THERMOHYDRO-DYNAMIC INSTABILITY IN FLUID-FILM BEARINGS
THERMOHYDRODYNAMIC INSTABILITY IN FLUID-FILM BEARINGS

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Dedicated to:
Yingyu, Jonathan, and Joyce Wang, and Karen, Maxwell, Milton, and Mason Khonsari
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Preface

The importance of rotor-bearing system instability due to oil whirl associated with fluid-film journal bearings has been recognized since its discovery was first reported by Newkirk and Taylor in 1925. While still not well understood, it remains to be a crucially important design consideration in many types of modern rotating machinery.

In this book, we aim to establish the appropriate instability criteria for a rotor-bearing system consisting of a rotor supported by fluid-film journal bearings. In addition to the conventional stability analysis based on linearized stiffness and damping coefficients, the Hopf bifurcation theory (HBT) is employed to describe the nature of important system characteristics that govern the behavior of instability such as the hysteresis phenomenon, stability envelope, dip phenomenon, and subcritical/supercritical bifurcation.

In practice, rotor flexibility, manufacturing imperfections such as residual shaft unbalance, and service-related imperfections (e.g., bearing bushing wear, shaft thermal bow, sag, and any uneven wear or rust) always exist. Moreover, there are operating conditions that require special considerations such as turbulent flows. In this book, the questions on how these factors affect the stability of a rotor-bearing system are answered systematically.

Aside from the running speed, bearing load, and oil grade, there are other operating parameters of a rotor-bearing system affect the system stability. These include, for example, the oil inlet temperature, inlet pressure, and inlet position. The influences of these operating parameters on the stability of rotor-bearing system are also discussed and general design guidelines are provided at the end of each section.

The material presented in this book is largely derived from a series of published research articles by the authors. Here, the concepts are presented in a self-contained
and coherent manner for use not only by academic researchers but also by the practising mechanical engineers and vibration analysts. It is hoped that readers find the presented methodologies and design guidelines useful for treating any rotor-bearing system with any specific set of operating parameters in an effort to improve system performance and guard against failures.

J. K. Wang
M. M. Khonsari
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1

Fundamentals of Hydrodynamic Bearings

Hydrodynamic (fluid film) bearings are used extensively in different kinds of rotating machinery in the industry. Their performance is of utmost importance in chemical, petrochemical, automotive, power generation, oil and gas, aerospace turbo-machinery, and many other process industries around the globe.

Hydrodynamic bearings are generally classified into two broad categories: journal bearings (also called sleeve bearings) and thrust bearings (also called slider bearings). In this book, we exclusively focus our attention on journal bearings.

Figure 1.1a shows a schematic illustration of a rotor bearing system, which consists of a shaft with a central disk symmetrically supported by two identical journal bearings at both ends. Figure 1.1b shows the geometry and system coordinates of the journal rotating in one of the two identical journal bearings. To easily identify the bearing’s physical wedge effect and annotate the multiple parameters of a rotor bearing system, the clearance between the journal and the bearing bushing is exaggerated. $\theta$ is the circumferential coordinate starting from the line going through the centers of the bearing bushing and the rotor journal. $\phi$ is defined as the system attitude angle. $e$ is the rotor journal center eccentricity from the center of the bearing bushing. $W$ represents the vertical load imposed on the shaft and supported by the bearing. $p$ is the hydrodynamic pressure applied by the thin fluid film onto the journal surface. $f$ is the hydrodynamic force obtained by integrating the hydrodynamic pressure $p$ generated around the journal circumference.
In most cases, except in a floating ring configuration, the bearing bushing is fixed and the rotor rotates at the speed of $\omega$ inside the bearing bushing. In Figure 1.1, the journal center position $O_j$ is described as $(e, \phi)$ relative to the center $O_b$ of the fixed journal bearing.

Radial clearance $C$ is defined as the clearance between the bearing and the rotor journal (i.e., $C = R_b - R_j$, where $R_b$ is the inside radius of the bearing bushing and $R_j$ the outside radius of the journal bearing).

Figure 1.1  (a) Model of a rotor supported by two identical journal bearings; (b) geometry and system coordinates of a journal rotating in a fluid film journal bearing.
is the radius of the rotor journal). In terms of this radial clearance, the journal center eccentricity from the bearing center can be normalized as $\varepsilon = e/C$. The dimensionless parameter $\varepsilon$ is called eccentricity ratio. Due to the physical constraint of the bearing bushing, the rotor journal must be designed to operate inside of the bearing bushing, that is, $0 \leq \varepsilon \leq 1$. Therefore, the journal center position $O_j$ within the fluid film journal bearing can be redefined as $(Ce, \phi)$. When $\varepsilon = 0$, the center of the shaft $O_j$ coincides with the center of the bearing bushing $O_b$, and the fluid film bearing is theoretically incapable of generating hydrodynamic pressure by wedge effect and its corresponding load-carrying capacity is nil. When $\varepsilon = 1$, the shaft comes into intimate contact with the inner surface of the bushing, and depending on the operating speed, bearing failure becomes imminent due to the physical rubbing between the shaft and the bushing.

Based on the above physics, the important concept of rotor bearing clearance circle is introduced to easily describe the rotor journal position within any hydrodynamic journal bearing. Figure 1.2a shows the rotor bearing clearance circle in both polar and Cartesian coordinate systems. The radius of the clearance circle is equal to the radial clearance $C$ defined earlier and the center of the clearance circle is the bearing center $O_b$. The journal center $O_j$ is always either within or on the clearance circle. In other words, it will never go beyond the clearance circle due to the physical constraint of bearing bushing. Figure 1.2b shows the dimensionless rotor bearing clearance circle in both polar and Cartesian coordinate systems.

The fundamental equation that governs the pressure distribution in a hydrodynamic bearing was first introduced by Osborne Reynolds in 1886. In this chapter, we begin by describing the Reynolds equation and provide closed-form analytical solutions for two simplified extreme cases commonly known as the short and long bearing solutions. At the end, a brief discussion will be provided to address the numerical methods to solve the Reynolds equation for finite-length journal bearings.

### 1.1 Reynolds Equation

The Reynolds equation assuming that thin-film lubrication theory holds for a perfectly aligned journal bearing system lubricated with an incompressible Newtonian fluid is given by Equation 1.1.

$$\frac{\partial}{R^2 \partial \theta} \left( G_\theta \frac{h^3}{\mu} \frac{\partial p}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( G_z \frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = \frac{\omega \partial h}{2 \partial \theta} + \frac{\partial h}{\partial t} \tag{1.1}$$

where $z$ is the axial coordinate with the origin at the mid-width of the journal bearing.
Figure 1.2 (a) Dimensional and (b) dimensionless rotor bearing clearance circles
Detailed derivation of the Reynolds equation is available in tribology textbooks (see for example, Khonsari and Booser, 2008). In Equation 1.1, \( \theta \) is the circumferential coordinate and \( z \) is the axial coordinate perpendicular to the paper in Figure 1.1, \( R \) is the journal radius, \( \mu \) is the fluid viscosity, and the fluid film thickness \( h \) is given by Equation 1.2. The parameters \( G_\theta \) and \( G_z \) are the turbulent coefficients given by Equations 1.3 and 1.4 (See Hashimoto and Wada (1982) and Hashimoto et al. (1987)).

\[ h = C(1 + \varepsilon \cos \theta) \]  
\[ G_\theta = \frac{1}{12(a_\theta + b_\theta \varepsilon \cos \theta)} \]  
\[ G_z = \frac{1}{12(a_z + b_z \varepsilon \cos \theta)} \]

where \( a_\theta = 1 + 0.00069\text{Re}^{0.95}, \ a_z = 1 + 0.00069\text{Re}^{0.88}, \ b_\theta = 0.00066\text{Re}^{0.95}, \ b_z = 0.00061\text{Re}^{0.88}, \) and \( \text{Re} = \rho R \omega C/\mu \) is the Reynolds number. The turbulent coefficients \( G_\theta \) and \( G_z \) given by Equations 1.3 and 1.4 agree well with those given by Ng and Pan (1965).

On the left-hand side of Reynolds Equation 1.1, the first term \( \frac{\partial}{\partial \theta} \left( G_\theta h^3 \frac{\partial p}{\partial \theta} \right) \) is the pressure-induced flow in the circumferential direction while the second term \( \frac{\partial}{\partial z} \left( G_z h^3 \frac{\partial p}{\partial z} \right) \) is the pressure-induced flow in the axial direction. On the right-hand side, the first term \( \frac{\omega h \partial h}{2 \partial \theta} \) is the physical wedge effect in the circumferential direction between the bearing bushing and the rotor journal, and the second term \( \frac{\partial h}{\partial t} \) is the normal squeeze action of the fluid film in the radial direction.

Under the simplified isothermal assumption and neglecting the pressure influence on the fluid viscosity (i.e., constant fluid viscosity throughout the fluid film), the Reynolds Equation 1.1 can be simplified to

\[
\frac{\partial}{R^2 \partial \theta} \left( G_\theta h^3 \frac{\partial p}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( G_z h^3 \frac{\partial p}{\partial z} \right) = \frac{\omega \mu}{2} \frac{\partial h}{\partial \theta} + \mu \frac{\partial h}{\partial t}
\]  

(1.5)

For a steady-state fluid film, the fluid film thickness \( h \) is not a function of time, that is, \( \partial h/\partial t = 0 \). Then, the Reynolds equation can be further reduced to

\[
\frac{\partial}{R^2 \partial \theta} \left( G_\theta h^3 \frac{\partial p}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( G_z h^3 \frac{\partial p}{\partial z} \right) = \frac{\omega \mu}{2} \frac{\partial h}{\partial \theta}
\]  

(1.6)
while for laminar flow \((a_\theta = 1, a_z = 1, b_\theta = 0, b_z = 0)\), the simplified Reynolds equation (Eq. 1.1) can be rewritten as

\[
\frac{\partial}{R^2 \partial \theta} \left( \frac{h^3 \partial p}{\mu \partial \theta} \right) + \frac{\partial}{\partial z} \left( \frac{h^3 \partial p}{\mu \partial z} \right) = 6\omega \frac{\partial h}{\partial \theta} + 12 \frac{\partial h}{\partial t}
\]  

(1.7)

Therefore, for a rotor bearing system with steady-state and laminar fluid film, Equation 1.8 presents the further reduced but commonly used Reynolds equation.

\[
\frac{\partial}{R^2 \partial \theta} \left( \frac{h^3 \partial p}{\mu \partial \theta} \right) + \frac{\partial}{\partial z} \left( \frac{h^3 \partial p}{\mu \partial z} \right) = 6\omega \frac{\partial h}{\partial \theta}
\]  

(1.8)

Reynolds Equation 1.1 is a time-dependent second-order partial differential equation. To predict the pressure distribution through solving the Reynolds equation, in addition to the initial condition, four boundary conditions are needed in terms of the geometrical parameters \(\theta\) and \(z\). For steady-state Reynolds equations such as Equations 1.6 and 1.8, only the four boundary conditions are needed to define the pressure distribution.

### 1.1.1 Boundary Conditions for Reynolds Equation

In most hydrodynamic bearing applications, the fluid lubricant flows out of the bearing at ambient pressure. In other words, the gauge pressure at the geometrical boundary is equal to 0. Inside the bearings, since a conventional fluid lubricant cannot withstand negative pressure, it cavitates if the liquid pressure falls below the atmospheric pressure.

Depending on how to define and handle the cavitation region, there are three classical types of boundary conditions: full-Sommerfeld boundary conditions (cavitation is fully neglected and \(p = 0\) when \(\theta = 2\pi\)), half-Sommerfeld boundary conditions (also called Gümbel boundary conditions, i.e., \(p = 0\) when \(180^\circ \leq \theta \leq 360^\circ\)), and Reynolds boundary conditions (also called Swift–Stieber boundary conditions, i.e., both pressure and pressure gradient approach 0 where cavitation begins). All three classical types of boundary conditions assume that the fluid film starts at \(\theta = 0\). The detailed definitions of these boundary conditions will be introduced in the related chapters that follow. For further reference, Khonsari and Booser (2008) have given a complete summary of these boundary conditions on both their implications and limitations. In recent years, by combining the Reynolds boundary condition with some new experimental findings on when and how the fluid film starts, a more complete type of boundary conditions (Reynolds–Floberg–Jakobsson or RFJ boundary conditions) has been derived and applied successfully into different applications (Wang and Khonsari, 2008). The RFJ boundary conditions will be discussed in Section 1.3.