

Computational Risk Management

Sifeng Liu
Yingjie Yang
Jeffrey Forrest

Grey Data Analysis

Methods, Models and Applications

 Springer

Computational Risk Management

Risks exist in every aspect of our lives and risk management has always been a vital topic. Most computational techniques and tools have been used for optimizing risk management and the risk management tools benefit from computational approaches. Computational intelligence models such as neural networks and support vector machine have been widely used for early warning of company bankruptcy and credit risk rating, operational research approaches such as VaR (value at risk) optimization has been standardized in managing markets and credit risk, agent-based theories are employed in supply chain risk management and various simulation techniques are employed by researchers working on problems of environmental risk management and disaster risk management. Investigation of computational tools in risk management is beneficial to both practitioners and researchers. The Computational Risk Management series is a high-quality research book series with an emphasis on computational aspects of risk management and analysis. In this series research monographs, conference proceedings are published.

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Foreword I

It gives me great pleasure to introduce this 8th edition of *Grey System Theory and Its Applications* by Prof. Sifeng Liu. The theory of grey systems was first introduced in 1982 by J.L. Deng (1933–2013) at Huazhong University of Science and Technology; it established a relatively new approach for addressing poorly defined problems with a high level of greyness or uncertainty. The theory enables one to model, analyse, monitor, and control such partially defined systems by generating, excavating, and extracting useful information from what is available. It built on the work of Dr. Lotfi A. Zadeh, who introduced the concept of fuzzy sets in the 1960s that in turn led to breakthroughs in neural networks and soft computing.

Grey System Theory actually combines two critical and overarching areas. The first concerns systems which attempt to synthesize the various components or subsystems into an overall functioning system or system of systems. Systems theory attempts to make transparent the deep connections and interactions among objects and events, all leading to the enrichment and progress of science and technology. Many of the historically difficult, hard-to-solve problems in the different scientific fields have been successfully resolved through the application of systems theory and its allied methodologies, including information theory, cybernetics, combinatorics, and genetics. The second concerns the greyness or uncertainty level that is implicit in all natural or man-made systems. Indeed, most modelling techniques assume the existence of uncertainty or stochasticity, as defined by either empirical evidence or assumed distributions, including fuzzy sets.

Grey System Theory, then, provides a realistic approach to modelling, analysing, monitoring, and controlling systems. Professor Sifeng Liu has greatly extended, if not expanded, Prof. Deng's earlier efforts. In the 1980s, he put forward a series of new models and concepts, including sequence operator, absolute degree of grey incidence, grey cluster evaluation model with fixed weight, and positioned coefficient of grey matrix. In the 1990s, he proposed a buffer operator and its axiom, generalized degree of grey incidence, grey number and measurement of its information content, drifting and positioning solution, the grey econometrics model GM(1,1), the grey Cobb–Douglas model, etc. More recently, he proposed the concept

of general grey numbers, the grey algebraic system based on a kernel and degree of greyness, and different variations of the model GM (1,1).

The widespread recognition and application of grey system theory reflect its growing acceptance. A number of universities from around the world have adopted Prof. Sifeng Liu's monographs, both in Chinese and English, as their textbooks. In 2002, he won the World Organization of Systems and Cybernetics (WOSC) Prize. In 2008, as a pre-eminent Chinese scholar, he was elected an Honorary WOSC Fellow. In 2013, after a strict review by the European Commission, he was selected to be a Marie Curie International Fellow, thus honouring him as the first such Fellow with grey systems expertise.

As a systems scientist and engineer, I am honoured to write this preface for the 8th edition of *Grey System Theory and Its Applications*. I look forward to its widespread dissemination and its promulgation of grey system applications in science and engineering.

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of Engineering, University of Miami, Coral Gables, FL, USA

Note Prof. James M. Tien prepared this note for 8th edition of *Grey System Theory and Its Applications* (in Chinese) by the same authors, published in 2016. With his permission, it is printed here as a foreword for this current book.

Foreword II

Grey Systems: Theory and Applications

Written by Sifeng Liu and Yi Lin

Springer-Verlag: Berlin, Heidelberg

2010, 379 pages, ISBN 978-3-642-16158-2 (cloth)

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Professors Sifeng Liu and Yi Lin have written another pioneering book on the important topic of grey systems. In 2006, the same authors wrote the well-received book entitled “Grey Information: Theory and Practical Applications” which was also published by Springer-Verlag. I am pleased to say that their second book on Grey Systems constitutes a significant expansion and improvement in their previous fine book. Accordingly, if you already possess a copy of the 2006 book, you can make a worthwhile academic investment by obtaining a copy of their recent book in order to be cognizant of the latest ideas and advancements in the crucial field of grey systems.

The question that naturally arises is why grey systems are of such great import at this point in history. The answer is quite straightforward: many challenging problems facing society consist of interconnected complex systems of systems exhibiting high uncertainty and having few measurements. For example, in order to effectively combat climate change, one must understand as much as possible the complex interactions among natural systems such as atmospheric, oceanic, geological, and hydrological systems, with societal systems including energy production, industrial, agricultural, and city systems. The deep uncertainty involved with these interconnected systems of systems and their potential emergent behaviour, coupled with a dearth of observations, mean that formal tools for handling this uncertainty are in high demand. Fortunately, an arsenal of mathematically based methodologies and techniques have been developed over the years: a rich variety of probabilistic-based tools, fuzzy sets founded by Lotfi Zadeh, rough sets started by Z. Pawlak, information-gap modelling perfected by Yakov Ben-Haim, uncertainty theory developed by Baoding Liu, and grey systems established by Julong Deng in 1982. The foregoing and other approaches to describing uncertainty are based upon

different axioms and are thereby highly complementary for tackling a wide variety of uncertain situations.

Grey systems are purposefully designed for modelling uncertain systems, or systems of systems, problems having small samples, and low-quality information. Grey systems are capable of dealing with partially known information through generating, excavating, and extracting useful information from what is available. How this is accomplished is explained in depth in the timely grey systems book of Professors Liu and Lin.

In their contemporary textbook, Liu and Lin systematically present the theory and practice of grey systems. In fact, the excellent ideas and applications contained in their book are based upon the authors' many years of developing theoretical concepts, applying their methods to real-world applications, testing and refining their new techniques with actual data, carrying out stimulating research with their students and colleagues, teaching their students about their exciting work, and delivering research papers at international conferences around the globe. Their comprehensive book contains the latest theoretical and applied advances created by the authors and other scholars around the world in order to place the readers at the forefront of international research in grey systems.

The main body of their book contains ten well-explained and interconnected chapters: Introduction to Grey Systems Theory, Basic Building Blocks, Grey Incidence and Evaluation, Grey Systems Modeling, Discrete Grey Prediction Models, Combined Grey Models, Grey Models for Decision Making, Grey Game Models, Grey Control Systems, and Introduction to Grey Systems Modeling Software. Moreover, this book includes a computer software package developed for grey systems modelling to permit both researchers and practitioners to use the new methodologies. Their book concludes with three appendices. The first appendix compares grey systems theory and interval analysis while revealing the fact that interval analysis is a part of grey mathematics. The second presents an array of different approaches to studying uncertainties. Finally, the last appendix shows how uncertainties occur using a general systems approach.

This book contains a wealth of mathematical results, techniques, and algorithms which are presented by the authors for the first time. These contributions include an axiomatic system of buffer operators and a series of weakening and strengthening operators; axioms for measuring the greyness of grey numbers; general grey incidences (grey absolute incidence, grey relative incidence, grey comprehensive incidence, grey analogy incidence, and grey nearness incidence); discrete grey models; fixed weight grey cluster evaluation; and grey evaluation methods based on triangular whitenization weight functions, multi-attribute intelligent grey target decision models, applicable range of the $G(1,1)$, grey econometrics (G-E), grey Cobb-Douglass (G-C-D), grey input-output (G-I-O), and grey game models (G-G).

In their well-written book, Drs. Liu and Lin do a thorough job in their presentation of many difficult technical concepts. The authors are able to convince the readers of their book regarding the power and usefulness of their new theory by presenting many interesting examples of practical applications to real-life problems. The challenging practical problems addressed in their book include urban economic

planning, downtown traffic design, natural disaster prediction, relative strength evaluation of a state, investment projection of a company, and employee performance evaluation.

The depth and scope of the advancements in grey systems covered in this book, in conjunction with clarity of explanation, make this seminal book attractive to researchers, students, teachers, and practitioners working in many different fields. These areas of endeavour include image processing, video processing, multimedia security, computer vision, machinery, control, agriculture, water resources, medicine, astronomy, earth science, economics, and management. I personally found grey systems useful for accurately forecasting wastewater time series for which there is a scarcity of data. I intend to keep a copy of this valuable book easily accessible in my university office and purchase more copies of the book for use by my students.

Keith William Hipel
Ph.D., P.Eng., FIEEE, FINCOSE, FCAE, FEIC, FRSC, FAWRA,
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Note Prof. Keith William Hipel prepared this note for one of the earlier book by the same authors, published in 2010. It is published in *Grey Systems: Theory and Application*, 2011, Vol. 1, No. 3. With his permission, it is printed here as a foreword for this current book.

Foreword III

With human knowledge maturing and scientific exploration deepening and largely expanding in the course of time, mankind finally realizes the fundamental fact that due to both internal and external disturbances and limitations of human and technical sensing organs, all information received or collected contains some kind of uncertainty. Accompanying the progress of science and technology and the aforementioned realization, our understanding about various kinds of uncertainties has gradually been deepened. Attesting to this end, in the second half of the twentieth century, the continual appearance of several influential and different types of theories and methods on unascertained systems and information has become a major aspect of the modern world of learning. Each of these new theories was initiated and followed up by some of the best minds of our modern time.

In their recent book, entitled “Grey Information: Theory and Practical Applications”, published in its traditionally excellent way by Springer, Profs. Sifeng Liu and Yi Lin presented in a systematic fashion the theory of grey system, which was first proposed by J.L. Deng in the early 1980s and enthusiastically supported by hundreds of scientists and practitioners in the following years. Based on the hard work of these scholars in the past (nearly) thirty years, scholars from many countries currently are studying and working on the theory and various applications of this fruitful scientific endeavour. With this book published by such a prestigious leading publisher of the world, it can be expected that more scientific workers from different parts of the world will soon join hands and together make grey system and information a powerful theory capable of bringing forward practically beneficial impacts to the advancement of the human society.

This book focuses on the study of such unascertained systems that are known with small samples or “poor information”. Different of all other relevant theories on uncertainties, this work introduces a system of many methods on how to deal with grey information. Starting off with a brief historical introduction, this book carries the reader through all the basics of the theory. And, each important method studied is accompanied with a real-life project the authors were involved in during their professional careers.

Many of the methods and techniques the reader will learn in this book were originally introduced by the authors. They show how from our knowledge based on partially and poorly known information can be obtained to accurate descriptions and effective controls of the systems of interest. Because this book shows how the theory of grey system and information was established and how each method could be practically applied, this book can easily be used as a reference by scholars who are interested in either theoretical exploration or practical applications or both. I recommend this book highly to anyone who has either a desire or a need to learn.

July 2007

Prof. Dr. Dr. h.c. mult. Hermann Haken
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Note Prof. Hermann Haken prepared this note for one of the earlier book by the same authors, published in 2006. It is published in Grey Systems: Theory and Application, 2011, Vol. 1, No. 1. With his permission, it is printed here as a foreword for this current book.

Foreword IV

I am much interested and impressed by Dr. Sifeng Liu and Dr. Yi Lin's recently published monograph on grey information, dealing with the theory and practical applications.

This book encompasses many aspects of mathematics under the aegis of uncertain information. I am greatly in favour of this attitude, concerning the uncertainty of information, which has been mine since a long time ago. Also, this book focuses on practice and aims at explorations of new knowledge. It is a comprehensive, all-in-one exposition, detailing not only with the theoretical foundation but also real-life applications. Because of this characteristic of quality and usefulness, Liu and Lin's book possesses the value of the widest possible range of reference by the workers and practitioners from all corners of natural and social sciences and technology.

In this book, Liu and Lin present the theory of grey information and systems starting on such background information as the relevant history, an attempt to establish a unified information theory, the basics of grey elements, and reaching all the most advanced topics of the theory. Complemented by many first-hand and practical project successes, the authors developed an organic theory and methodology of grey information and grey system, dealing with errors. In fact, there is much more to tell about error than about truth. Error (inexactitude) can be met everywhere and truth (exactitude) nowhere. But inexactitude contains a part of the truth. Greyness is the field we live in. Extremes, as whiteness and blackness, are inaccessible, but very useful, ideal concepts.

With the publication of such a book that contains not only a theory, aspects of magnificent real-life implications and explorations of new research, but also the

history, the theorization of various difficult concepts, and directions for future works, there is no doubt that Drs. Liu and Lin have made a remarkable contribution to the development and applications of systems science.

June 2007

Prof. Robert Vallée
Member of the French-speaking Academy
of Engineers, President of the World Organisation
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Paris-Nord, Paris, France

Note This note is a book review written by Prof. Robert Vallée for one of the earlier book by the same authors, published in 2006. It is published in *Kybernetes: The International Journal of Cybernetics, Systems and Management Science*, 2008, Vol. 37, No. 1. With his permission, it is printed here as a foreword for this current book.

Preface

In this book, we answer the calls of the readers of our previous publications and systematically present the main advances in grey system theory and applications. By following our readers' feedback and suggestions, this volume introduces the most recent research results and updates on what is presented in our earlier books. In particular, the following content, which represents the authors' recent research, is highlighted in the book: general grey numbers and their operations, grey incidence models based on similarity and closeness, three-dimensional degree of grey incidence models, grey evaluation models based on centre-point mixed possibility functions, grey evaluation models based on endpoint mixed possibility functions, original difference grey model (ODGM), even difference grey model (EDGM), multi-attribute intelligent grey target decision models, weight vector group of kernel clustering, and the weighted coefficient vector of kernel clustering for decision-making. We also attach software designed for grey system modelling, which was developed by Bo Zeng using Visual C#, the widely employed C/S software tool. This user-friendly software allows users to conveniently input and/or upload data and clearly distinguish module functions. Also, the software has the ability to present users with operational details, as well as periodic and partial results. Additionally, users can adjust the levels of computational accuracy based on their practical needs.

During the writing of this book, we prioritized theoretical simplicity and clarity to make it easy for the reader to follow the main arguments made. With a good number of practical applications, we intended to illustrate the methodology of grey system theory and modelling techniques so that we could emphasize the practical applicability of grey system thinking. We drew on the most recent research developments from various research groups around the world, and tried to present the most complete picture of this new area of scientific endeavour in a concise manner.

The overall planning and organization of topics contained in this book were carried out by Sifeng Liu, who also authored Chaps. 1, 4, 6, and 10. Yingjie Yang produced Chaps. 2, 3, and 11, Jeffrey Forrest composed Chaps. 7 and 8, Naiming

Xie wrote Chap. 9, and Chap. 12 and the attached computer software were developed by Zeng Bo, Zhigeng Fang, Yaoguo Dang, Lirong Jian, and Chunhua Su and colleagues also worked with the authors to refine some of the book's content. Sifeng Liu was responsible for unifying the terms used throughout the book and for finalizing the manuscript.

Finally, we would like to encourage you to communicate with us and send us any comments you might have about this book. It is only by working together, as a team, that we can grow and mature as researchers. Sifeng Liu can be reached at sfliu@nuaa.edu.cn and sifeng.liu@dmu.ac.uk. Yingjie Yang can be reached at yyang@dmu.ac.uk and Jeffrey Forrest at jeffrey.forrest@sru.edu or jeffrey.forrest@iigss.net.

Nanjing, China
April 2016

Sifeng Liu

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Over the years, our research has been highly commended by many first class scholars, such as Julong Deng, the founder of grey system theory, L.A. Zadeh, the founder of fuzzy mathematics, Hermann Haken, the founder of synergetics, Robert Vallee, the president of the World Organization of Cybernetics and Systems, James Tien, former vice-president of IEEE and member of the American Academy of Engineering, K.W. Hipel, the president of The Academy of Science of the Royal Society of Canada, Jifa Gu, former president of the International Federation for Systems Research, and academicians of the Chinese Academy of Sciences and the Chinese Academy of Engineering, such as Qun Lin, Da Chen, Chunsheng Zhao,

Haiyan Hu, Suzi Yang, Youlun Xiong, Zhongtuo Wang, Shanlin Yang, Guozhi Xu, and others. Xuesen Qian, a world-renowned scientist, has also congratulated us for our achievements in this area of research. Indeed, such positive feedback on our work has given us the impetus to develop this book.

Finally, many colleagues and administrators have supported us in the process of writing this book, as the authors have consulted many scholars and a wide range of relevant published literature throughout the development of this project. In particular, they are deeply in debt to Dr. Caroline Moraes, who proofread all chapters of the book; Prof. Dash Wu, the editor of a book series who has encouraged us to publish the book; and Wei Zhao and Hannah Qiu, who provided great editorial guidance through to the final publication of this book. The authors would like to express their appreciation to them all.

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Sifeng Liu received Ph.D. in systems engineering from Huazhong University of Science and Technology, China, in 1998. Dr. Liu is currently a distinguished professor of Nanjing University of Aeronautics and Astronautics and a research professor of De Montfort University. He is serving as the founding director of Institute for Grey Systems Studies, the founding president of International Association of Grey Systems and Uncertainty Analysis, the founding chair of TC of IEEE SMC on Grey Systems, and the founding president of Grey System Society of China. Professor Liu is also serving as the founding editor of Grey Systems Theory and Application (Emerald) and the editor in chief of The Journal of Grey Systems (Research Information). He had worked at Slippery Rock University in Slippery Rock and New York Institute of Technology in New York, USA; Sydney University in Sydney, Australia; and De Montfort University in Leicester, UK, as a visiting professor. And he led the College of Economics and Management, NUAA, from 2001 until 2012.

Professor Liu's main research activities are in grey system theory and applications. He has directed more than 50 research projects from China, UK, UN, and EU. He has published over 600 research papers and 26 books by Science Press, Springer-Verlag, Taylor & Francis Group, and John Wiley & Sons, Inc. He is currently serving as the editor of the book series of "Grey Systems" published by Science Press. His works cited 32 thousand times by others and had translated to Korean, German, and Romanian. Over the years, he has been awarded 18 provincial and national prizes for his outstanding achievements in scientific research and applications. In 2002, Dr. Liu was recognized by the World Organization of Systems and Cybernetics. Sifeng Liu's H-index is 55.

Dr. Liu has won several accolades such as the "National Excellent worker of science and technology", "National Excellent Teacher", "National Advanced Individual for Returnee", "Expert Enjoying Government's Special Allowance", and "National Expert with Prominent Contribution".

Prof. Yingjie Yang is currently a full professor in Computational Intelligence at the Centre for Computational Intelligence of De Montfort University, Leicester, UK. He received the B.Sc. (Hons.), M.Sc., and Ph.D. degrees in engineering from Northeastern University, Shenyang, China, in 1987, 1990, and 1994, respectively. He was awarded his Ph.D. degree in computer science at Loughborough University, Loughborough, UK, in 2008. Before joining De Montfort University in 2002, Dr. Yang had worked at Beijing Jiaotong University, Nottingham Trent University, and Loughborough University for various research projects. He has published over 100 papers on grey systems, fuzzy sets, rough sets, neural networks, and their applications to civil engineering, transportation, and environmental engineering. His research interests include the representation and modelling of various uncertainties and the application of computational intelligence to real-world problems. Dr. Yang is the executive president of International Association of Grey Systems and Uncertainty Analysis, a senior member of IEEE Systems, Man and Cybernetics Society, a co-chair of IEEE SMC Technical Committee on Grey Systems and the vice-chair of the Task Force on Competitions for Fuzzy Systems Technical Committee of IEEE Computational Intelligence Society. He is also a member of the Analytics and Risk Technical Committee of IEEE Systems Council. He has been involved in over 15 research projects with total fund value exceeding £1 million from various funding sources including EU FP6, EU FP7, European Space Agency, Royal Society, EPSRC, Royal Academy of Engineering, Leverhulme Trust, and De Montfort University. He is currently focusing on extended fuzzy sets, grey systems, rough sets, and their applications to decision support and risk analysis. Dr. Yang has co-chaired several international conferences/special sessions, and been involved as programme committee member in over 80 international conferences. Dr. Yang is serving as associate editors in 5 international academic journals, including IEEE Transaction on Cybernetics, Journal of Intelligent and Fuzzy Systems, World Scientific Journal, Journal of Grey Systems, and Journal of Grey System: Theory and Applications. His pioneer work on factor analysis using neural networks has been adopted by many researchers since its publication more than 10 years ago. His work on the integration of neural networks with GIS is an early attempt in this field. His work on extended fuzzy sets and grey systems has been well received in research community.

Prof. Jeffrey Forrest earned all his educational degrees in pure mathematics. His Ph.D. degree was granted in 1988 by Auburn University, Alabama; and he did one-year postdoctoral research in statistics at Carnegie Mellon University, Pittsburgh, during 1990–1991. Dr. Forrest is currently a specially appointed professor in economics, finance, and systems science of Nanjing University of Aeronautics and Astronautics; a specially appointed professor in mathematics and systems science of National University of Defense Technology, China; and a tenured professor of mathematics at Slippery Rock University of Pennsylvania. Dr. Forrest is a founder and the present president of the International Institute for General Systems Studies (IIGSS), a non-profit organization registered in PA in the mid-1990s. Since 1984, Forrest has had over 300 research papers and over

30 monographs and edited volumes published by a large array of prestigious publishers, such as Springer, World Scientific, Kluwer Academic (currently part of Springer), Academic Press (currently part of Elsevier), Wiley, Taylor and Francis, and Meteorological Press. He serves or served on the editorial boards of 11 professional journals, including *Kybernetes: The International Journal of Systems, Cybernetics and Management Science*, *Journal of Systems Science and Complexity*, and *International Journal of General Systems*. He is currently a co-editor of the book series “Systems Evaluation, Prediction and Decision-Making”, published by CRC Press, an imprint of Taylor and Francis.

Some of Dr. Forrest’s research was funded by the United Nations, the State of Pennsylvania, the National Science Foundation of China, and the German National Research Centre for Information Architecture and Software Technology. Over the years, Dr. Forrest’s scientific achievements have been recognized by various professional organizations and academic publishers. In 2001, he was inducted into the honorary fellowship of the World Organisation of Systems and Cybernetics. In 2016, he was elected to fellowship of the International Association of Grey Systems and Uncertainty Analysis. His research interests are wide ranging, covering areas such as mathematical and general systems theory and applications, foundations of mathematics, data analysis, predictions, economics and finance, management science, and philosophy of science.

Abstract

This book systematically presents the fundamental methods, models, and practical application techniques of grey system theory. It is a result of the authors' combined expertise in theoretical exploration, real-life application, and teaching in this research area, which the authors have developed over the past thirty years. This book covers up-to-date theoretical and applied advances in grey systems from across the world and vividly presents the reader with the overall picture of this new theory and its frontier research.

This book contains 12 chapters, including Introduction to Grey System Theory, The Novel Framework of Grey System Theory, Grey Numbers and their Operations, Sequence Operators and Grey Data Mining, Grey Incidence Analysis Models, Grey Cluster Evaluation Models, Series of GM Models, Combined Grey Models, Techniques for Grey System Forecasting, Grey Models for Decision-Making, Techniques for Grey Control, and Introduction to the Software of Grey System Modeling. This is the first book of its kind to address general grey numbers and their operations, the axiomatic system of buffer operators and a series of weakening and strengthening operators, absolute degree of grey incidence model, relative degree of grey incidence model, synthetic degree of grey incidence model, grey incidence models based on similarity and closeness, three-dimension degree of grey incidence models, grey fixed weight clustering model, grey evaluation models based on mixed possibility functions, original difference grey model (ODGM), even difference grey model (EDGM), discrete grey model (DGM), multi-attribute intelligent grey target decision models, weight vector group of kernel clustering, and the weighted coefficient vector of kernel clustering for decision-making.

This book will be appropriate as a reference and/or textbook for graduate students or high-level undergraduate students, majoring in areas of science, technology, agriculture, medicine, astronomy, earth science, economics, and management. It can also be utilized as a reference book by researchers and technicians in research institutions, business entities, and government agencies.

Chapter 1

Introduction to Grey Systems Research

1.1 Appearance and Growth of Grey Systems Research

On the basis of dividing the spectrum of scientific and technological endeavors into fine sections, the overall development of modern science has shown the tendency of synthesis at a high level. This higher level synthesis has led to the appearance of various studies of systems science with their specific methodological and epistemological significance. Systems science reveals deep and intrinsic interconnections between objects and events, and has greatly enriched the overall progress of science and technology. Many of the historically difficult problems in different scientific fields have been resolved successfully along with the appearance of systems science and its specific branches. Furthermore, because of the emergence of various new areas in systems science, our understanding of nature and the laws that govern objective evolutions has been gradually deepened. At the end of the 1940s, there appeared systems theory, information theory, and cybernetics. Toward the end of 1960s and the start of 1970s, there appeared the theory of dissipative structures, synergetics, catastrophe, and bifurcations. Then, in the mid to late 1970s, new transfield and interfiled theories of systems science such as the ultracircular theory and dynamic systems theory emerged.

Due to both the existence of internal and external disturbances and the limitations of our understanding, when investigating systems the available information tends to contain various kinds of uncertainty and noises. Along with the development of science, technology and the progress of the mankind, our understanding of systems' uncertainties has been gradually deepened and the research of uncertain systems has reached a new height. During the second half of the 20th century, the seemingly non-stoppable emergence of various theories and methodologies of uncertain systems has been particularly significant in the areas of systems science and systems engineering. For instance, L.A. Zadeh established fuzzy mathematics in the 1960s (Zadeh 1965), J.L. Deng developed grey systems theory (Deng 1982) and Z. Pawlak advanced rough set theory in the 1980s (Pawlak 1982). These works

represent some of the most important efforts in uncertain systems research of this time period and provide the theories and methodologies for describing and dealing with uncertain information from different angles.

Grey systems theory, established by Julong Deng in 1982, is a relatively new methodology that focuses on the study of problems involving small samples and poor information. It deals with uncertain systems that contain partially known information through generating, excavating, and extracting useful information from what is available. Through this process, systems' operational behaviors and their laws of evolution can be correctly described and effectively monitored. In the natural world, uncertain systems with small samples and poor information exist commonly. This fact determines the wide applicability of grey systems theory.

1.2 Development History and Current State

In 1982, Professor Julong Deng's paper titled "The Control Problems of Grey Systems" was the first paper on grey systems to be published in the *Systems and Control Letters* journal (Deng 1982a). In that same year, Professor Deng also published "Grey Control System" in Chinese and the paper was published by the *Journal of Huazhong University of Science and Technology* (Deng 1982b). The publication of these two seminal articles indicated that a new and cross-sectional discipline named grey system theory came into the world.

In 1989, *The Journal of Grey System* was launched by Research Information Ltd in the UK. Currently, this publication is indexed by Mathematical Review of the United States, Science Citation Index, and other important indexing agencies from around the world. In 1997, a Chinese publication named *Journal of Grey System*, was launched in Taiwan, China, and it was only in 2004 that this publication began to be published in English. Additionally, in 2011 Emerald launched a new journal named *Grey Systems: Theory and Application*, edited by the faculty of the Institute for Grey System Studies at Nanjing University of Aeronautics and Astronautics. There are currently over one thousand different professional journals in the world that have published papers in grey systems theory, many of which are top journals in a variety of fields. As of this writing, many journals and publishers such as the journal of the Association for Computing Machinery (USA), *Communications in Fuzzy Mathematics* (Taiwan, China), *Kybernetes: The International Journal of Systems and Cybernetics*, *Transaction of Nanjing University of Aeronautics and Astronautics*, China Ocean Press, Chinese Agricultural Science Press, Henan University Press, Huazhong University of Science and Technology Press Co. Ltd, IEEE Press, Springer-Verlag have respectively published special issues or proceedings on grey system theory.

Numerous universities around the world have set up grey system theory curriculums. For example, in Nanjing University of Aeronautics and Astronautics (NUAA), the curriculums of the grey system theory are found not only in PhD and Master's programs, but also in undergraduate programs of many disciplines across

the university, as an elective module. In 2008, the grey system theory course of NUAA was selected one of the national level model courses in China. In 2013, the same course was selected as the national excellent resource sharing course, which became a free open learning resource for all grey system hobbyists.

Many universities are recruiting and funding doctoral and postdoctoral researchers in grey system theory and its application. Examples include Huazhong University of Science and Technology, Nanjing University of Aeronautics and Astronautics, Southeast University, Wuhan University of Technology, Fuzhou University, Shantou University, America Central Florida University, Nebraska-Lincoln University, Canada Waterloo University, Toronto University, De Montfort University, Spain Pablo de Olavide University, Turkey Bogazici University, Cape Town University in South Africa, Romania Bucharest Economics University, Japan Kanagawa University and many universities in Taiwan. There are also tens of thousands of graduate students and PhDs currently engaged in scientific research applying grey system thinking and methods across the world.

Numerous publishing agencies such as Science Press, Defense Industries Press, Huazhong University of Science and Technology Press Co. Ltd, Jiangsu Science and Technology Press, Shandong People's Press, Science and Technology Literature Press of China, China Science and Technology Book Press of Taiwan, Gaoli Books Limited Company of Taiwan, ASE Press of Romania, Japan Polytechnic Press, IIGSS Academic Press, CRC of Taylor & Francis Group, Springer-Verlag, Springer-Verlag London Ltd, and John Wiley & Sons, Inc. have published hundreds of academic works on grey systems, in many different languages including Chinese, Traditional Chinese, English, Japanese, Korean, Romanian, and German.

Additionally, over the years a group of cutting-edge disciplines such as grey hydrology, grey geological geology, grey breeding, and grey medical science has emerged. Following this, many national and local science funding agencies are actively supporting grey system research. There are hundreds of research projects on grey systems and their applications currently receiving support from the National Natural Science Foundation of China, The European Commission, The Royal Society, Leverhulme Trust, as well as Canada, Spain, and Romania national funds.

Since 2000, 18 regional domestic conferences on grey system theory and its applications have been held. Such conferences have been supported by The Leverhulme Trust, Institute for Grey System Studies, Nanjing University of Aeronautics and Astronautics, De Montfort University, Wuhan University of Technology, Educational Society of Pudong, Shanghai, and China Center of Advanced Science and Technology; the latter directed by Mr. Tsung-Dao Lee, a Nobel Prize winner, and two of the former presidents of the Chinese Academy of Sciences, Mr. Zhou Guangzhao and Mr. Lu Yongxiang. As a result, grey system theory has attracted a large number of young scholars to such events.

Many special sessions and tracks on grey system theory have been organized at significant international conferences such as International Conference on Uncertain System Modeling, International Conference on System Forecast and Control,