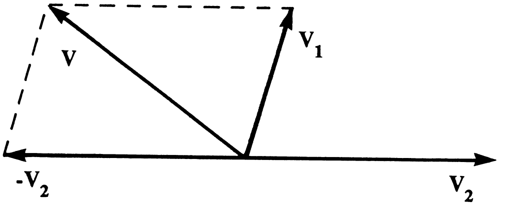


Fig. 1.2 Subtracting vectors.

parallelogram law to add the vectors $\mathbf{V_1}$ and $-\mathbf{V_2}$. Figure 1.2 illustrates this.

The result of vector addition or subtraction can be obtained by scale drawing of the parallelogram and measurement of the resulting resultant, or by calculation. With calculation useful relationships are the *cosine rule* and the *sine rule*. For the triangle shown in figure 1.3, the cosine rule gives

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ b^2 &= a^2 + c^2 - 2ac\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned} \qquad [1]$$

and the sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad [2]$$

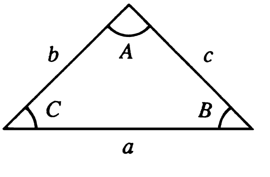


Fig. 1.3 Cosine and sine rules.

1.1.2 Resolution of vectors

It is often useful to be able to replace a single vector by two other vectors, generally at right angles to each other. The single vector is said to have been resolved into its components. It is done by using the parallelogram law in reverse, i.e. starting with the diagonal and finding the two vectors which would fit as sides of the parallelogram. Thus the magnitudes of the components of vector **V** in figure 1.4 are, in the x-direction

$$V_x = V\cos\theta \qquad [3]$$

and in the y-direction

$$V_y = V\sin\theta \qquad [4]$$

Vector addition and subtraction can often be simplified by resolving each vector into components and then adding or subtracting by simple arithmetic the vectors in each of the two directions. The resulting two components can then be recombined by means of the parallelogram law to give the resultant.

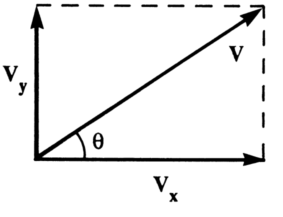


Fig. 1.4 Resolution of vectors.

1.2 Force as a vector

Forces cannot be directly observed; only their effects can be seen, these being the acceleration of an object when acted on by a resultant force or the distortion of a body acted on by a pair of forces. The concept of force arises from *Newton's laws*. These can be stated as follows.

1 A body will remain at rest, or continue to move in a straight line with a constant velocity, if there is no resultant force acting on it.
2 If a resultant force acts on a body then it will accelerate in the direction of the force with an acceleration proportional to the magnitude of the force.
Or alternatively:
If a resultant force acts on a body then it will change its momentum in the direction of the force with a rate of change of momentum proportional to the magnitude of the force.
3 If one body exerts a force on a second body then the second body will exert an equal and opposite force on the first, i.e. to every action there is an opposite and equal reaction.

The first law defines the condition for a body to be in what is termed *equilibrium*. The second law explains what happens when there is no equilibrium. The third law defines the way two bodies interact. The second law can be written as the equations

$$F = ma \qquad [5]$$

$$F = \frac{\mathrm{d}(mv)}{\mathrm{d}t} \qquad [6]$$

where m is called the mass of the body, a the acceleration and v the velocity, with mv being the linear momentum.

In equation [5], of the three quantities F, m and a the units of any two can be chosen arbitrarily and used to determine the unit of the third. The system of units used, the SI system, has the mass specified in kg and acceleration in m/s^2. Consequently the unit of force is defined in terms of these two units as kg m/s^2. This unit is given the name of the newton (N).

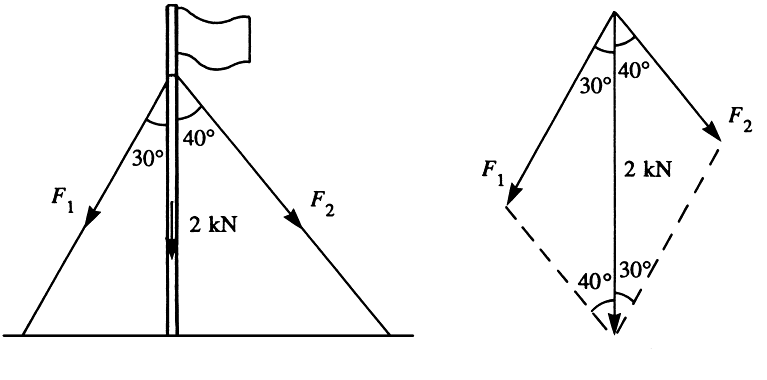
Forces, since they have both magnitude and direction, are vector quantities. They thus have to be added or subtracted by vector means. The following example illustrates this.

Example

A flag pole is held in a vertical position by two wire stays attached to the same point on the pole. If the two stays are at angles of 40° and 30° to the pole and the resultant force is to be 2 kN along the vertical axis of the pole, what are the forces acting in each stay?

Figure 1.5(a) shows the pictorial situation and figure 1.5(b) the parallelogram of forces for the concurrent forces acting at the point where the stays are attached to the flag pole. Using the sine rule with the parallelogram

$$\frac{F_1}{\sin 40°} = \frac{2}{\sin(180° - 30° - 40°)}$$



(a) (b)

Fig. 1.5 Example 1.

Hence $F_1 = 1.4\,\text{kN}$.
Similarly

$$\frac{F_2}{\sin 30^\circ} = \frac{2}{\sin(180^\circ - 30^\circ - 40^\circ)}$$

Hence $F_2 = 1.1\,\text{kN}$.

Alternatively we could have obtained the result by resolving each of the stay forces into directions along the pole and at right angles to it. Along the pole

$$2 = F_1 \cos 30^\circ + F_2 \cos 40^\circ$$

and at right angles to it

$$F_1 \sin 30^\circ = F_2 \sin 40^\circ$$

or $F_1 = 1.29F_2$. Hence, substituting this in the first equation gives

$$2 = 1.29F_2 \cos 30^\circ + F_2 \cos 40^\circ$$

and hence $F_2 = 1.1\,\text{kN}$ and $F_1 = 1.4\,\text{kN}$.

1.2.1 Weight

The *weight* of a body at some location can be defined as the force that is necessary to stop it accelerating. A weightless body is thus one that is freely falling. However, it is customary to take the weight as being equal to the gravitational force. In the absence of any other force the gravitational force will cause the body to accelerate, the acceleration being known as the *acceleration due to gravity* g. At the surface of the earth, the acceleration due to gravity is about $9.8\,\text{m/s}^2$. We can thus write, for a mass m,

$$\text{weight} = W = mg \qquad [7]$$

Example

A truck weighing 10 000 N stands on an incline that has a gradient of 1 in 50 (i.e. it descends by 1 m vertically for every 50 m along the hill). What is the size of the gravitational force component down the hill?

The weight of the truck is 10 000 N vertically. This can be resolved into two components, one at right angles to the hill and the other parallel to it. The parallel component for a hill which makes an angle of θ with the horizontal is $W \sin\theta$. Thus, since $\sin\theta = 1/50$, the parallel component is $10\,000/50 = 200\,\text{N}$.

1.3 Non-concurrent forces

Consider the situation where two or more forces act on a single body and the forces are not concurrent. A simple situation is a see-saw where forces, the weights of people, are applied at each end of a pivoted beam (figure 1.6). A simple experiment on such a system shows that the see-saw will balance when

$$F_1 d_1 = F_2 d_2$$

If force F_2 is removed, i.e. one person gets off the see-saw, then the see-saw begins to rotate about its pivot. The cause of the rotation is the product $F_1 d_1$. This quantity is called the *turning moment*. The turning moment of a force about some axis is thus defined as being the product of the force and its perpendicular distance from the axis.

For balance, the anticlockwise moment of one force must be balanced by the clockwise moment of the other force. This is known as the *principle of moments*.

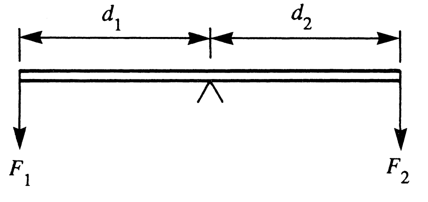


Fig. 1.6 See-saw system.

Example

For the bracket shown in figure 1.7, what is the moment of the 100 N force about the axis through point A?

AB is the perpendicular distance between the line of action of the force and the axis through A. Thus the moment is the product of the force and AB. The distance AB can be computed from the figure. However, an alternative way, and often simpler way, of considering the problem is to resolve the 100 N force into its components 100 sin 60° and 100 cos 60° and determine the moments for each component. These are $0.200 \times 100 \sin 60° = 17.3$ N m and

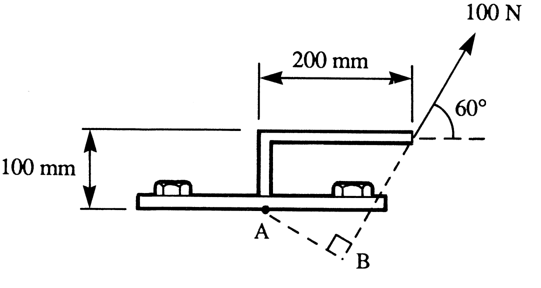


Fig. 1.7 Example.

$0.100 \times 100 \cos 60° = 5.0$ N m. The 17.3 N m moment gives an anticlockwise moment while the 5.0 N m moment gives a clockwise moment. The total moment is thus 12.3 N m anticlockwise.

1.3.1 Couple

A pair of equal and opposite forces that are not acting through the same point are known as a *couple*. Thus, for example, when a driver is using both hands to turn the steering wheel of a car, two non-concurrent forces are applied and hence a couple. Figure 1.8 illustrates this. The turning moment of the couple in the figure about the axis through the wheel centre is $FR + FR = 2FR$. The moment of the couple is thus the magnitude of one of the forces multiplied by their distance apart, the distance being measured perpendicular to one of the forces.

The moment of the couple does not contain any reference to the distances of the forces from an axis. Thus a couple has the same moment about any axis, indeed there is no need to specify an axis.

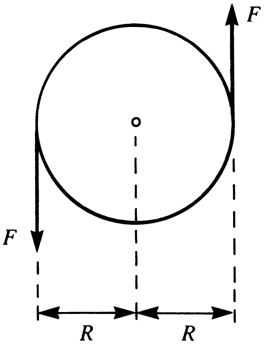


Fig. 1.8 Couple.

1.3.2 Centre of gravity

The weight of a body is an example of a distributed force in that any body can be considered to be made up of a number of particles, each having weight. However we can replace all the various weight forces of an object by a single weight force with a magnitude equal to the sum of the magnitudes of all the constituent weight forces and acting at a particular point known as the *centre of gravity*.

Consider a body made up of a large number of small elements, each having weight. The weights are non-current forces. The total moment of all the elements about some axis is

$$\text{total moment} = \delta w_1 x_1 + \delta w_2 x_2 + \delta w_3 x_3 + \dots$$

where δw_1 is the weight of element 1 which is a perpendicular distance x_1 from the axis, δw_2 the weight of element 2 and x_2 its perpendicular distance from the axis, etc. If a single weight W is to be used to replace all these distributed forces, then for it to have the same effect we must have

$$W\overline{x} = \text{sum of all the } \delta wx \text{ terms, i.e. } \Sigma\delta wx$$

where $\overline{x}$ is the distance of W from the axis. The centre of gravity is thus a distance from the axis of

$$\overline{x} = \frac{\Sigma\delta wx}{W} \qquad [8]$$

For symmetrical homogeneous objects the centre of gravity is located at the geometrical centre. For composite objects it can be obtained by considering the object to be made up of the constituent parts, each having its weight acting through its centre of gravity. For objects containing holes or cut-outs, the object can be considered to be a composite object with the hole being treated as a negative mass.

When considering a section of a constant cross-section item such as a beam the term *centroid* is often used instead of centre of gravity. This is because, if the beam is of constant cross-section, we are concerned with locating the point in the cross-section which specifies the axis throughout the length of the beam along which the centre of gravity will lie. This point is termed the centroid and is the geometrical centre of the cross-section. If the weight per unit length of a constant cross-section beam is w then equation [8] becomes

$$\overline{x} = \frac{w\Sigma\delta Ax}{wA} = \frac{\Sigma\delta Ax}{A} \qquad [9]$$

Equation [9] gives the position of the centroid within the cross-section area. The product of an element of area δA and its distance x from some axis is known as the *first moment of area*.

Example

Determine the position of the centre of gravity relative to point X for the homogeneous section shown in figure 1.9.

Consider the section as being a composite homogeneous object made up of three parts A, B and C. Part C has an area 150×30 mm^2 with its centre of gravity 15 mm from X. Part B has an area of 100×70 mm^2 with its centre of gravity 80 mm from X. Part A has an area of 150×30 mm^2 with its centre of gravity 145 mm from X. The total section has an area of $2 \times 150 \times 30 + 100 \times 70$ mm^2. Thus if the weight per unit section area is w then equation [8] gives

$$\overline{x} = \frac{(150 \times 30 \times 15 + 100 \times 70 \times 80 + 150 \times 30 \times 145)w}{(2 \times 150 \times 30 + 100 \times 70)w}$$

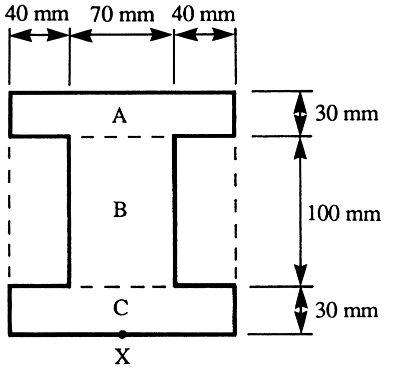


Fig. 1.9 Example.

Thus $\overline{x} = 80\,\text{mm}$.

Alternatively we could have considered the object as being a rectangular section 150 mm by 160 mm, centre of gravity 80 mm from X, with two rectangular pieces 100 mm by 40 mm, centres of gravity 80 mm from X, missing. Then

$$\overline{x} = \frac{(150 \times 160 \times 80 - 100 \times 40 \times 80 - 100 \times 40 \times 80)w}{(150 \times 160 - 100 \times 40 - 100 \times 40)w}$$

Thus, as before, $\overline{x} = 80\,\text{mm}$.

Alternatively we could have realised that the object is homogeneous and symmetrical and thus the centre of gravity would be at its geometrical centre which can be seen from inspection of the section to be 80 mm from X. The problem is really a determination of the position of the centroid of the section.

Example

Determine the position of the centroid for a triangular-shaped area.

Consider a small area segment of the triangle shown in figure 1.10. It has a width δy and length x, hence its area is $x\delta y$. By similar triangles

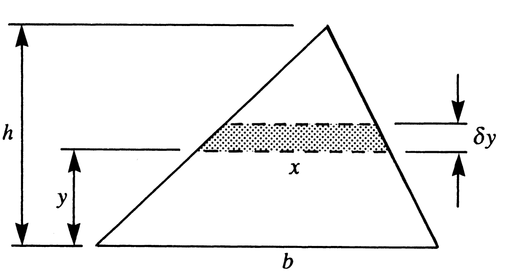


Fig. 1.10 Example.

$$\frac{x}{b} = \frac{h-y}{h}$$

Thus the area of the small segment can be written as

$$\delta A = \frac{b(h-y)\delta y}{h}$$

The sum of all the area moments is the sum of all the $y\delta A$ terms, i.e.

$$\text{sum of area moments} = \int_0^h \frac{yb(h-y)}{h}\mathrm{d}y = \frac{bh^2}{6}$$

The total area of the triangle is $bh/2$ and so the position of the centroid is given by equation [9] as

$$\bar{x} = \frac{bh^2/6}{bh/2} = \frac{h}{3}$$

1.4 Free-body diagrams

The term free-body diagram is used for a diagram showing all the external forces acting on a body. The important word is 'all', in that all the active forces and the reactive forces should be included so that the forces acting on the body can be considered in isolation from the surroundings. In the case of a composite body or a structure, free-body diagrams might be drawn for the body as a whole in isolation from its surroundings and for each component part considered in isolation from the rest of the body and the surroundings.

For example, consider the structure shown in figure 1.11(a). The free-body diagram for joint C will be all the forces acting on joint C and thus will be as shown in figure 1.11(b). Such a diagram for joint C enables us to determine the resultant force on joint C or whether joint C is in equilibrium (see section 1.5.3 for an example of this).

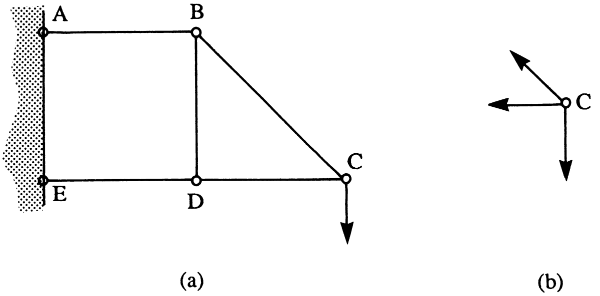


Fig. 1.11 (a) Structure, (b) free-body diagram for joint C.

1.5 Forces in equilibrium

Consider the conditions necessary for the equilibrium of a particle. A particle is an entity which is just a point, having no physical dimensions. Thus all the forces acting on a particle will be concurrent. According to Newton's first law, for equilibrium there must be no resultant force. This means that the vector sum of all the forces acting on the particle must be zero.

In many problems involving coplanar forces, it is convenient to resolve all the forces acting on a particle into two mutually perpendicular directions. Then for equilibrium the algebraic sum of the forces in each of the directions must be zero.

1.5.1 Triangle and polygon of forces

When two forces act at a point, for them to be in equilibrium the forces must be equal in size, opposite in direction and act in the same straight line.

When three forces act at a point, for them to be in equilibrium they must be all in the same plane and if the forces are represented in magnitude and direction by arrow-headed lines, then these lines when taken in the order of the forces must form a triangle. This is known as the *triangle of forces*.

When more than three forces act at a point, they will be in equilibrium if they all lie in the same plane and if the forces are represented in magnitude and direction by arrow-headed lines, then these lines when taken in the order of the forces must form a polygon, i.e. a closed shape. This is known as the *polygon of forces*.

1.5.2 Equilibrium of structures

Consider the conditions necessary for the equilibrium of a structure, i.e. a body having physical dimensions with forces being able to be applied at different points on the body. Figure 1.12(a) shows a simple body acted on by three forces of sizes F_1, F_2 and F_3 acting at different points on it in the directions shown in the figure. The force system can be simplified by inserting a pair of forces of F_3 and $-F_3$ at B (figure 1.12(b)). The force F_3 at C and $-F_3$ constitute a couple with a moment of $M_1 = F_3 d_1$, where d_1 is the perpendicular distance between the lines of action of the forces. We can thus replace these forces by the moment M_1 (figure 1.12(c)). We can perform a similar process with the force F_1, inserting additional forces of F_1 and $-F_1$ at B and then replacing the F_1 force at A and $-F_1$ force at B by a couple with a moment $M_2 = F_1 d_2$, where d_2 is the perpendicular distance between the lines of action of the forces.

Thus the force system in figure 1.12(a) can be replaced by the equivalent force system shown in figure 1.12(e). We now have three forces acting at the point B and two moments. For equilibrium the resultant force at B must be zero. This means that the algebraic sums of the vertical components and the

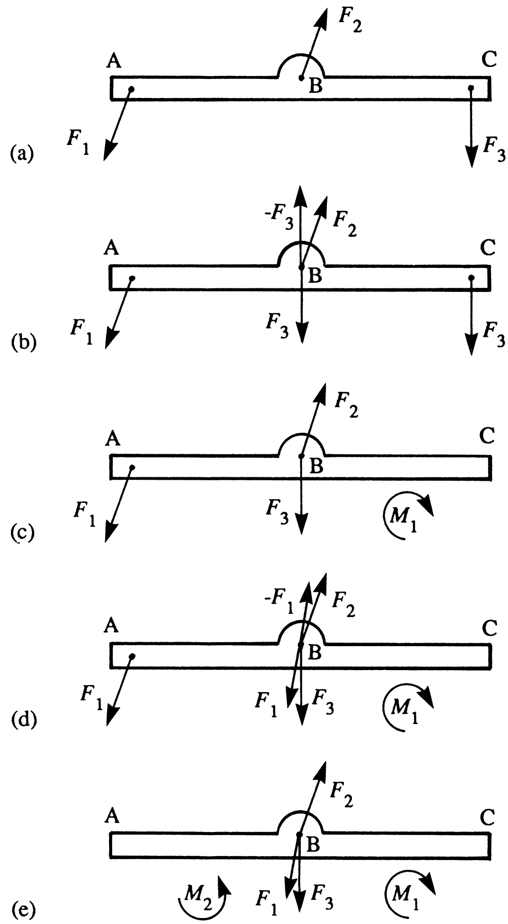


Fig. 1.12 Equilibrium.

horizontal components of the forces must both be zero. Also there must be no resultant turning moment. Both these statements when translated back to the original force system in figure 1.12(a) give the conditions for equilibrium as:

1 the algebraic sum of the vertical components of all the forces must be zero;
2 the algebraic sum of the horizontal components must be zero;
3 the moments about any axis must be zero.

1.5.3 Support reactions

Reaction forces occur at supports or points of contact between bodies (Newton's third law). Thus, for example, with a see-saw resting across a

(a)
(b)
(c)
(d)

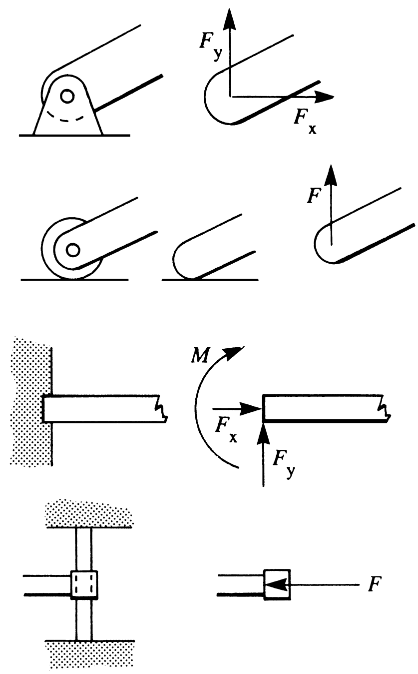


Fig. 1.13 Types of support, (a) pin connection, (b) roller or smooth contacting surface, (c) built-in or fixed support, (d) collar on smooth rod.

pivot, as in figure 1.6, the see-saw exerts a force on the pivot and the pivot exerts an opposing reaction on the see-saw. The types of reaction that occur at a support depend on the type of support concerned. A support develops a reactive force on the supported member if the support prevents a translational movement of the member, and it develops a reactive couple if it prevents rotation of the member.

Figure 1.13 shows some common forms of supports and the reactions on free-body diagrams of the members. At a pin connection, translation motion is prevented in any direction and so the reactive force can be considered to have two components at right angles to each other. The pin connection does however permit free rotation about the pin and so there is no reactive couple. A roller support, or the member resting on a smooth surface with no frictional forces involved, permits translational movement only in the direction at right angles to the surface and so there is just a reactive force at right angles to the surface. With a built-in or fixed support, both translational movement in any direction and rotational movement are not permitted. Thus there are two components of the reactive force and a reactive couple.

Example

Figure 1.14(a) shows a jib crane with a beam AB of length 5 m and mass

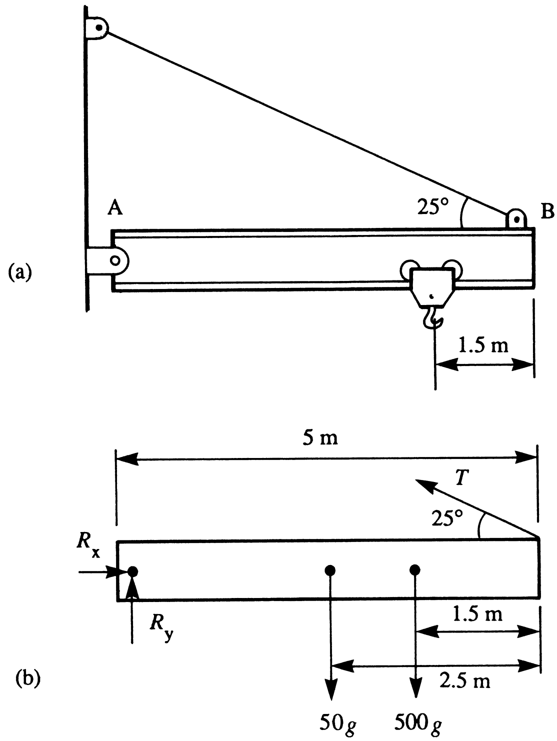


Fig. 1.14 Example.

10 kg/m, lifting a mass of 500 kg. Determine the tension in the supporting cable and the force on the pin at A.

Figure 1.14(b) shows the free-body diagram for the beam. The weight of the beam is considered to act at its centre point. Because there is a pin-joint at A there will be two reactive force components but no couple. For equilibrium, the horizontal components of all the forces will be zero, i.e.

$$R_x = T \cos 25° \qquad [10]$$

The vertical components will also be zero, i.e.

$$R_y + T \sin 25° = 50\,g + 500\,g \qquad [11]$$

The moments about some axis will be zero. Thus, taking moments about A, and assuming the pin is effectively on the end surface of the beam and the supporting cable is effectively connected to the centre line of the beam,

$$50\,g \times 2.5 + 500\,g \times 3.5 = T \times 5 \sin 25°$$

Hence $T = 8.7$ kN. Substituting this value in equation [10] gives $R_x = 7.9$ kN, and in equation [11] gives $R_y = 1.7$ kN. These two reactive force components can be combined to give a resultant reactive force of $\sqrt{(1.7^2 + 7.9^2)} = 8.1$ kN at an angle of $\tan^{-1}(1.7/7.9) = 12.1°$ to the beam.

Example

A homogeneous smooth ball of weight 500 N rests between two smooth surfaces at 30°, as shown in Fig. 1.15(a). What are the reactive forces at the points where the sphere touches each of the surfaces?

Figure 1.15 shows the system and the free-body diagram for the sphere. We have three forces acting at a point. Thus, for equilibrium the vertical components will be zero, i.e.

$$R_A \cos 60^\circ = 500$$

Hence $R_A = 1000$ N. The horizontal components will also be zero, thus

$$R_B = R_A \sin 60^\circ$$

Hence $R_B = 866$ N.

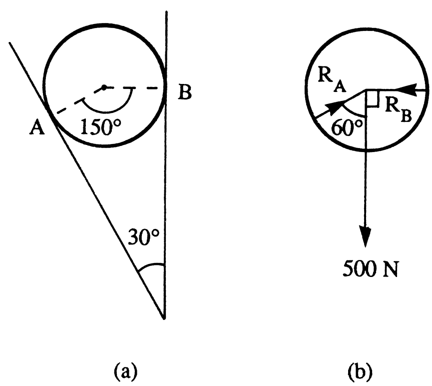


Fig. 1.15 Example.

1.5.4 Pin-jointed structures

The members of pin-jointed structures can only carry axial forces, no moments being transmitted at the joints. Thus it is possible to determine the equilibrium of the various parts of such structures by considering each joint individually as a free body. Such a method is called the *method of joints*. At any such joint the algebraic sum of the forces in the x-direction is zero and the sum in the y-direction is also zero. We could, alternatively, apply the triangle or polygon of forces at each joint.

The method of joints is useful when the forces in all the members of a structure are required. An alternative method, called the *method of sections*, is preferred when the forces are required in only one or a few members. The structure is considered to be cut into two pieces by a section through the member under consideration. Both parts of the structure can then be treated as structures in equilibrium and free-body diagrams drawn. The forces in the sectioned members can then be found by resolution of the forces or taking moments about a suitable point.

A member is said to be in tension if the forces applied to it have stretched it, and in compression if they have compressed it. If the forces were removed, then the member in tension would shorten, the member in compression would lengthen. Thus the internal forces in a tensile member are pulling on the pins at its ends while those in a compressive member are pushing on the pins. The convention is adopted of labelling the forces in a tensile member as being positive and those in a compressive member as negative. A member in tension is called a *tie*, a member in compression a *strut*.

Example

Determine, using the method of joints, the forces in each of the members of the plane pin-jointed bridge truss shown in figure 1.16 and the reactions at the supports, one being a pin joint and the other a roller.

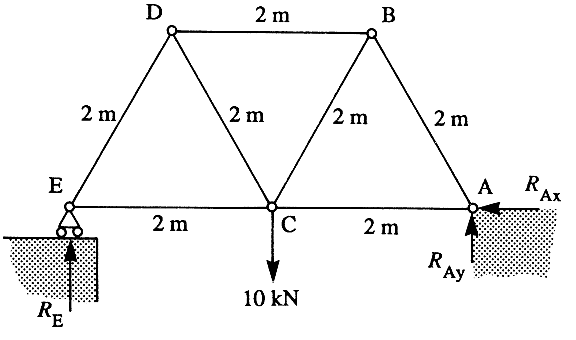


Fig. 1.16 Example.

Considering the free-body situation for the truss as an entity, the situation is essentially just a beam supported at each end and supporting a central load (figure 1.17(a)). Taking moments about end A of the beam gives

$$4R_E = 10 \times 2$$

Hence $R_E = 5$ kN. Since we must have $R_E + R_{Ay} = 10$, then $R_{Ay} = 5$ kN. The reaction R_{Ax} is zero, since there are no other horizontal forces acting on the beam.

Figure 1.17(b) shows the free-body diagram for the joint at E. For the vertical components of the forces

$$F_{ED} \sin 60° = 5$$

Hence $F_{ED} = 5.8$ kN. For the horizontal components

$$F_{ED} \cos 60° = F_{EC}$$

Hence $F_{EC} = 2.9$ kN.

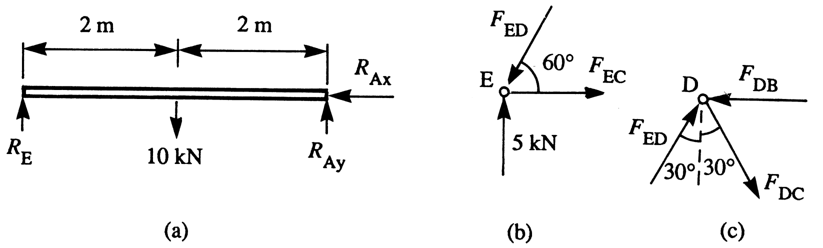


Fig. 1.17 Example.

Figure 1.17(c) shows the free-body diagram for the joint at D. For the vertical components of the forces

$$F_{ED}\cos 30° = F_{DC}\cos 30°$$

Thus $F_{DC} = 5.8\,\text{kN}$. For the horizontal components

$$F_{DB} = F_{DC}\cos 60° + F_{ED}\cos 60°$$

Thus $F_{DB} = 5.8\,\text{kN}$.

Each of the other joints can be considered in a similar way or we can recognise that the structure is symmetrical and so $F_{ED} = F_{BD} = F_{BA} = 5.8\,\text{kN}$, $F_{DC} = F_{BC} = 5.8\,\text{kN}$ and $F_{EC} = F_{AC} = 2.9\,\text{kN}$. Members ED, BD and BA are struts, DC, BC, EC and AC are ties.

Example

Determine, using the method of sections, the force acting in the member BD of figure 1.16.

Figure 1.18 shows the free-body diagrams for the structure in figure 1.16 sectioned through the member BD. In general, no more than three members with unknown forces should be cut by the section since only three equations can be generated, a moment equation and forces in the vertical and horizontal

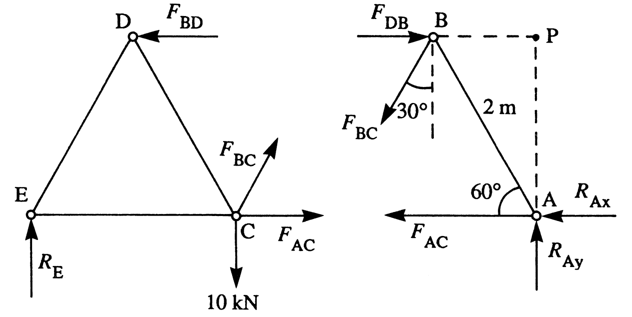


Fig. 1.18 Example.

directions, and so only three unknowns can be solved. The free-body diagrams for the two parts will then include just the external forces acting on each part and the internal forces of the cut members.

Consider the right-hand section. Taking moments about point P.

$$F_{AC} \times 2\sin 60° + R_{Ax} \times 2\sin 60° = F_{BC}\cos 30° \times 2\cos 60° \quad [12]$$

For the vertical components of the forces to be in equilibrium, then

$$R_{Ay} = F_{BC}\cos 30° \quad [13]$$

for the horizontal components to be in equilibrium

$$F_{DB} = F_{BC}\sin 30° + F_{AC} + R_{Ax} \quad [14]$$

The reaction forces can be found as in the previous example, thus $R_{Ax} = 0$ and $R_{Ay} = 5\,\text{kN}$. Thus equation [13] gives $F_{BC} = 5.77\,\text{kN}$. Equation [12] gives

$$1.732\,F_{AC} = 0.866\,F_{BC}$$

Thus $F_{AC} = 2.89\,\text{kN}$. Equation [14] then gives $F_{DB} = 5.77\,\text{kN}$.

Problems

(1) Determine the resultant force acting on the bolt shown in figure 1.19 when it is subject to the forces shown acting along the wires.

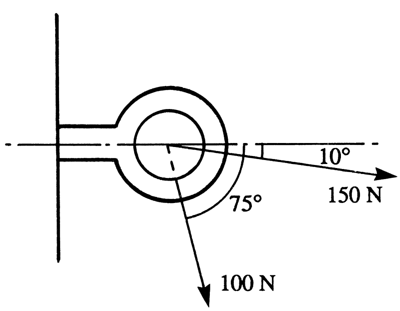


Fig. 1.19 Problem 1.

(2) Forces of 40 N, 80 N and 100 N act at a point A in the directions AB, AC and AD respectively, where ABCD is a square. What is the resultant force acting at A?

(3) A load of mass 12 kg is lifted by two ropes connected to the same point on the load and making angles of 20° and 30° on opposite sides of the vertical. What are the tensions in the ropes?

(4) A bolt is acted on by a force of 120 N acting along a wire at 30° to the surface to which the bolt is attached. What are the components