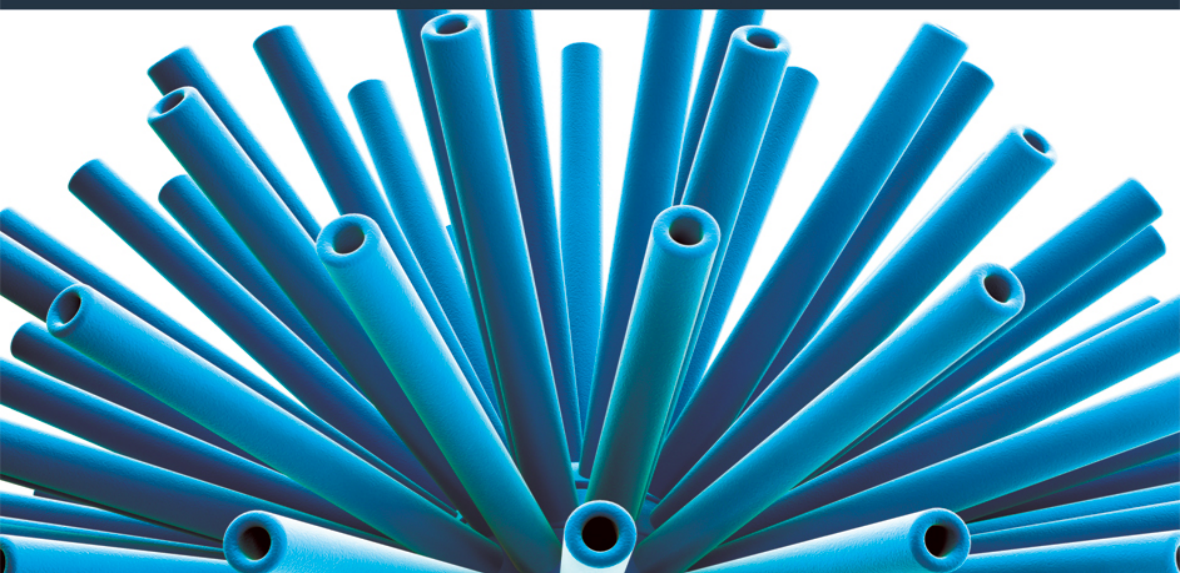


NANOSCIENCE AND NANOTECHNOLOGY SERIES

NANOTECHNOLOGIES FOR ENERGY RECOVERY SET



Volume 3

Electrostatic Kinetic Energy Harvesting

**Philippe Basset, Elena Blokhina
and Dimitri Galayko**

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Nanotechnologies for Energy Recovery Set

coordinated by
Pascal Maigné

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Contents

Preface	ix
Introduction: Background and Area of Application	xi
Chapter 1. Introduction to Electrostatic Kinetic Energy Harvesting	1
Chapter 2. Capacitive Transducers	7
2.1. Presentation of capacitive transducers	7
2.2. Electrical operation of a variable capacitor	11
2.3. Energy and force in capacitive transducers	12
2.3.1. Energy of a capacitor	12
2.3.2. Force of the capacitor	14
2.3.3. Capacitive transducers biased by an electret layer	17
2.4. Energy conversion with a capacitive transducer	20
2.5. Optimization of the operation of a capacitive transducer	21
2.6. Electromechanical coupling	23
2.7. Conclusions	24
2.8. Appendix: proof of formula [2.32] for the energy converted in a cycle	24
Chapter 3. Mechanical Aspects of Kinetic Energy Harvesters: Linear Resonators	27
3.1. Overview of mechanical forces and the resonator model	27
3.1.1. Linear resonator as the main model of the mechanical part	27
3.1.2. The nature and effect of the transducer force	30
3.1.3. Remarks on mechanical forces	33
3.2. Interaction of the harvester with the environment	36

3.2.1. Power balance of KEHs	36
3.2.2. Efficiency of KEHs	40
3.3. Natural dynamics of the linear resonator	42
3.3.1. Behavior of the resonator with no input	42
3.3.2. Energy relation for the resonator with no input	44
3.3.3. Forced oscillator and linear resonance	45
3.3.4. Periodic external vibrations	49
3.3.5. Energy relation for a forced resonator	50
3.4. The mechanical impedance	52
3.5. Concluding remarks	54
Chapter 4. Mechanical Aspects of Kinetic Energy Harvesters: Nonlinear Resonators	55
4.1. Nonlinear resonators with mechanically induced nonlinearities	55
4.1.1. Equation of the nonlinear resonator	55
4.1.2. Free oscillations of nonlinear resonator: qualitative description using potential wells	60
4.1.3. Free oscillations of nonlinear resonator: semi-analytical approach	62
4.1.4. Forced nonlinear resonator and nonlinear resonance	63
4.2. Review of other nonlinearities affecting the dynamics of the resonator: impact, velocity and frequency amplification and electrical softening	68
4.3. Concluding remarks: effectiveness of linear and nonlinear resonators	71
Chapter 5. Fundamental Effects of Nonlinearity	75
5.1. Fundamental nonlinear effects: anisochronous and anharmonic oscillations	75
5.2. Semi-analytical techniques for nonlinear resonators	79
5.2.1. Normalized form of nonlinear resonators	79
5.2.2. Anharmonic oscillations demonstrated by straightforward expansion	81
5.2.3. Anisochronous oscillations demonstrated by the LPM	84
5.2.4. Multiple scales method	88
5.2.5. Nonlinearity of a general form	91
5.3. Concluding remarks	95
Chapter 6. Nonlinear Resonance and its Application to Electrostatic Kinetic Energy Harvesters	97
6.1. Forced nonlinear resonator and nonlinear resonance	97
6.1.1. Analysis of forced oscillations using the multiple scales method	97
6.1.2. Forced oscillations with a general form of nonlinear force	102

6.2. Electromechanical analysis of an electrostatic kinetic energy harvester	105
6.2.1. Statement of the problem	105
6.2.2. Mathematical model of the constant charge circuit	106
6.2.3. Steady-state nonlinear oscillations	109
6.2.4. Dynamical effects and bifurcation behavior	113
6.2.5. Other conditioning circuits	115
6.3. Concluding remarks	119
Chapter 7. MEMS Device Engineering for e-KEH	121
7.1. Silicon-based MEMS fabrication technologies	121
7.1.1. Examples of bulk processes	122
7.1.2. Thin-film technology with sacrificial layer	123
7.2. Typical designs for the electrostatic transducer	124
7.2.1. Capacitive transducers with gap-closing electrode variation	125
7.2.2. Strategies on the stopper's location in gap-closing e-KEH	128
7.2.3. Capacitive transducers with overlapping electrode motion	130
7.3. e-KEHs with an electret layer	133
Chapter 8. Basic Conditioning Circuits for Capacitive Kinetic Energy Harvesters	135
8.1. Introduction	135
8.2. Overview of conditioning circuit for capacitive kinetic energy harvesting	136
8.3. Continuous conditioning circuit: generalities	138
8.3.1. Qualitative discussion on operation of the circuit	139
8.3.2. Analytical model in the electrical domain	140
8.4. Practical study of continuous conditioning circuits	141
8.4.1. Gap-closing transducer	141
8.4.2. Area overlap transducer	145
8.4.3. Simple conditioning circuit with diode rectifiers	148
8.5. Shortcomings of the elementary conditioning circuits: auto-increasing of the biasing	149
8.5.1. Appendix: listing of the Eldo netlist used to obtain the presented plots	152
Chapter 9. Circuits Implementing Triangular QV Cycles	155
9.1. Energy transfer in capacitive circuits	155
9.1.1. Energy exchange between two fixed capacitors	155
9.1.2. Case of a voltage source charging a capacitor	156
9.1.3. Inductive DC-DC converters	157
9.1.4. Use of a variable capacitor	161

9.2. Conditioning circuits implementing triangular QV cycles	163
9.2.1. Constant-voltage conditioning circuit	163
9.2.2. Constant-charge conditioning circuits	165
9.2.3. Analysis of the circuit implementing a constant-charge QV cycle	166
9.2.4. Practical implementation	169
9.3. Circuits implementing triangular QV cycles: conclusion	171
Chapter 10. Circuits Implementing Rectangular QV Cycles, Part I	173
10.1. Study of the rectangular QV cycle	173
10.2. Practical implementation of the charge pump	178
10.2.1. Evolution of the harvested energy	180
10.3. Shortcomings of the single charge pump and required improvements	182
10.3.1. Need for a flyback	182
10.3.2. Auto-increasing of the internal energy	183
10.4. Architectures of the charge pump with flyback	184
10.4.1. Resistive flyback	184
10.4.2. Inductive flyback	185
10.5. Conditioning circuits based on the Bennet's doubler	188
10.5.1. Introduction of the principle	188
10.5.2. Analysis of the Bennet's doubler conditioning circuit	191
10.5.3. Simulation of a Bennet's doubler	199
Chapter 11. Circuits Implementing Rectangular QV Cycles, Part II	203
11.1. Analysis of the half-wave rectifier with a transducer biased by an electret	203
11.2. Analysis of the full-wave diode rectifier with transducer biased by an electret	205
11.3. Dynamic behavior and electromechanical coupling of rectangular QV cycle conditioning circuits	210
11.4. Practical use of conditioning circuits with rectangular QV cycle	215
11.5. Conclusion on conditioning circuits for e-KEHs	216
Bibliography	217
Index	225

Preface

Miniaturization and efficiency are current trends in modern microelectronics. They will most likely continue to be for the next few years as we enter the age of the Internet of Things. Future technologies will rely on energy sources and, for this reason, energy harvesting is an extremely active, versatile and developing area that includes engineers and scientists from the field of electronics, microsystems and material science. While there are many different types of energy harvesting systems, we believe that electrostatic kinetic energy harvesters – devices that generate electricity from mechanical motion employing the capacitive (electrostatic) mechanism – are underrepresented in the literature. Although they are particularly compatible with microtechnology, they are also more complex compared to other kinetic energy harvesters.

This book is the summary of collaboration between three research groups from Université Paris-Sorbonne, University College of Dublin and Université Paris-Est on kinetic energy harvesters carried out between 2007 and 2015. Although this book is focused on energy harvesting employing the electrostatic transduction, we believe that this allowed us to write a complete and self-sufficient study. The book covers all aspects necessary to understand and design an efficient harvester, including linear and nonlinear resonators, electrostatic transaction principles, microfabrication processes and the design of conditioning electronics.

This book is primarily intended for Master's degree and PhD students who wish to discover the field of kinetic energy harvesting. It contains both chapters on fundamentals and chapters that present state-of-the-art results. We believe that some chapters will also be of interest to scientist and engineers involved in the design and development of kinetic energy harvesting.

Chapters 1, 2 and 7 discuss a capacitive energy harvester as a system, with additional chapters devoted to the operation in both the electrical and mechanical

domains. Chapters 3 through 6 discuss mechanical aspects of harvesters, and Chapters 8 through 11 are devoted to electronic conditioning circuitry. We have made a choice to present the material at a relatively high level of abstraction, limiting the discussion to the aspects that have the most impact on the global operation of the harvester. While this choice does not allow an extended discussion on some practical aspects of implementation, we are certain that this study provides a deep enough understanding of the role and function of each component in an energy harvester.

We sincerely thank all the PhD students and postdoctoral researchers who have contributed to our collaboration and research under our supervision: Mahmood Ayyaz Paracha, Andrii Dudka, Raphaël Guillemet, Peter Harte, Francesco Cottone, Eoghan O’Riordan, Armine Karami, Mohammed Bedier and Yingxian Lu. In particular, we are grateful to Eoghan O’Riordan, Peter Harte, Armine Karami, Yingxian Lu and Mohamed Bedier for helping prepare this book.

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January 2016

Introduction: Background and Area of Application

Sensing and data recording is appearing as the new paradigm of the 21st Century: smart cars, smart homes, smart buildings and cities are the objects of very extensive and active research. They all need a large number of communicating sensors (preferably wireless) for installation and operation cost reduction, or reliability improvement. The modern car has a few hundred sensors, and it is expected that the automotive industry will require 22 billion sensors by 2020. Today, most sensors are still powered using wires. Making them autonomous would ease the sensor installation and would, generally, offer a lighter and more reliable system. These ideas about smart environments benefit from the wireless sensor network (WSN) or the Internet of Thing (IoT) concepts. In WSNs, all the sensors are provided with an embedded energy source and an antenna to wirelessly transmit data measurements. This communication system usually takes place in a star network where each sensor communicates with a master node. A better implementation of the network can be arranged if each sensor communicates with the closest node in order to progressively propagate the measured information to reach the base station (Figure I.1). For the IoT, the main idea is that any item of daily life is able to communicate data through such a network.

For both the WSN and the IoT, independent and miniature power sources are usually preferable over wire powering. In the majority of cases, a battery is used, which may last from several days to several years, depending on its size and the application. However, there are applications where a battery is not suitable for some reasons: a harsh environment degrades the battery too fast, an inaccessible location makes the cost of the battery replacement too high, or, for example, a chemical battery may be incompatible with ecological requirements. In these cases, an appropriate solution would be to take energy from the ambient environment of the sensor. This is the modern concept of “energy harvesting”.

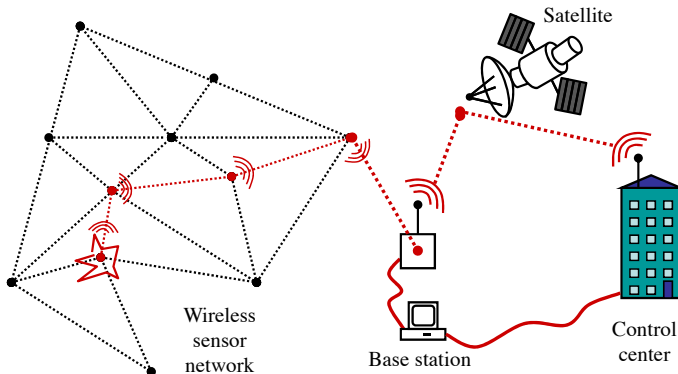


Figure I.1. *Principle of an ideal wireless sensor network (WSN) ecosystem: an event captured by an abandoned sensor node is transmitted through communications with other nodes nearby*

There are many ways to harvest ambient energy. The most mature and efficient technique is the use of photovoltaic cells converting ambient light into electricity. Another useful, but more complex technique, is the use of the Seebeck effect to obtain electrical power from a temperature gradient. However, in some cases where no light or temperature gradient are available in the environment, less conventional energy sources have to be envisaged. Kinetic energy, and more specifically vibrations, can provide great opportunities since these are present in the environment of many applications.

The idea of using kinetic energy to power a system is well known, such as with the dynamo invented in the late 19th Century. However, recent developments in material science and microelectronics allow us to envisage miniaturized systems combining a source of ambient mechanical energy and a sensor supplied by this source, possibly without any battery. Three families are generally distinguished for this conversion of mechanical energy into electricity, depending on the mode of transduction used: electromagnetic, piezoelectric and electrostatic. Each family has its advantages and drawbacks. In principle, we can say that electromagnetic transduction is the most effective in theory, but its performance drops as the device dimensions are scaled down. The piezoelectric transduction is efficient at all scales but requires constant stressing of an electroactive material, which raises reliability issues. Finally, the electrostatic transduction may be more complex to implement but is particularly suitable for miniaturized systems.

There are numerous applications for kinetic energy harvesting, even if there are few commercial products so far. For instance, mechanical structure health monitoring includes the monitoring of bridge oscillations, cracks in plane wings or changes in

train rail fixtures with the aim to avoid deadly accidents. Other common examples include mechanical vibrations of the heart (can power an implanted pacemaker) or tire pressure monitoring systems (can be fed with mechanical energy of a rotating wheel).

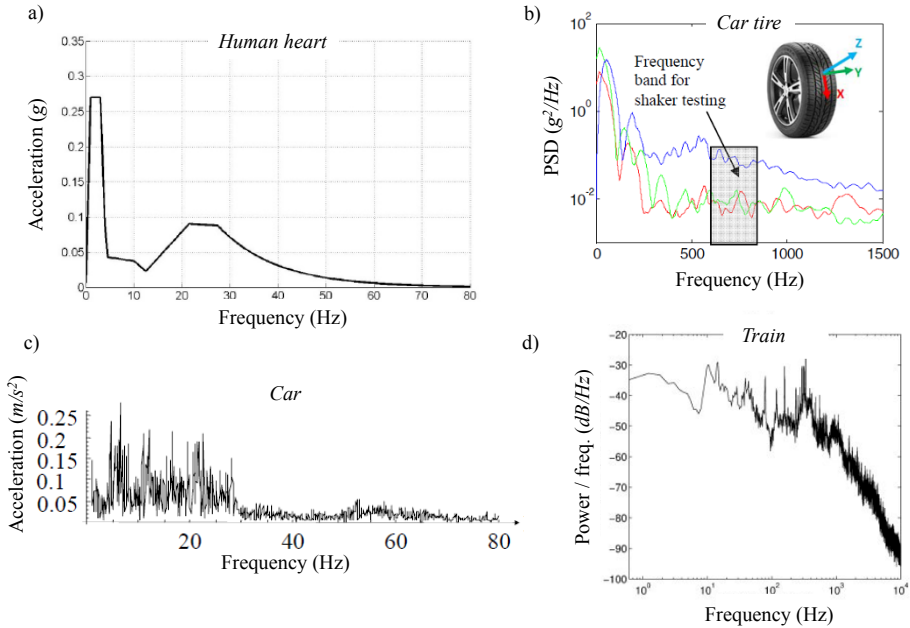


Figure I.2. Examples of different vibration spectra published in the scientific literature: a) typical shape of the acceleration spectrum in the right atrium of a human heart [DET 11], b) power spectral density of the acceleration measured on the inner surface of a car tire driving at 60 km/h [REN 13], c) acceleration spectrum of a car [DES 05b] and d) acceleration of a train [VOC 14]

Vibrations suitable for the generation of electricity can be of various forms. They can be periodic or non-periodic, harmonic or non-harmonic, regular or irregular, spread over a large frequency range or occurring at a single frequency. As shown in Figure I.2, each environment has a specific vibration frequency spectrum and it is very difficult, if not impossible, to design a generic KEH: each application needs a dedicated device in order to optimize the power yield.

Introduction to Electrostatic Kinetic Energy Harvesting

As a truly modern and complex system, the electrostatic kinetic energy harvester has several blocks, each performing its own function in the electrical or mechanical domains (Figure 1.1). We will briefly overview the role of the blocks in this introduction.

A large amount of energy can be produced by motion or vibrations in the mechanical domain. However it is not possible to use it directly in the electrical domain. Therefore, one would need a mechanism or a device that will transfer one form of energy (mechanical) to another (electrical). Such a device is called a *transducer*.

By definition, the transducer is a mechanism or a device that takes energy (power) in one form and converts it into another form. An electromechanical transducer is a very common and important type of transducers in modern microelectronics and microsystems: it converts mechanical energy into electrical form (and in some applications, vice versa, from the electrical to the mechanical domain). Thus, if one wishes to build a system that charges a storage capacitor in the electrical domain using the kinetic energy of some external motion in the mechanical domain, we should include an electromechanical transducer in such a system.

In the scheme of electrostatic kinetic energy harvesting shown in Figure 1.1, we use a variable capacitor as an electromechanical transducer. Let us consider a very simple case to understand its operation. Suppose that we have a parallel plate capacitor (Figure 1.2), with one of its two plates fixed and the other one movable. The distance

d between the plates is allowed to vary (due to any reason). The capacitance C_t of this capacitor depends on the distance d as follows:

$$C_t = \frac{\varepsilon_0 \varepsilon_r A}{d} \quad [1.1]$$

where ε_0 is the permittivity of free space, ε_r is the relative permittivity of the medium between the plates (usually air and so $\varepsilon_d \approx 1$) and A is the area of plates.

If the capacitor is charged to Q_t , the energy of the electric field stored in it is

$$W_t = \frac{Q_t^2}{2C_t} \quad [1.2]$$

Suppose now that we will manually move the movable plate farther from the fixed plate, keeping the charge Q_t constant. The new distance d_1 is greater than the original distance d : $d_1 > d$. The capacitance, according to equation [1.1], decreases and the stored energy, according to [1.2], increases.

In this example, we changed the energy stored in the electrical domain by manipulating a mechanical parameter (the distance d) of the transducer. This type of the electromechanical transducer is called *capacitive* (since it employs a capacitor) or *electrostatic*. There are other types of electromechanical transducers. Notably, electromagnetic and piezoelectric transducers are commonly used in kinetic energy harvesters, but they are out of the scope of this book.

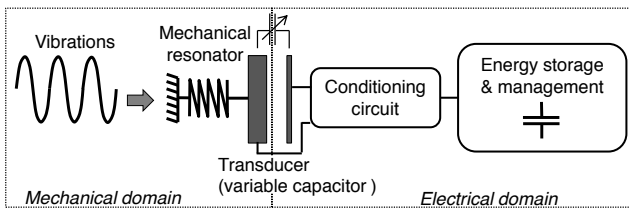


Figure 1.1. Generic high-level structure of an electrostatic kinetic energy harvester (KEH) that includes a resonator, a capacitive transducer and a conditioning circuit (the transducer couples the mechanical and the electrical domain)

We can reasonably assume that if we manage to attach one plate of such a capacitor to a vibrating or moving object, the external vibration will move the plate and change the distance d between the plates. This will affect the capacitance C_t and,

in turn, will change the electrical energy stored in the capacitor. The work required to change electrical energy will be *provided by external mechanical forces* in the form of vibrations or any kind of motion that possesses kinetic energy. Hence, it gives the name to the system – kinetic or vibration energy harvesting. Note that the capacitor must be charged (or we can say that a voltage must be applied to it). Otherwise, without a charge or voltage, its electrical energy is zero.

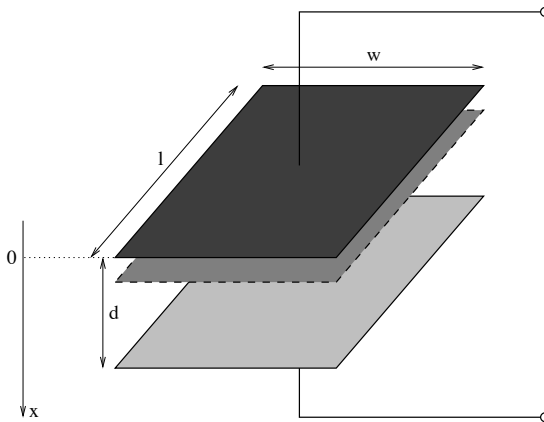


Figure 1.2. *Simplest electromechanical transducer in the form a variable parallel plates capacitor. The gap d of the capacitor can vary, which causes the variation of its capacitance*

Variable capacitors (and electromechanical transducers in general) are never attached directly to moving mechanical objects – this would be technologically impossible. There must be an intermediate system that will transfer external motion due to the environment to the motion of a capacitive transducer. This system will be a purely mechanical system as it involves the transformation of motion entirely in the mechanical domain.

How could we arrange a proper connection of a variable capacitor to a vibrating environment? Consider a system shown schematically in Figure 1.3. It has a simple parallel plate capacitor we just discussed, with one fixed electrode and one movable electrode. The movable electrode is suspended on a spring and also serves as a proof mass. The spring is attached to an enclosing frame. Now the frame is placed in the vibrating environment and experiences an acceleration due to external vibrations. Now, according to the laws of classical mechanics, the inertia of the mobile mass captures the energy from the external vibrations of the environment. The inertia generates the force $F_{ext}(t) = -mA_{ext}(t)$ in the mechanical domain where $A_{ext}(t)$ is the acceleration of the external vibrations. The inertia force causes the displacement of the proof mass with respect to the frame. Therefore, it also causes

the mechanical displacement of the electrode of the capacitive transducer that is required for energy conversion.

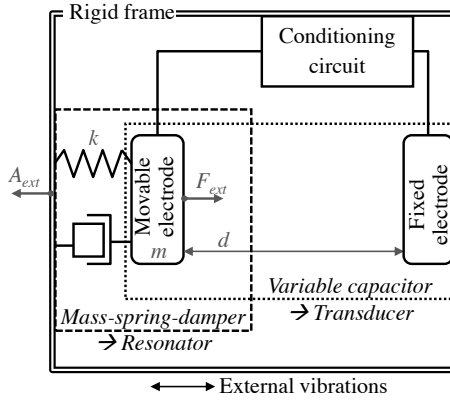


Figure 1.3. Schematic view of a KEH including a resonator, a transducer and a conditioning circuit

The mass and the spring form a *resonator*. There are always some losses due to dissipation (for example air damping) in a realistic mechanical system that we usually model as a damper. Therefore, we usually say that it is a mass–spring–damper system or a damped resonator. The aim of the resonator is to constrain the motion of the proof mass (electrode) and capture external vibrations in the most efficient way, usually through the phenomenon of *resonance*.

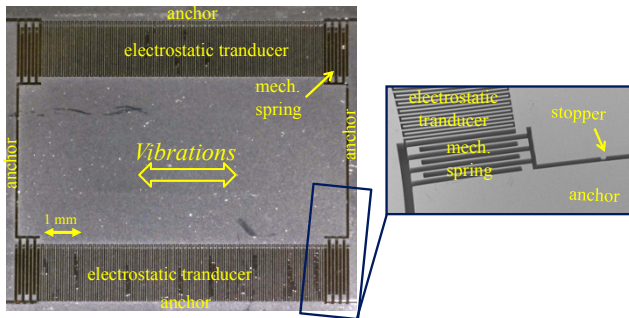


Figure 1.4. Photo of a MEMS resonator and an electrostatic transducer for an e-KEH described in [BAS 14]

Actual resonators do not look like the simplified structure as shown in Figure 1.3 with a proof mass on a spring. Realistic KEH resonators are distributed mechanical

structures suspended on elastic arms, sometimes of a very complex form. One example is shown in Figure 1.4. To describe vibrations in a resonator very accurately and precisely, we could model it using a partial differential equation or even a finite element based method. However, the 1D model shown in Figure 1.3 is a very useful approximation, simple and convenient for the use in analytical and semianalytic studies of KEHs. In order to use the 1D model, we provide the mass m of the resonator (proof mass) and its equivalent spring coefficient k [GRA 12, NAY 05].

In Figure 1.3, we show that the conducting proof mass is, at the same time, a part of the resonator and a part of the transducer. Thus, as seen from the figure, the variable capacitor couples the mechanical and the electrical domains. We mentioned that the variable capacitor must be charged (or a voltage must be applied to it) in order to obtain useful operation. Knowing the purpose of the resonator and the transducer, let us briefly describe the conversion of energy from the mechanical to the electrical domain. If the proof mass (movable electrode) is displaced by dx due to external vibrations, this causes a change dC_t in the capacitance. The energy of the transducer changes according to equation [1.2]:

$$W_t + dW_t = \frac{Q_t^2}{2(C_t + dC_t)} \approx \frac{Q_t^2}{2C_t} \left(1 - \frac{dC_t}{C_t}\right) \quad [1.3]$$

We will increase electrical energy (i.e. convert a portion of energy from the mechanical domain to the electrical domain) if $dW_t > 0$. Thus, dC_t must be negative, i.e. the capacitance C_t of the transducer must decrease. And vice versa, energy is converted from the electrical to the mechanical domain if $dW_t < 0$ and the capacitance increases, $dC_t > 0$. Therefore, energy conversion from the mechanical to the electrical domain occurs when the mobile electrode of a charged transducer moves in such a way that the capacitance of the transducer decreases [GAL 13b, DUD 14].

The transduction is obtained when an electrical force F_t generated by the transducer impedes the displacement of the resonator mass induced by external vibrations. For e-KEH, this force is the electrostatic force between the charged electrodes. The negative work of this force corresponds to the energy converted from the mechanical to the electrical domains. The generation of this force is controlled by the electrical state of the transducer (e.g. by the voltage across its terminals), which in turn is controlled by a *conditioning circuit*. The conditioning circuit operates entirely in the electrical domain. Later, the converted energy is managed, stored and supplied to the load.

In conclusion, in order to capture the motion of the environment, we use a mass–spring–damper system (resonator). The resonator drives the movable plate of a variable capacitor (transducer) changing its capacitance and therefore changing its

electrical energy. A conditioning circuit then controls the voltage applied to the transducer (or its charge) in order to ensure useful energy conversion. Converted energy is further managed and stored. The resonator operates in the mechanical domain while the conditioning circuit, the power management circuit and energy storage subsystem operate in the electrical domain. The variable capacitor (transducer) couples the mechanical and electrical domains.

Energy harvesting principles that have been briefly presented in this introductory chapter will be analyzed thoroughly in the following chapters of the book. Chapter 2 discusses capacitive transducers and electromechanical conversion achieved by these devices. Chapters 3 and 4 provide a discussion on the mechanical component of a kinetic energy harvester. Chapters 5 and 6 are focused on nonlinear effects arising in KEHs and provide methods for the analysis of nonlinear systems. Chapter 7 discusses the fabrication technology of a microelectromechanical system (MEMS) KEH employing electrostatic transduction. Chapters 8 through 11 explain the role of the conditioning circuit and discuss the architecture of main families of conditioning circuits.

Capacitive Transducers

Widespread use of capacitive transducers has become possible because of the miniaturization of electronic systems. Indeed, capacitive transducers are inefficient at macroscale, so capacitive transducers are mainly implemented with silicon-based microelectromechanical system (MEMS) technologies. Capacitive transducers are used either as sensors or actuators for the transfer of information between the mechanical and electrical domains.

For information processing, the functions describing relations between mechanical and electrical quantities should be linear. For that reason, the preferable mode of operation of a transducer is generally a small-signal mode, where the magnitude of dynamic quantities is small enough so that the nonlinear distortions are negligible.

However, the energy conversion sets very different constraints. Not only is the linearity of the conversion unimportant, but also in many cases nonlinear behavior of electrical and mechanical devices is unavoidable or even desirable. And since the energy conversion generally operates in a *large amplitude* mode, the linearized small-signal mode is not adequate for their behavior modeling. As we will see, a capacitive transducer is an intrinsically nonlinear device. Moreover, in the mode of the energy conversion, the capacitive transducer is associated with a conditioning electronic, which is also nonlinear.

This chapter presents basic information describing capacitive transducers used as converter of mechanical energy into electricity.

2.1. Presentation of capacitive transducers

A capacitive transducer is a physical capacitor whose geometry can change in time so as to affect the value of the capacitance. Although a capacitor can be of any

geometrical shape, in practice the most common is a parallel plate capacitor, whose geometry is given in Figure 2.1(a). Such a capacitor is constituted from a pair of parallel conductive planes (electrodes) spaced by some distance, called *gap*. The capacitance of such a device is:

$$C_t = \varepsilon_0 \varepsilon_r \frac{S}{d} \quad [2.1]$$

where d is the distance between the planes (the gap), S is the overlapping area of the planes, ε_0 is the permittivity of vacuum (a fundamental constant equal to 8.85×10^{-12} F/m) and ε_r is the dielectric constant of the material between the electrodes.

It is very important to remember that [2.1] is only valid if the linear dimensions of the overlapping area are large compared to the gap d . In this case, the essential part of the flux of the electric field is formed by the homogeneous electric field concentrated between the plates. Equation [2.1] neglects the *fringing* electric field that, however, becomes dominant if d is large compared to the dimensions of the overlapping area (see discussion on the area-overlap transducer later in this section).

The capacitance of a parallel plate capacitor is a function of three parameters, and a variation of any of them produces a change in the capacitance:

$$C_t(t) = \varepsilon_0 \varepsilon_r(t) \frac{S(t)}{d(t)}, \quad [2.2]$$

Most existing variable capacitors operate in air or in vacuum, so that $\varepsilon_r \approx 1$. However, there are exotic cases where the variation of the capacitance is produced by a motion of the dielectric material separating the electrodes (see Figure 2.1(b)), particularly in fluidic devices [BU 12]. A variable capacitor is usually obtained when one electrode of the capacitor moves with regard to the other. To simplify the analysis, it is usually considered that one electrode of the capacitor is fixed and the other moves. This is the most common configuration in energy harvesters, and will generally be assumed in this book, although there are many other applications of capacitive transducers where both electrodes are mobile [GAL 05].

In principle, the motion can be in any direction, but in the majority of capacitive transducers there are only two possible and exclusive kinds of motion: (1) electrodes move in their plane or (2) electrodes move along the axis normal to their planes. The choice of the motion mode is obtained by implementation of a particular geometry of capacitor, so that all undesirable directions of motion are blocked. Let us consider both cases:

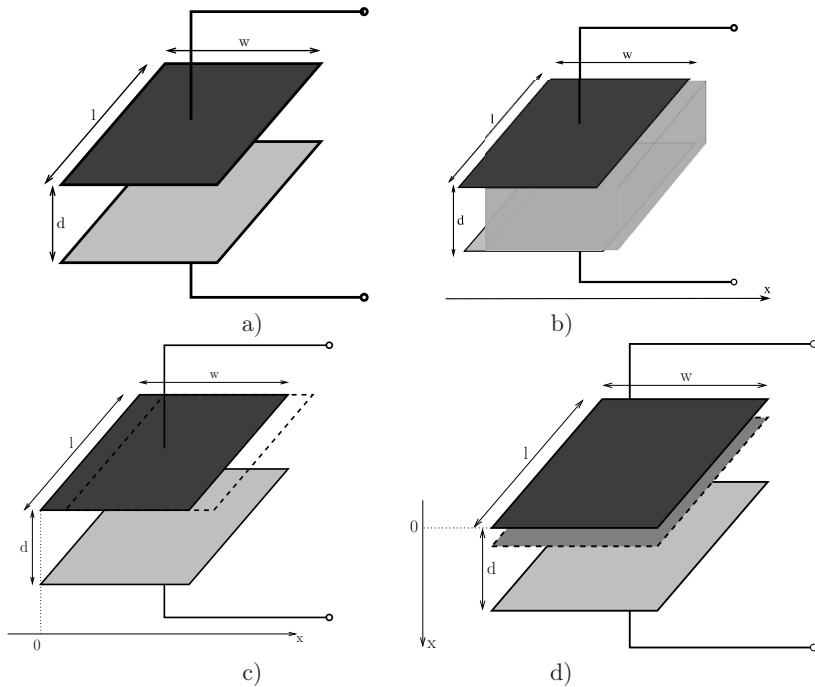


Figure 2.1. Diagram presenting the geometry of capacitive transducer: a) Basic geometry of a parallel plate capacitor, b) geometry of a capacitive transducer with a movable dielectric, c) geometry of a transducer with parallel motion of electrodes and d) geometry of a capacitive transducer with gap-closing geometry

1) *Parallel motion of electrodes*: In this case, the distance between the electrodes is kept constant, and the capacitance varies according to:

$$C_t(t) = \varepsilon_0 \frac{S(t)}{d} \quad [2.3]$$

Such a capacitor is called an *area overlap capacitor* (Figure 2.1(c)). The variation of the overlapping area can be related to the relative displacement x of the electrodes by a function $S(x)$, where the function $S(\cdot)$ depends on the geometry of the transducer. If the transducer electrodes have a rectangular shape and the mobile electrode moves along one of its sides, the function S is given by:

$$S(x) = l(x_0 \pm x), \quad [2.4]$$