MICROWAVE AMPLIFIER AND ACTIVE CIRCUIT DESIGN USING THE REAL FREQUENCY TECHNIQUE

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# Contents

<table>
<thead>
<tr>
<th>Foreword</th>
<th>vii</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preface</td>
<td>ix</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>xiii</td>
</tr>
</tbody>
</table>

## 1 Microwave Amplifier Fundamentals

1.1 Introduction 2
1.2 Scattering Parameters and Signal Flow Graphs 2
1.3 Reflection Coefficients 5
1.4 Gain Expressions 7
1.5 Stability 9
1.6 Noise 10
1.7 **ABCD** Matrix 14
   1.7.1 **ABCD** Matrix of a Series Impedance 14
   1.7.2 **ABCD** Matrix of a Parallel Admittance 15
   1.7.3 Input Impedance of Impedance Loaded Two-Port 15
   1.7.4 Input Admittance of Admittance Loaded Two-Port 16
   1.7.5 **ABCD** Matrix of the Cascade of Two Systems 16
   1.7.6 **ABCD** Matrix of the Parallel Connection of Two Systems 17
   1.7.7 **ABCD** Matrix of the Series Connection of Two Systems 17
   1.7.8 **ABCD** Matrix of Admittance Loaded Two-Port Connected in Parallel 17
   1.7.9 **ABCD** Matrix of Impedance Loaded Two-Port Connected in Series 19
   1.7.10 Conversion Between Scattering and **ABCD** Matrices 19
1.8 Distributed Network Elements 20
   1.8.1 Uniform Transmission Line 20
   1.8.2 Unit Element 21
   1.8.3 Input Impedance and Input Admittance 22
   1.8.4 Short-Circuited Stub Placed in Series 23
   1.8.5 Short-Circuited Stub Placed in Parallel 24
   1.8.6 Open-Circuited Stub Placed in Series 24
   1.8.7 Open-Circuited Stub Placed in Parallel 25
### Contents

1.8.8 Richard’s Transformation 25  
1.8.9 Kuroda Identities 28  

References 35  

#### 2 Introduction to the Real Frequency Technique: Multistage Lumped Amplifier Design 37  
2.1 Introduction 37  
2.2 Multistage Lumped Amplifier Representation 38  
2.3 Overview of the RFT 40  
2.4 Multistage Transducer Gain 41  
2.5 Multistage VSWR 43  
2.6 Optimization Process 44  
2.6.1 Single-Valued Error and Target Functions 44  
2.6.2 Levenberg–Marquardt–More Optimization 46  
2.7 Design Procedures 48  
2.8 Four-Stage Amplifier Design Example 49  
2.9 Transistor Feedback Block for Broadband Amplifiers 57  
2.9.1 Resistive Adaptation 57  
2.9.2 Resistive Feedback 57  
2.9.3 Reactive Feedback 57  
2.9.4 Transistor Feedback Block 58  
2.10 Realizations 59  
2.10.1 Three-Stage Hybrid Amplifier 59  
2.10.2 Two-Stage Monolithic Amplifier 62  
2.10.3 Single-Stage GaAs Technology Amplifier 64  

References 64  

#### 3 Multistage Distributed Amplifier Design 67  
3.1 Introduction 67  
3.2 Multistage Distributed Amplifier Representation 68  
3.3 Multistage Transducer Gain 70  
3.4 Multistage VSWR 71  
3.5 Multistage Noise Figure 73  
3.6 Optimization Process 74  
3.7 Transistor Bias Circuit Considerations 75  
3.8 Distributed Equalizer Synthesis 78  
3.8.1 Richard’s Theorem 78  
3.8.2 Stub Extraction 80  
3.8.3 Denormalization 82  
3.8.4 UE Impedances Too Low 83  
3.8.5 UE Impedances Too High 85  
3.9 Design Procedures 88  
3.10 Simulations and Realizations 92  
3.10.1 Three-Stage 2–8 GHz Distributed Amplifier 92  
3.10.2 Three-Stage 1.15–1.5 GHz Distributed Amplifier 94  
3.10.3 Three-Stage 1.15–1.5 GHz Distributed Amplifier (Noncommensurate) 94  
3.10.4 Three-Stage 5.925–6.425 GHz Hybrid Amplifier 96  

References 99
## Contents

### 4 Multistage Transimpedance Amplifiers

4.1 Introduction  
4.2 Multistage Transimpedance Amplifier Representation  
4.3 Extension to Distributed Equalizers  
4.4 Multistage Transimpedance Gain  
4.5 Multistage VSWR  
4.6 Optimization Process  
4.7 Design Procedures  
4.8 Noise Model of the Receiver Front End  
4.9 Two-Stage Transimpedance Amplifier Example  
References  

### 5 Multistage Lossy Distributed Amplifiers

5.1 Introduction  
5.2 Lossy Distributed Network  
5.3 Multistage Lossy Distributed Amplifier Representation  
5.4 Multistage Transducer Gain  
5.5 Multistage VSWR  
5.6 Optimization Process  
5.7 Synthesis of the Lossy Distributed Network  
5.8 Design Procedures  
5.9 Realizations  
5.9.1 Single-Stage Broadband Hybrid Realization  
5.9.2 Two-Stage Broadband Hybrid Realization  
References  

### 6 Multistage Power Amplifiers

6.1 Introduction  
6.2 Multistage Power Amplifier Representation  
6.3 Added Power Optimization  
6.3.1 Requirements for Maximum Added Power  
6.3.2 Two-Dimensional Interpolation  
6.4 Multistage Transducer Gain  
6.5 Multistage VSWR  
6.6 Optimization Process  
6.7 Design Procedures  
6.8 Realizations  
6.8.1 Realization of a One-Stage Power Amplifier  
6.8.2 Realization of a Three-Stages Power Amplifier  
6.9 Linear Power Amplifiers  
6.9.1 Theory  
6.9.2 Arborescent Structures  
6.9.3 Example of an Arborescent Linear Power Amplifier  
References  

### 7 Multistage Active Microwave Filters

7.1 Introduction  
7.2 Multistage Active Filter Representation  
References  

101

121

151

181
It has been a privilege for me to read through the manuscript of the *Microwave Amplifier and Active Circuit Design Using the Real Frequency Technique*. I found that the authors have been very thorough in putting together this outstanding book containing a unique blend of theory and practice. I sensed that the authors are very passionate about the design of microwave circuits, which is also evidenced from their other recent books.

The book provides an extensive use of an impedance matching methodology, known as the real frequency technique (RFT), in numerous applications. The topics treated include the RFT itself, design of a wide variety of multistage amplifiers, active filters, passive equalizers for radar pulse shaping, and antenna impedance matching applications. All topics are self-contained and also include practical aspects. Extensive analysis and optimization methods for these topics are discussed. The design techniques are well explained by means of solved examples.

The book is divided into nine chapters covering the basics of amplifiers, an overview of RFT, multistage distributed amplifiers, the use of RFT to design trans-impedance microwave amplifiers, the optimization of equalizers employing lossy distributed networks, the use of RFT to design multistage power amplifiers, the design of multistage active filters, the design of equalizers for radar pulse shaping, and antenna impedance matching. To solve impedance matching-related design problems using RFT from specifications to realization of the end product, the book provides a unique integration of analysis/optimization aspects. I found that the book is well balanced and treats the material in depth. With emphasis on theory, design, and practical aspects applied to numerous day-to-day applications, the book is suitable for graduate students, teachers, and design engineers.

Congratulations Profs. Jarry and Beneat on this excellent book that I am confident will be very well received in the RF and microwave community for many years to come.

Inder J. Bahl
Roanoke, VA
November 2015
Microwave and radio-frequency (RF) amplifiers play an important role in communication systems, and due to the proliferation of radar, satellite, and mobile wireless systems, there is a need for design methods that can satisfy the ever-increasing demand for accuracy, reliability, and fast development times. This book provides an original design technique for a wide variety of multi-stage microwave amplifiers and active filters, and passive equalizers for radar pulse shaping and antenna return loss applications. This technique is referred to as the real frequency technique (RFT). It has grown out of the authors own research and teaching and as such has a unity of methodology and style, essential for a smooth reading.

The book is intended for researchers and RF and microwave engineers but is also suitable for an advanced graduate course in the subject area. Furthermore, it is possible to choose material from the book to supplement traditional courses in microwave amplifier design.

Each chapter provides complete representation and characterization of the multistage or passive equalizer structure as well as the design methodology. We hope that this will provide the researcher with a set of approaches that he/she could use for current and future microwave amplifier designs. We also emphasize the practical nature of the subject by summarizing the design steps and giving numerous examples of amplifier realizations and measured responses so that RF and microwave engineers can have an appreciation of each amplifier in view of their needs. This approach, we believe, has produced a coherent, practical, and real-life treatment of the subject. The book is therefore theoretical but also experimental with over 18 microwave amplifier realizations.

The book is divided into nine chapters. In Chapter 1 recalls fundamental equations and definitions useful for understanding the design of the microwave amplifiers presented here.

Chapter 2 provides a complete description of the RFT as it is first used to design multistage lumped amplifiers. The chapter starts with the multistage amplifier representation made of field-effect transistors (FETs) and equalizers defined using their scattering matrices. In this introductory chapter, the equalizers are assumed to be realizable as lumped ladder structures with transmission zeroes at 0 or infinity. The equalizers are optimized to improve the overall transducer gain and voltage standing wave ratio (VSWR). The complete expressions for multistage transducer gain and VSWR are provided and further explained in Appendix A. The RFT uses a progressive optimization of the equalizers, leading to a small number of parameters to optimize simultaneously. This is a great
advantage of the RFT over typical design techniques that require simultaneous optimization of all unknown parameters.

The RFT uses a practical implementation of the Levenberg–Marquardt optimization method and is summarized in Chapter 2 and detailed in Appendix B. The optimization is performed over hundreds of frequency data points and is numerical in nature so that measured scattering parameters of the transistors can be used. The chapter then presents a complete step-by-step example of the design of a four-stage amplifier. Intermediary and final results are provided. A first extension of the technique is shown. It consists in using a transistor feedback circuit instead of the standalone transistor to increase the useful bandwidth of the amplifier. The chapter concludes with reporting two realizations designed using this technique in the range of 0–6.7 GHz, and a realization intended for 45 MHz to 65 GHz operation.

Chapter 3 extends the RFT to the case multistage distributed amplifiers. The equalizers are made of quarter-wave transmission lines. A new variable and formalism are introduced that are better suited for the distributed medium. The modified RFT is able to optimize and synthesize the transmission lines for transducer gain, VSWR, and also for multistage noise figure. This chapter shows how the effects of the bias circuit can be combined with the scattering parameters of the transistor to present a more accurate representation of the structure. A method is provided to solve the realization problem of having too high or too low characteristic impedance requirements. The chapter concludes with realizations of a 1.15–1.5 GHz, a 2–8 GHz, and a 5.925–6.425 GHz amplifier.

Chapter 4 presents the modifications to the RFT to design trans-impedance microwave amplifiers that are used for example in the case of photodiodes acting as high impedance current sources. It must provide a flat gain for a load charge impedance of 50 Ω. Contrary to the previous cases, the RFT performs progressive optimization of the equalizers from load to source (i.e., right to left). It uses admittance and impedance matrices rather than solely relying on scattering matrix representations. A technique based on peaking inductor is described, and it is sued to reduce the noise at the input of the amplifier, critical for the overall noise figure. Results for lumped and distributed examples from 3 to 7 GHz are given.

In Chapter 5, a method is presented for optimizing equalizers made of a lossy distributed network. Compared to the previous RFT for broadband multistage amplifiers, parallel resistors at the gate and drain of the transistors become part of the RFT optimization process. The chapter sets the foundations for this new medium and topology. An image network technique is used for defining the properties of the equalizers as well as the synthesis technique needed for transmission lines with lossy junctions. The chapter presents two hybrid realizations of broadband lossy distributed amplifiers. The first is optimized for gain and VSWR from 0.1 to 5 GHz and the second from 0.1 to 9 GHz.

In Chapter 6, it is shown how the RFT can be used in the case of multistage power amplifiers. It is shown how added power is optimized in addition to gain and VSWR. The technique requires interpolating large-signal scattering parameters in two dimensions: frequency and power. The optimization is done from load to source as was the case in Chapter 4. The technique is used in the realization of a single-stage 2.25 GHz power amplifier and in the case of a three-stage 2.245 GHz power amplifier. The chapter also provides a method and examples for the design of linear multistage power amplifiers using arborescent structures.

Chapter 7 describes how to use the RFT to design multistage active filters. In this chapter, the transducer gain and VSWR are optimized in the passband, while finite transmission zeros can be placed in the stopband to provide the desired selectivity of the filter. The technique also optimizes the group delay in the passband, an important feature for radar and digital communications. Three active filter examples are presented. The first is a low-pass case with two transmission zeros using two transistors and three equalizers operating in the 0.1–5 GHz band. The second is a band-pass
case with four transmission zeroes using one transistor and two equalizers with a 3–7 GHz pass band. The third is a band-pass case with two transmission zeroes with a 7.7–8.1 GHz pass band. This last case was realized in monolithic microwave integrated circuit (MMIC) technology.

Chapter 8 shows the flexibility of the RFT to solve a variety of microwave circuit design problems. In this chapter, the RFT is modified to optimize arbitrary responses using a structure made solely of passive equalizer. In this case, a transistor is shown to act as a “dummy passive device” when providing well-chosen scattering parameter data to represent it. Therefore with very little modification, the optimization capabilities of the traditional RFT can be used for passive structures. This chapter shows how the RFT no longer optimizes a constant transducer gain but rather optimizes a specific forward transmission curve for radar applications provided as a set of data points in a frequency band.

Two examples are provided. The first equalizer has an order six and used no transmission zeroes and the second equalizer has an order six and used two transmission zeroes to improve the optimization results in the stopband.

Finally, Chapter 9 describes a possible method for the synthesis of microwave antennas. The RFT is used to optimize the matching network placed between antenna and receiving/transmitting circuitry. The optimization focuses on improving the return loss in the receiving and transmitting bands.

The organization of the book is summarized in Figure 0.1. After a review chapter with important amplifier and microwave notations and equations, the central element is the RTF. The RFT is introduced in Chapter 2 for the design of multistage lumped amplifiers and is then modified to solve numerous problems in microwave amplifier design (Chapters 3–7), and adapted to solve more general microwave circuit design problems (Chapters 8 and 9).
It is believed that this book will provide the reader with an appreciation of the RFT as a tool that can be applied successfully in many disciplines.

We would like to acknowledge the contributions of our past and present research students whose collaboration has resulted in much of the material in the book. In particular, we would like to mention Professor Eric Kerherve; Associate Professors E.H. El Hendaoui, P. M. Martin, and A. Perennec; and Engineers L. Courcelle, M. Hazouard, M. Lecouve, and A. Olomo.

The book is based on the authors’ research under the sponsorship of France Telecommunications Research & Development (CNET), National Center of Spatial Studies (CNES), ALCATEL SPACE, PHILIPS OMMC, Commissariat Energie Atomique (CEA), PLESSEY (GB).

The work resulted in approximately nine contracts with different agencies and companies.

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Acknowledgments

The authors are deeply indebted to Dr Inder J. Bahl (USA) editor of the International Journal of RF and Microwave Computer-Aided Engineering from Wiley. This book could not have been written without his help, and he is acknowledged with gratitude.

Their sincere appreciation extends to the Publisher of Wiley-Interscience and all the staff at Wiley involved in this project for their professionalism and outstanding efforts.

Pierre Jarry thanks his colleagues at the University Bordeaux Sciences, including Professor Eric Kerherve and Assistant Professors Nathalie Deltimple and Jean-Marie Pham. He thanks as well Professor Yves Garault who introduced Microwave and Telecommunications at the University of Limoges. Finally, he expresses his deep appreciation to his wife Roselyne and to his son Jean-Pierre for their tolerance and support.

Jacques Beneat is very grateful to Norwich University, a place conducive to trying and succeeding in new endeavors. He particularly thanks the Senior Vice President for Academic Affairs, Dr. Guiyou Huang, and the Director of the David Crawford School of Engineering, Professor Steve Fitzhugh, for their encouragements in such a difficult enterprise.

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1

Microwave Amplifier Fundamentals

1.1 Introduction
1.2 Scattering Parameters and Signal Flow Graphs
1.3 Reflection Coefficients
1.4 Gain Expressions
1.5 Stability
1.6 Noise
1.7 ABCD Matrix
  1.7.1 ABCD Matrix of a Series Impedance
  1.7.2 ABCD Matrix of a Parallel Admittance
  1.7.3 Input Impedance of Impedance Loaded Two-Port
  1.7.4 Input Admittance of Admittance Loaded Two-Port
  1.7.5 ABCD Matrix of the Cascade of Two Systems
  1.7.6 ABCD Matrix of the Parallel Connection of Two Systems
  1.7.7 ABCD Matrix of the Series Connection of Two Systems
  1.7.8 ABCD Matrix of Admittance Loaded Two-Port Connected in Parallel
  1.7.9 ABCD Matrix of Impedance Loaded Two-Port Connected in Series
  1.7.10 Conversion Between Scattering and ABCD Matrices
1.8 Distributed Network Elements
  1.8.1 Uniform Transmission Line
  1.8.2 Unit Element
  1.8.3 Input Impedance and Input Admittance
  1.8.4 Short-Circuited Stub Placed in Series
  1.8.5 Short-Circuited Stub Placed in Parallel
  1.8.6 Open-Circuited Stub Placed in Series
  1.8.7 Open-Circuited Stub Placed in Parallel
  1.8.8 Richard’s Transformation
  1.8.9 Kuroda Identities
References
1.1 Introduction

In many high-speed applications, there is a need for microwave amplifier circuits. For example, satellite communications can be used when radio signals are blocked between two terrestrial transceiver stations as shown in Figure 1.1. The satellite then acts as a repeater, and the signal being repeated must be amplified before being sent back.

Important amplifier characteristics are center frequency and span of the pass band, gain, stability, input and output matching to the rest of the communication system, and noise figure [1].

At microwave frequencies, a common amplification component that has minimum noise is a field effect transistor (FET) as shown in Figure 1.2.

In this book, the FET will be typically modeled as a two-port network, where the input is on the gate and the output is on the drain. The source is mainly used for biasing of the transistor.

1.2 Scattering Parameters and Signal Flow Graphs

At high frequencies, voltages and currents are difficult to measure directly. However, scattering parameters determined from incident and reflected waves can be measured with resistive terminations. The scattering matrix of a two-port system provides relations between input and output reflected waves $b_1$ and $b_2$ and input and output incident waves $a_1$ and $a_2$ when the structure is

![Figure 1.1](image)  
**Figure 1.1** High-speed signals must be amplified in a satellite repeater.

![Figure 1.2](image)  
**Figure 1.2** A FET modeled as a two-port network.
terminated on its characteristic impedance $Z_0$ as shown in Figure 1.3. Typically the reference source and load $Z_0$ used in commercial network analyzers is 50 $\Omega$.

In the case of a two-port system, the equations relating incident and reflected waves and the scattering parameters are given by

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

(1.1)

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

(1.2)

The incident and reflected waves are related to the voltages and currents in Figure 1.3.

$$a_1 = \frac{V_1 + Z_0I_1}{2\sqrt{Z_0}}$$

(1.3)

$$a_2 = \frac{V_2 + Z_0I_2}{2\sqrt{Z_0}}$$

(1.4)

$$b_1 = \frac{V_1 - Z_0I_1}{2\sqrt{Z_0}}$$

(1.5)

$$b_2 = \frac{V_2 - Z_0I_2}{2\sqrt{Z_0}}$$

(1.6)

The parameter $S_{11}$ is the input reflection coefficient and is the ratio of input reflected wave over input incident wave when the output incident wave is equal to zero. The output incident wave $a_2$ is equal to zero when the output of the system is connected to the characteristic impedance $Z_0$:

$$S_{11} = \frac{b_1}{a_1} \bigg|_{a_2 = 0}$$

(1.7)

The parameter $S_{21}$ is the forward transmission coefficient and is the ratio of the output reflected wave over the input incident wave when the output incident wave is equal to zero:

$$S_{21} = \frac{b_2}{a_1} \bigg|_{a_2 = 0}$$

(1.8)
The parameter $S_{22}$ is the output reflection coefficient and is the ratio of the output reflected wave over the output incident wave when the input incident wave is equal to zero. The input incident wave $a_1$ is equal to zero when the input of the system is connected to the characteristic impedance $Z_0$:

$$S_{22} = \frac{b_2}{a_2} \bigg|_{a_1 = 0}$$ (1.9)

The parameter $S_{12}$ is the reverse transmission coefficient and is the ratio of the input reflected wave over the output incident wave when the input incident wave is equal to zero:

$$S_{12} = \frac{b_1}{a_2} \bigg|_{a_1 = 0}$$ (1.10)

The two-port network and scattering parameters can be modeled using the signal flow graph representation of Figure 1.4.

A useful tool when defining system gains using signal flow graphs is the Mason gain formula [2]. It provides the gain $T$ of a system between a source node and an output node:

$$T = \frac{\sum T_k \Delta_k}{\Delta}$$ (1.11)

with

$$\Delta = 1 - \sum L_i + \sum L_i L_j - \sum L_i L_j L_k \cdots$$

where

- $T_k$ is the gain of the $k$th forward path between the source node and the output node
- $\sum L_i$ is the sum of all individual loop gains
- $\sum L_i L_j$ is the sum of two loop gain products of any two nontouching loops
- $\sum L_i L_j L_k$ is the sum of three loop gain products of any three nontouching loops
- $\Delta_k$ is the part of $\Delta$ that does not touch the $k$th forward path

Figure 1.4 Signal flow graph representation of a two-port network.
1.3 Reflection Coefficients

As shown in Figure 1.5, the input reflection coefficient when the output is connected to characteristic impedance \( Z_0 \) can be expressed in terms of the input impedance \( Z_{IN} = V_1/I_1 \) by replacing \( a_1 \) and \( b_1 \) by their expressions in terms of voltages and currents:

\[
S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} = \frac{V_1 - Z_0 I_1}{2\sqrt{Z_0}} = \frac{Z_{IN} - Z_0}{2\sqrt{Z_0}} \frac{Z_{IN} - Z_0}{Z_{IN} + Z_0} = \frac{Z_{IN} - Z_0}{Z_{IN} + Z_0} (1,12)
\]

The input impedance can be expressed in terms of the input reflection coefficient by

\[
Z_{IN} = Z_0 \left( \frac{1 + S_{11}}{1 - S_{11}} \right) (1,13)
\]

Figure 1.6 defines additional reflection coefficients when the two-port is terminated on arbitrary loads \( Z_G \) and \( Z_L \).

Reflection coefficient of the source:

\[
\rho_G = \frac{a_1}{b_1} = \frac{Z_G - Z_0}{Z_G + Z_0} (1,14)
\]

Figure 1.5 Input reflection coefficient and input impedance.

Figure 1.6 Reflection coefficients of a two-port when terminated on arbitrary loads.
Reflection coefficient of the load:

\[ \rho_L = \frac{a_2}{b_2} = \frac{Z_L - Z_0}{Z_L + Z_0} \]  

(1.15)

Input reflection coefficient of the two-port when output loaded on \( \rho_L \):

\[ \rho_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\rho_L}{1 - S_{22}\rho_L} \]  

(1.16)

Output reflection coefficient of the two-port when input loaded on \( \rho_G \):

\[ \rho_{out} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12}S_{21}\rho_G}{1 - S_{11}\rho_G} \]  

(1.17)

For example, the expression of the input reflection coefficient when loaded on \( \rho_L \) is obtained by first using the general scattering parameter definition of the two-port:

\[ b_1 = S_{11}a_1 + S_{12}a_2 \]
\[ b_2 = S_{21}a_1 + S_{22}a_2 \]

Then, using the relation between incident and reflected waves \( a_2 = \rho_L b_2 \) gives

\[ b_1 = S_{11}a_1 + S_{12}\rho_L b_2 \]
\[ b_2 = S_{21}a_1 + S_{22}\rho_L b_2 \]

then from the second equation, \( b_2(1 - S_{22}\rho_L) = S_{21}a_1 \) and \( b_2 = \frac{S_{21}}{(1 - S_{22}\rho_L)}a_1 \), so that

\[ b_1 = S_{11}a_1 + S_{12}\rho_L \frac{S_{21}}{1 - S_{22}\rho_L}a_1 = \left( S_{11} + \frac{S_{21}S_{12}\rho_L}{1 - S_{22}\rho_L} \right)a_1 \]

and

\[ \rho_{in} = \frac{b_1}{a_1} = \frac{S_{11} + S_{21}S_{12}\rho_L}{1 - S_{22}\rho_L} \]

The voltage standing wave ratio (VSWR) is given in terms of a reflection coefficient \( \rho \) by

\[ \text{VSWR} = \frac{1 + |\rho|}{1 - |\rho|} \]  

(1.18)

The input VSWR for the two-port in Figure 1.6 is therefore

\[ \text{VSWR}_{in} = \frac{1 + |\rho_{in}|}{1 - |\rho_{in}|} \]  

(1.19)
The output VSWR for the two-port in Figure 1.6 is therefore

\[
\text{VSWR}_{\text{OUT}} = \frac{1 + |\rho_{\text{out}}|}{1 - |\rho_{\text{out}}|}
\]

(1.20)

### 1.4 Gain Expressions

Figure 1.7 shows the different reflection coefficients used to define various power gains.

The transducer power gain can be computed using the signal flow graph and the Mason gain formula as shown in Figure 1.8.

There is one forward path from node \( b_G \) to node \( b_2 \). The path gain of this path is \( T_1 = 1 \times S_{21} = S_{21} \).

There are three individual loops: \( \rho_G S_{11}, \rho_L S_{22} \), and \( \rho_G S_{21} \rho_L S_{12} \).

This gives

\[
\Delta = 1 - \sum L_i + \sum L_i L_j - \sum L_i L_j L_k \cdots = 1 - [S_{11} \rho_G + S_{22} \rho_L + S_{12} S_{21} \rho_G \rho_L] + [S_{11} \rho_G S_{22} \rho_L\rho_G \rho_L] - 0
\]

and

\[
T = \frac{\sum T_k \Delta_k}{\Delta} = \frac{T_1 \times (1 - 0)}{1 - S_{11} \rho_G - S_{22} \rho_L - S_{12} S_{21} \rho_G \rho_L + S_{11} \rho_G S_{22} \rho_L\rho_G \rho_L}
\]

![Figure 1.7 Gain definitions.](image1)

![Figure 1.8 Signal flow graph representation for defining the gain.](image2)
The transducer power gain is defined as

\[ G_T = \frac{\text{power delivered to the load}}{\text{maximum available power from the source}} \]

so that

\[ G_T = \frac{|b_2|^2 (1 - |\rho_L|^2)}{|b_G|^2} = |T|^2 \left(1 - |\rho_G|^2\right) \left(1 - |\rho_L|^2\right) \]

and

\[ G_T = \left(1 - |\rho_G|^2\right) \left(1 - |\rho_L|^2\right) \frac{|S_{21}|^2}{|1 - S_{11}\rho_G - S_{22}\rho_L + S_{11}S_{22}\rho_G\rho_L - S_{12}S_{21}\rho_G\rho_L|^2} \]

(1.21)

Note that \( G_T \) can also be written as

\[ G_T = \frac{\left(1 - |\rho_G|^2\right) \left(1 - |\rho_L|^2\right) |S_{21}|^2}{|(1 - S_{11}\rho_G)(1 - S_{22}\rho_L) - S_{12}S_{21}\rho_G\rho_L|^2} \]  

(1.22)

or as

\[ G_T = \frac{\left(1 - |\rho_G|^2\right) \left(1 - |\rho_L|^2\right) |S_{21}|^2}{|1 - \rho_G|\rho_L|^2 |1 - S_{22}\rho_L|^2} \]  

(1.23)

or as

\[ G_T = \frac{\left(1 - |\rho_G|^2\right) \left(1 - |\rho_L|^2\right) |S_{21}|^2}{|1 - S_{11}\rho_G|^2 |1 - \rho_L|\rho_{out}|^2} \]  

(1.24)

Note that when \( S_{12} = 0 \), the transducer power gain reduces to the unilateral transducer power gain \( G_{TU} \) given by [3]

\[ G_{TU} = G_G G_0 G_L \]

where

\[ G_G = \frac{1 - |\rho_G|^2}{|1 - S_{11}\rho_G|^2}, \quad G_0 = |S_{21}|^2 \quad \text{and} \quad G_L = \frac{1 - |\rho_L|^2}{|1 - S_{22}\rho_L|^2} \]
In this case, \( G_G \) represents the losses in the source, \( G_0 \) is the intrinsic gain, and \( G_L \) represents the losses in the load and can be modeled as in Figure 1.9.

The maximum unilateral gain occurs when there is perfect matching of source and load impedances. For maximum unilateral gain, one would match the source to the input of the transistor by making \( \rho_G = S_{11}^* \) and match the load to the output of the transistor by making \( \rho_L = S_{22}^* \).

The maximum unilateral gain \( G_{TU \, \text{MAX}} \) is then given by

\[
G_{TU \, \text{MAX}} = \frac{1}{1 - |S_{11}|^2} \frac{|S_{21}|^2}{1 - |S_{22}|^2}
\]  

(1.25)

The available power gain is defined as

\[
G_A = \frac{\text{maximum power the amplifier can deliver to the load}}{\text{maximum available power from the source}}
\]

and it is given by

\[
G_A = \frac{1 - |\rho_G|^2}{|1 - S_{11} \rho_G|^2 \left(1 - |\rho_{out}|^2\right)}
\]  

(1.26)

Note that when the input is perfectly matched then \( Z_G = Z_0 \) and \( \rho_G = 0 \) and

\[
\rho_{out} = S_{22} + \frac{S_{12} S_{21} \rho_G}{1 - S_{11} \rho_G} = S_{22}
\]

so that the available power gain becomes

\[
G_A = \frac{|S_{21}|^2}{1 - |S_{22}|^2}
\]  

(1.27)

1.5 Stability

The stability of the amplifier depends on the scattering parameters of the transistor but also on the matching networks and terminations [4].

For the two-port shown in Figure 1.10, \( \rho_{in} \) is the input reflection coefficient of the transistor when output loaded on \( \rho_L \) and \( \rho_{out} \) is the output reflection coefficient of the transistor when input
The system is said to be unconditionally stable if the amplitude of $\rho_{\text{in}}$ and $\rho_{\text{out}}$ are less than unity for all the real parts of load impedance $Z_L$ and source impedance $Z_G$:

$$\forall Z_L (\text{with } \text{Re}\{Z_L\} > 0); \ |\rho_{\text{in}}|^2 < 1$$

$$\forall Z_G (\text{with } \text{Re}\{Z_G\} > 0); \ |\rho_{\text{out}}|^2 < 1$$

It can be shown that for unconditional stability one must satisfy three conditions:

$$\begin{align*}
K &> 1 \\
B_1 &> 0 \\
B_2 &> 0
\end{align*}$$

(1.28)

where

$$K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}S_{21}|}$$

(1.29)

$$B_1 = 1 - |S_{22}|^2 - |S_{12}S_{21}|$$

(1.30)

$$B_2 = 1 - |S_{11}|^2 - |S_{12}S_{21}|$$

(1.31)

It is seen that these conditions only depend on the scattering parameters of the transistor. When these three conditions are met the amplifier can be connected to the loads without risk of becoming unstable and producing oscillations.

### 1.6 Noise

Figure 1.11 shows an active two-port between input impedance $Z_G$ and load impedance $Z_L$.

The noise in the amplifier can be characterized by the noise figure $F$ defined by

$$F = F_{\text{min}} + \frac{R_n}{G_G} |Y_G - Y_{G\text{min}}|^2$$

(1.32)

where

- $F_{\text{min}}$ is the minimum noise figure obtained when $Y_G = Y_{G\text{min}}$
- $R_n$ is the equivalent noise resistance of the active device
$Y_{G_{\min}}$ is the source admittance that makes the noise figure minimum.

$Y_G$ is the source admittance such that $Y_G = G_G + jB_G$.

The Section 1.7 consists of the rewritten noise figure formula in terms of reflection coefficients rather than in terms of admittances.

Referring to Figure 1.11, the reflection coefficient $\rho_G$ from the source admittance is given by

$$\rho_G = \frac{Y_0 - Y_G}{Y_0 + Y_G}$$

(1.33)

where $Y_0$ is the characteristic admittance used.

This gives the source admittance in terms of the source reflection coefficient such that

$$Y_G = \frac{1 - \rho_G}{1 + \rho_G} Y_0$$

(1.34)

In (1.32), taking $Y_G = Y_{G_{\min}}$ makes the noise figure become minimum. This translates to a reflection coefficient $\rho_{G_{\min}}$, where the noise figure is at a minimum such that

$$\rho_{G_{\min}} = \frac{Y_0 - Y_{G_{\min}}}{Y_0 + Y_{G_{\min}}}$$

(1.35)

$$Y_{G_{\min}} = \frac{1 - \rho_{G_{\min}}}{1 + \rho_{G_{\min}}} Y_0$$

(1.36)

Then, replacing $Y_{G_{\min}}$ and $Y_G$ by their expressions in terms of $\rho_G$ and $\rho_{G_{\min}}$ gives us

$$|Y_G - Y_{G_{\min}}|^2 = Y_0^2 \left| \frac{1 - \rho_G}{1 + \rho_G} - \frac{1 - \rho_{G_{\min}}}{1 + \rho_{G_{\min}}} \right|^2$$

$$= Y_0^2 \left| \frac{1 - \rho_G + \rho_{G_{\min}} - \rho_G \rho_{G_{\min}} - (1 - \rho_{G_{\min}} + \rho_G - \rho_G \rho_{G_{\min}})}{(1 + \rho_G)(1 + \rho_{G_{\min}})} \right|^2$$

and

$$|Y_G - Y_{G_{\min}}|^2 = Y_0^2 \left( \frac{2(\rho_{G_{\min}} - \rho_G)}{(1 + \rho_G)(1 + \rho_{G_{\min}})} \right)^2 = 4Y_0^2 \frac{|\rho_{G_{\min}} - \rho_G|^2}{|1 + \rho_G|^2 |1 + \rho_{G_{\min}}|^2}$$

Figure 1.11 Active device and source and load impedances.
So that the noise figure can first be expressed as

$$ F = F_{\text{min}} + \frac{4R_n}{G_G} Y_0^2 \frac{\left| \rho_G - \rho_{G_{\text{min}}} \right|^2}{\left| 1 + \rho_G \right|^2 \left| 1 + \rho_{G_{\text{min}}} \right|^2} $$

Then, one expresses $G_G$ as the real part of the source admittance in terms of reflection coefficients:

$$ G_G = \text{Re}\{Y_G\} = \frac{Y_G + Y_G^*}{2} = \frac{Y_0}{2} \left[ \frac{(1 - \rho_G) + (1 - \rho_G^*)}{1 + \rho_G} \right] $$

$$ = \frac{Y_0}{2} \left[ \frac{1 - \rho_G + \rho_G^* - \rho_G^2 + 1 + \rho_G - \rho_G^* - \rho_G^2}{1 + \rho_G} \right] $$

and

$$ G_G = \frac{Y_0}{1 - \left| \rho_G \right|^2} \frac{1}{1 + \rho_G} $$

so that

$$ F = F_{\text{min}} + \frac{4R_n}{Y_0} Y_0^2 \frac{1}{1 - \left| \rho_G \right|^2} \frac{\left| \rho_G - \rho_{G_{\text{min}}} \right|^2}{\left| 1 + \rho_G \right|^2 \left| 1 + \rho_{G_{\text{min}}} \right|^2} $$

and the noise figure is given in terms of the source reflection parameter $\rho_G$ and the optimum source reflection parameter $\rho_{G_{\text{min}}}$ by

$$ F = F_{\text{min}} + \frac{4R_n Y_0}{1 - \left| \rho_G \right|^2} \frac{\left| \rho_G - \rho_{G_{\text{min}}} \right|^2}{\left( 1 - \left| \rho_G \right|^2 \right) \left| 1 + \rho_{G_{\text{min}}} \right|^2} \quad (1.37) $$

Typically, the manufacturer provides the three parameters $\rho_{G_{\text{min}}}$, $F_{\text{min}}$, and $r_n = R_n/R_0$, the normalized equivalent noise resistance. Note that the reflection coefficient $\rho_{G_{\text{min}}}$ is complex and is often given as magnitude and phase. Note that these parameters do change with frequency so they are provided in table form.

Next, we provide the noise figure corresponding to a cascade of active devices. In Figure 1.12, a first active device is characterized by a gain $G_1$ and noise figure $F_1$, and a second active device is characterized by a gain $G_2$ and noise figure $F_2$.

![Figure 1.12](image.png)  
**Figure 1.12** Noise figure of the cascade of two active devices.
It can be shown [5] that the noise figure $F_2'$ corresponding to the cascade of the two systems is given by

$$F_2' = F_1 + \frac{1}{G_1}(F_2 - 1)$$

(1.38)

and the gain $G'_2$ of the cascaded system is given by

$$G'_2 = G_1 G_2$$

(1.39)

The system is then placed in cascade with a third active device characterized by a gain $G_3$ and noise figure $F_3$ as shown in Figure 1.13.

Then, the noise figure $F_3'$ corresponding to the cascade of the three systems is given by

$$F_3' = F_2' + \frac{1}{G_2}(F_3 - 1)$$

and the gain $G'_3$ of the cascaded system is given by

$$G'_3 = G'_2 G_3$$

We repeat this approach for the case of a cascade of $k$ active devices as shown in Figure 1.14.

In this case, the noise figure $F'_k$ corresponding to the cascade of the $k$ systems is given by

$$F'_k = F'_{k-1} + \frac{1}{G'_{k-1}}(F_k - 1)$$

(1.40)

and the gain $G'_k$ of the cascaded system is given by

$$G'_k = G'_{k-1} G_k$$

(1.41)