Hydraulic Modeling
Hydraulic Modeling

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"I appreciate the ability of creating analogies, which, if they are spirited and rational, take us beyond the limits of what the nature wished to open for us, allowing us to anticipate facts before we see them."

J. L. D'Alembert, "Encyclopedia", (1751)
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Hydraulic problems, put forward by practice, are very complicated, varied and numerous. This is because there are hydraulic phenomena in a wide range of processes, being typical for all sectors of public activity. Engineering hydraulics acts as the basis of not only water supply, navigation and hydroenergetics – but the sciences, for which water movement is the main subject of study. And for heat and nuclear power energetics, hydraulic problems, associated with energy transformation, are no less important than problems of thermodynamics and nuclear physics.

The oldest branch of energetics - wind energetics - operates with the laws of aerodynamics at air speeds much lesser than the sound velocity, when these laws are similar to the laws of hydraulics. The scales of modern energetics are so, that its interaction with the environment has not only local, but also global character. When assessing the impact of energetics on the environment by discovering restrictive measures for limitation of both negative and positive consequences of this impact, it is necessary to provide the forecast of different hydraulic phenomena. The complexity of hydraulic energy problems is determined by the fact that these problems have borderlines with many other sciences: dynamics of structures, thermodynamics, soil mechanics, meteorology, ecology, etc.
Hydraulics is going through a special period of its long history associated with the widespread introduction of numerical methods based on the use of modern computers. The use of computers has greatly expanded the range of hydraulic problems that can be solved by calculating without conduction of laboratory research, which has created the opportunity to solve complicated problems (especially lying on the borders with other sciences), which until recently were beyond the technical capabilities of hydraulics. However, increased implementation of numerical methods (numerical modeling) has not led to a decrease of significance in laboratory tests (physical modeling). This situation is connected with the incompleteness of mathematical models of hydraulic phenomena in many practically important cases. There are some technical reasons due to which the physical modeling has some advantages over the numerical modeling in solving concrete problems. Optimal hydraulic research combines both numerical and physical modeling. Hydraulic (physical) modeling, the wide use of which began many decades ago, continues to improve, as does its connection with the development of numerical modeling. Computers in the structure of physical experiments are used to improve techniques of laboratory research and for the purposes of control over them. Complication of hydraulic problems leads to the necessity of solving fundamental issues of modeling, which are not sufficiently developed. These issues include the problem of approximate hydraulic similarity. Ideas of similarity of natural phenomena, which are based, in particular, on physical modeling, go back to the days of Leonardo da Vinci. Galileo, Newton and Fourier. However, the present doctrine of similarity has been formed in the second half of the 19th and early 20th centuries. Among scientists, whose works have formed the basis of the similarity theory and the theory of dimensions (Bertrand, Buckingham, Federman), the most significant place is occupied by Russian specialists - T.A. Afanasieva-Erenfest and V.L. Kirpichev. Some general questions of the similarity theory have been developed by the USSR scientists - M.V. Kirpichev [63], L. I. Sedov [138] A. A. Guhman [45] and others. First works on hydraulic and related thermodynamic modeling have appeared in literature in the 30-ies of the 20th century. Along with the famous work of F. Eisner “Experimental hydraulic structures and open channels”, translated into Russian with additions of S. A. Egorov and B. A. Fidman, the original works of Soviet authors - S.V. Izbash [55], A.P. Zegzhda [52], P.K. Konakov [66], I. I. Levi [76] have been published.

Interest in hydraulic modeling has resulted in the appearance of a great number of works in this field [177, 179, 182, 204, 216, 218, 226, 227] during the last 40 years. In 1971, the authors published a book [91], which outlines the results related to general issues of hydraulic modeling. However, their
practical application was associated mainly with new specialized types of hydraulic models - pressure models of open channels. In 1984, the authors published the more complete work, which is now available in English, with some modifications and additions.

Along with general problems of the physical modeling theory set out in ch.1, close attention is paid throughout the book to the approximate similarity of modern hydrodynamics tendencies with physical modeling [28, 64, 91, 193, 205]. The book shows that conditions of the approximate similarity are significantly associated with the accepted mathematical description of modeled phenomenon, and that makes it necessary to address mathematical models of fluid flows (Chapter 2), precessing presentation of the approximate similarity theory (Chapter 3). Chapters 4 - 8 contain proposals for hydraulic modeling at the decision of specific problems for determination of the currents in natural and artificial channels parameters, study of the dam spillways, estimation of deformations of channels and processes in hydraulic tanks, hydraulic transport and slag-disposal systems, assessment of efficiency of heat exchangers and water-coolers of thermal and nuclear power plants, identification of consequences of the discharge of warm waters into natural water reservoirs and streams, and many other problems. Chapter 9, focused on the dynamic loads and structures reaction, has not been published in the edition of 1984.

Along with practical recommendations on modeling, the illustrations of general methods and approximate similarity methods are given in the book. Among the hydraulic modeling objects, those selected were from authors with the most experience.

For the most part, the materials used in the 1984 version of the book have accumulated from many years of work by two authors in the Scientific research sector of the “Hydroproject” Institute, which is named after S.Y. Zhuk. Formation of the book material is distributed among them as follows:

V.M. Lyatkher – Chapters 2, 4, 9, Sections 5.4, 6.1, 6.2; 8.2 and 8.3;
A.M. Proudovsky – Chapters 1, 3, 7, Sections 6.3, 6.4, 8.1, 8.4 and 8.5;
Chapter 5, except Section 5.4, was co-written by the authors.

The book was translated into English by V.M. Lyatkher due to the untimely death of A.M. Proudovsky. Additional references on literature in the given publication are executed as a separate list with the index A.
1.1 Modeling as the Method of Cognition

Modeling is one of the most important ways of cognition. When using this method the object of cognition (the original, nature, prototype) is realized through studying of its substitute, which is called the model. If the nature of $N$ is characterized by a number of $X_N \in N$, values, then when modeling such an object $M$ is created, the characteristics $X_M \in M$ of which are determined according to the nature values:

$$X_M \rightarrow X_N$$

The term “model” in the theory of knowledge is rather ambiguous, making classifying models complicated. For our purposes it is important to divide models as substantive and symbolic. The first of them are material objects, whose characteristics are somehow corresponding to the characteristics of the nature. The second represent notation symbols (diagrams, graphs, drawings, formulas, words, and so on). The most important type of symbolic models are the mathematical (logical-mathematical) models, realized by the means of mathematical language and logic.
Mathematical models of fluid flows include continuity equations, conservation of momentum equations, energy conservation and condition equations in one or another form. For multiphase fluids the equations are written for each phase with the inclusion of terms that characterize the interaction of phases. The equations contain some schematization of hydraulic phenomena. The quality of mathematical models depends on how successful the schematization is: some models can be better than others, but they all have some degree of uncertainty and inaccuracies, as they are based on a hypothesis. Quality assessment of a mathematical model (relevancy of underlying hypotheses) is the comparison of values, corresponding to the hypothesis used, with the measured characteristics of the real phenomena to which they relate. Of course, there are new challenges associated with the estimation of accuracy of measurement results, which usually cannot be executed without using the model of the phenomenon. Thus, the problem of estimation of the model is very complicated in principle.

Formulation of the mathematical model does not exhaust the modeling process. To predict the phenomenon it is necessary to establish a correspondence between the values \( X''_{N_o} \), characterizing the problem specification and values of \( X'_{N_o} \), forming its solution. Such a correspondence is set up by a certain operator \( O \):

\[
X'_{N_o} = O(X''_{N_o}) \tag{1.1}
\]

The operation \( O \) is executed using different computational tools. The way of realization of the operator is the main characteristic for classification of modeling in engineering. On this basis it is possible to allocate the first two large classes.

Modeling of the first class combines the cases in which the operator is represented in the form of an assistant operators function:

\[
0 = \Phi(0_1, 0_2, \ldots, 0_n) \tag{1.2}
\]

The solution is resulted from consecutive impact of operators from the set \( \{O_i\} \) on basic values and the intermediate results according to plan, designed in accordance with the type of the \( \Phi \) function. In this case we speak of a computational process. The order of operations, providing representation of the source data in the solution of this problem is called the algorithm of its solution. Modeling, using computational process can be called numerical modeling.

The principal place among computing devices, used in modeling and belonging to this class, is occupied by digital electronic computing
machines (computers). It is easy to notice that the operations carried out by a computer at “manual” solving of the physical problem, are a particular case of numerical modeling.

The second class modeling uses methods and tools, implying the direct, carried out in one time representation of quantities included in the problem specification, its solution. These means include analog devices, and each of them represents a physical model of a particular class of mathematical problems. This model allows carrying out direct measurement of the considered characteristics, corresponding to these or that basic values. Modeling using analog means is called analog modeling.

The process of analog modeling of a phenomenon can be described by the following scheme:

\[ \text{phenomenon (nature)} \rightarrow \text{mathematical model (system of mathematical equations)} \rightarrow \text{physical model (analog medium)} \rightarrow \text{nature} \]

The physical model included in this scheme is commonly called the phenomenon model. It is clear that this phenomenon model corresponds to the studied phenomenon so its mathematical model is accurate.

Thus, in case of analog modeling, the physical model and nature are related through formally similar mathematical model. Here phenomena in kind and on the model may have different physical nature in general case. The fact that various phenomena have the same mathematical description is not associated with the random formal analogy of the mathematical apparatus. It is concerned with the unity of regularities of processes of different kinds of material motion.

The models representing the phenomenon of the same kind show characteristics which you want to predict, and constitute the special type of analog means. With the understanding that further such analog devices shall be called physical models, unlike the analogs in those cases, the nature and the model represent heterogeneous phenomena.

The main difference of the physical model and the analogous is that the physical model’s natural characteristics correspond to the homogeneous (one-nominal) characteristics (speed - the speed, pressure-pressure, etc.), and the analog portion of them (or all) can comply with heterogeneous characteristics. In this sense, the flow of fluid may be physical model (routine hydraulic and aerodynamic modeling) and analog (gas hydraulic, gravity-elastic analogy).

A physical model of the hydraulic phenomenon is usually called a hydraulic model and the modeling with the use of such models – hydraulic
modeling. This definition is inaccurate, as any hydraulic modeling phenomena refers to hydraulics and can be called hydraulic. Here, however, the author decided not to break the tradition and, moreover, this term in the title of the book.

1.2 Hydraulic and Numerical Modeling

Hydraulic and numerical modeling of hydraulic phenomena shall be compared now. If hydraulic modeling is a traditional method of hydraulic engineering, successfully used in the last century, numerical modeling with the use of computers - is the latest achievement. The range of problems solved by means of numerical modeling is constantly expanding. It opens new perspectives in solving complex hydraulic problems. Optimal for the development of applied hydromechanics is to use the advantages of hydraulic and numerical modeling in combination.

Between hydraulic and numerical modeling has a great community, namely, that, first, they are based on a mathematical model of the phenomenon, reflecting the most significant side of the studied object, and, secondly, when modeling the object significantly schematized. Differences in hydraulic and numerical modeling can be caused by the following circumstances. Hydraulic model is a highly specialized tool, while computers can be a solution for the most diverse problems. The only condition is the possibility of receiving the decision, in their algorithmic solvability of a problem on the selected set of operators \( \{O_i\} \). Modern computers have successful from this point of view, the set of elementary operators.

The main advantage of hydraulic models is that with their help it is possible to find the solution of problems even in cases where the mathematical model of the phenomenon are not complete and take advantage of the numerical modeling impossible. As is known, up to now there is not a closed system of equations describing the averaged velocity field and pressure turbulent flow. However, if we assume that these equations have the same form for nature and models, it is possible to determine the conditions of conversion of measurement results on models in nature (see Section 1.3).

Another advantage of hydraulic models is the relative simplicity of the complex conditions of the solution uniqueness. Uniqueness conditions (first of all the geometric shape of the boundaries) for natural flows are especially complicated. To set the channel form on a computer, you need to put into it a lot of figures characterizing at least from the bottom to the length and width of the stream. In addition, you need to enter the
operation of the “smoothing” of these data. On hydraulic model channel form a relatively simple set when building the model.

The choice of method of modeling in each case is determined by many circumstances, among which are the paramount principle limits the possibilities of its use. For hydraulic modeling the main of such restrictions is the incompatibility of the defining criteria of similarity (see Section 3.1), and for the numerical modeling is not a closed system of equations that constitute a mathematical model of the phenomenon. Along with the principal constraints exist practical ones. In the case of hydraulic modeling these include limited laboratory resources (size of experimental facilities, pumping capacity, availability of materials and so on), which does not allow fulfillment of the similarity conditions. And in the case of numerical modeling constraints include characteristics of computing machinery (storage capacity, processing speed) that are inadequate to the problem set. These factors should be taken into account such requirements as specified accuracy of the research, the simplicity of its implementation, the required time and cost, visualization of results, the possibility of refining the methodology, taking into consideration the field data, etc.

Account of these circumstances provides an opportunity to highlight areas of pre-emptive use of modeling techniques, what made, for example, in [181]. The share of numerical modeling, at present, it is advisable to refer mainly to the solution of one-dimensional and two-dimensional problems, with relatively simple boundary conditions and the great length of the studied objects, and also problems of filtration in a porous material. Primary areas of using hydraulic modeling are local spatial problems and problems related to relatively poorly studied hydraulic phenomena.

But within the studied object, you can select different areas to explore appropriate to apply different methods of modeling. In some cases, complex real-world phenomenon can be split into simpler phenomena, modeling, which is advisable to be implemented separately in the appropriate way. Therefore, when solving part of the problems, it is rational to implement both numerical and hydraulic modeling simultaneously, and that is used in the idea of “hybrid” models.

In fact, hybrid modeling includes all the cases of implementation of different methods of modeling in one study. However, it can be realized in another way. Segmentation of large areas of currents on extensive areas, suitable for one-dimensional or two-dimensional schematization and relatively small areas of complex spatial flows (areas of facilities location, water discharge and intakes and so on) has become routine in practice of recent years. The first of them are studied using numerical modeling, the second of them - on hydraulic model. The models relation is realized through conditions on the
boundaries of an area. This research was conducted in the time scale of a hydraulic model, and computers also provide the hydraulic model control (setting of conditions on its boundaries), collection and processing of experimental data.

Great achievements of recent years in the field of numerical modeling, success in increasing the capabilities of computer technology and the reduction of its cost, it would seem, should have led to the displacement of the subject modeling of the process of research. However, the current practice of hydraulic research indicates that in our days the physical modeling has not lost its significance. The international community of technicians in the hydraulics sphere clearly formed opinion about the need for complex problems using both these methods of modeling within one study [19a and 90a].

The earliest proposal on the concurrent implementation of physical and numerical modeling in hydraulics was the proposal to divide the considered object (flow) on areas, in studying of which different modeling techniques are used [99a]. Here the broader area is investigated using numerical modeling, and an area, directly containing the relatively complex phenomenon, forecasting of which can be difficult in case of numerical modeling, is reproduced on the physical model. Conditions of uniqueness for the physical model are determined by numerical modeling. Such organization of studies was named the “hybrid modeling.”

In our practice successful use of numerical modeling for determination of boundary conditions of the physical model with an abrupt limitation of possibility of appointment of the dimensions of this model refers to the study of the water intake conditions to the HPP water inlets from a deep reservoir. The results of this study are shown in Figure 1.1. Characterization of the flow on the major part of the considered area of the reservoir was executed by the means of numerical modeling. However used, the numerical model is no assessment of funnel development ahead of the HPP underground water intakes. This problem is solved on the physical model.

Figure 1.2 represents an example the research scheme of distribution of thermal pollution from the NPP, located on the coastal area in the presence of tidal and wind currents [195], in which the described above technique is used.

The separate implementation of hydraulic and numerical modeling, where the whole process of research should be related to hybrid modeling, is also possible. Such a case occurs when using a numerical model are determined boundary conditions that are then played on a hydraulic model of steady flow. Non-simultaneous use of numerical and hydraulic modeling occurs when due numerical modeling adjustments are made to the results of hydraulic modeling, which is not reproduced certain effects
Figure 1.1 Conditions of water abstraction from the deep reservoir in the HPP water-inlets.

Figure 1.2 Scheme of thermal pollution from nuclear power plants in the coastal area "hybrid" modeling.

(for example, Coriolis forces). In the practice of hydraulic research and there are other cases of distribution between the hydraulic and numerical modeling parts of the phenomenon. For example, in [184] the study reported the load on the vessel in the lock chamber, in which components of the load are defined in different ways, and then made their superposition.

The comparison of the results of the use of such traditional reception of data research proves the appropriateness of the proposed method (Figure 1.3). In Figure 1.3 \( F' = F/(\rho g b l_0 e_0) \), where \( F \) – component of load, \( b \) and \( l_0 \) – average width and length of the vessel, \( e_0 \) – vessel draught in still water.
Many practically important cases of numerical modeling do not prove the convergence of the obtained solutions. If a researcher does not have the necessary in-situ data, the estimation of convergence can be made according to the results of the problem on hydraulic models, which are considered private exact solution of the problem of numerical modeling. Hydraulic models can be used not only to check convergence, but also to clarify the wording of the mathematical model underlying the numerical modeling.

When using hydraulic models to check the results of numerical modeling of the hydraulic model used in the beginning of the study, its main part is made with the use of numerical modeling as a faster and less labour-intensive means. Hydraulic model after the necessary comparisons can be left to check the most responsible decisions. On the other hand, numerical modeling can be used to obtain the basic decisions necessary for the evaluation of the hydraulic conditions of similarity (see Section 1.3). It follows that the ever-expanding use in hydraulic systems of numerical modeling should not undermine the hydraulic modeling.

We propose to use the term “hybrid modeling” in relation to the entire research process when sharing as physical (subject), and numerical modeling. General scheme of research in the use of hybrid modeling are presented in Figure 1.4.
Figure 1.4 Combined implementation of physical and numerical modeling in hydroengineering studies.

A number of fundamental and technical circumstances may lead to inaccuracies in both physical and numerical modeling solutions. Testing of the obtained solutions should be performed by comparing the corresponding modeling results by different methods. Figure 1.5 represents the comparison of forecasts of the spatial currents in the area of bucket water intake from the channel. The research is carried out both on the physical model in geometric scale of 1:20 and by numerical modeling on the basis of the spatial mathematical model. The comparison provides confidence in
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Figure 1.5 Surface currents obtained from numerical models and surface currents trajectories, obtained on the physical model.

the reliability of numerical modeling. When using numerical modeling it is easy to obtain the more detailed data.

Good coincidence of results of physical and numerical modeling is also demonstrated in Figure 1.6, where information is given on the surface currents in the upstream hydroelectric station on the approach to the HPP underground water intakes.

1.3 Dimensions of Quantity

A physical model can be presented as analog device allowing to solve the equations that describe the considered phenomenon. It would seem that in order to solve the equation, you need to know his views. Indeed, the equations view of communication, describing the phenomenon must be
known to select a phenomenon that plays in the model; on the basis of the equations, communications establishes the rules of modeling (similarity criteria are found).

Consideration of the phenomenon specifics allows for attributing it to the possibly narrower class, and that facilitates modeling. For example, transition from the wide class of “mechanical movement” to the “continuous medium motion,” and further to the “fluid flow,” “dropping fluid flow,” “pressure turbulent flow of the droplet fluid,” etc., allows reduction of the number of variables included in the equations, describing the phenomenon.

However, due to the complexity of the phenomena of nature, its lack of knowledge or lack of suitable mathematical methods of its description may be incomplete. Does modeling in this case solve the problem, having a closed system of equations to solve it? This, at first glance, somewhat paradoxical question can be answered affirmatively. This possibility is limited until those cases when the model has the same phenomenon as in kind (on the model and in kind - homogeneous phenomenon). In these conditions, we may assume that both on the model and in kind phenomena are described by the same equations. Then the rules of modeling, providing achievement of natural similarity on the model, can be established on the basis of the theory of dimensions conclusions.

![Diagram](image-url)

**Figure 1.6** Conditions of water abstraction from the deep reservoir to the HPP water intakes.
The possibility of solving the problem in the absence of the fully formulated mathematical model may cause doubts in the generality of the scheme of physical modeling, represented above, in accordance with which the physical model and the nature are related through their mathematical model. In fact, there are no exceptions here. This question is considered in [66], which shows “that the dimensional analysis is essentially the analysis of the equations describing the studied phenomenon in General.” The smaller the community, the more complete mathematical model of the phenomenon, the more real the implementation of the modeling due to the possibility to evaluate the consequences of the necessary simplifications of similarity. The success of the theory of dimensions to modeling depends primarily on the correctness of the choice of values describing the phenomenon. The concept of the values dimensions forms the basis of the theory of dimensions.

The values, numeric values of which depend on the system of units, are called dimensional, or named. If there is no such dependence, then the quantities are called dimensionless, or abstract. Physical quantities are allied. So if some of them are accepted as “fundamental” and the units are established for them, the units of all other quantities will be expressed in the units of the fundamental quantities in a certain way. Units of fundamental quantities are called basic or primary, and the units of all others are called derived units, or secondary units. In practice in mechanical problems it is enough to establish units for three values, to express through them the units of all the others.

If there are three main units, the units of other mechanical values are obtained from their definitions. The expression derived units via the main unit is called the dimension. The dependence of units of derived values of the items of property values can be represented in the form of formulas, called formula dimension. In this formula the main unit dimensions are as symbols \([X]\). About the dimension we can talk only with regard to a particular system of units. When using International system of units (ISU) units are symbols of units of length \([l]\), time \([t]\) and weight \([m]\). Formula dimensions of physical quantities overlook the power of a monomial. In the ISU the formula for the dimension of any mechanical values of \(X\) can be represented as

\[
[X] = [l]^{m_1}[m]^{m_2}[t]^{m_3}
\]

The number of basic units is not necessarily equivalent to three. You can borrow more basic units. For example, if you set independently of each unit four values: time, length, mass and force, the Newton’s equation will be:
\[ F = c_1 ma \]

where \( c_1 \) - is the constant, having \([c_1] = [F][t]^2 \times [m]^{-1}[l]^{-1}\) dimension

You can choose independently \( K \) each basic units (\( K \geq 3 \)). However, you’ll need to enter \((K–3)\) additional dimensional constants. The main units can be rented and less than three. In this case, you must enter \((3–K)\) dimensional constant, and consider them as an absolute constant.

In the study of certain classes of phenomena, it is advisable to use the most typical as the basic unit of those values. The basic units can be different in different private problems. The dimension values can be dependent and independent. Dimension \( K \) is a value called independent if each of them can be presented as a combination of the others. For example, dimensions of energy \([m][l]^2[t]^{-2}\), length \([l]\) and velocity \([l][t]^{-1}\) should be considered as independent because none of them can be obtained by a combination of dimensions of other two units.

The constraint equation, that reflects the objective physical law, which does not depend on the choice of system units, can be generally written as:

\[ \varphi(X_1, \ldots, X_N) = 0 \quad (1.3) \]

The relationship between \( N \) dimensional quantities, \( K \) of which have independent dimensions, not changing with the change of the system unit, can be replaced by the relationship between \( N–K \) dimensionless quantities:

\[ \varphi(1, 1, \ldots, 1, \pi_1, \ldots, \pi_{N-K}) = 0 \quad (1.4) \]

or

\[ \varphi_1(\pi_1, \ldots, \pi_{N-K}) = 0 \quad (1.5) \]

This conclusion of the theory of dimensions is called the Pi-card theorem (Pi theorem). Practical value of the Pi theorem lies primarily in the fact that it can help significantly reduce the number of considered values. Thus simplifies the execution of the study. In particular, if all the values included in the communication, except one, have independent dimensions, use of Pi-theorem, this dependence is completely determined with accuracy up to a constant multiplier. Indeed, if \( N=K+1 \), then

\[ \varphi_1 \left( \frac{X_{K+1}}{X_1^{m_1}X_2^{m_2} \ldots X_K^{m_K}} \right) = 0 \]

or

\[ X_{K+1} = c_1 X_1^{m_1}X_2^{m_2} \ldots X_K^{m_K} \quad (1.6) \]
where $C_j$ - is the dimensionless constant, which is the root of the equation (1.6), and the exponents $m_{i_1}, m_{i_2}, \ldots, m_{i_k}$ can be easily found from the condition

$$\left[ \frac{X_{K+1}}{X_1^{m_{i_1}} X_2^{m_{i_2}} \ldots X_K^{m_{i_K}}} \right] = \left[ X_1 \right]^0 \left[ X_2 \right]^0 \ldots \left[ X_K \right]^0$$

(1.8)

Constant $C_j$ can be defined either by experience or theoretically, solving the corresponding mathematical problem.

Theory of dimensions greatly facilitates physical experiment: reducing the number of variables included in the communication, even one several times reduces the number of experiments needed to establish a link. But it is clear that with only the theory of dimensions it is impossible to determine the functional relationship between the dimensionless quantities for one and the same system of governing parameters, on which the conclusions of the theory of dimensions only depend, there can be different constraint equations (in particular, equations of motion). However, the constraint equations contain the potential of finding solutions mathematically.

If for full analytical solution of the problems of the use of the conclusions of the theory of dimension not enough, then when modeling on these conclusions can be based establish the similarity conditions. The Buckingham’s formulation of the Pi theorem reflects this perspective of using theories of dimension: the similarity condition of identical phenomena group, defined by the number of values of $N$ total quantity, consists in the identity of the values of $N$—3 dimensionless $\pi_i$ combinations.

Here we should pay attention to the fact that we are talking about the same phenomena, as the fact that mapped phenomena are determined by the same values does not give grounds for attributing them to like. The same phenomena imply that they are described by the equations of communication that have the same type, i.e. the first condition third theorem similarity is automatically performed. The identity of $p$-members represents the third condition of similarity (consisting of identity criteria of similarity), as they are dimensionless combination of all dimensional quantities, one way or another affect the desired solution.

The identity of part of the $\pi$-members provides the second condition of the phenomenon similarity: the requirements of similarity of conditions of uniqueness of the solution because the system of values characterizing the phenomenon must contain the values used to specify conditions of uniqueness.