Network Reliability
**Scope**: A true performance of a product, or system, or service must be judged over the entire life cycle activities connected with design, manufacture, use and disposal in relation to the economics of maximization of dependability, and minimizing its impact on the environment. The concept of performability allows us to take a holistic assessment of performance and provides an aggregate attribute that reflects an entire engineering effort of a product, system, or service designer in achieving dependability and sustainability. Performance should not just be indicative of achieving quality, reliability, maintainability and safety for a product, system, or service, but achieving sustainability as well. The conventional perspective of dependability ignores the environmental impact considerations that accompany the development of products, systems, and services. However, any industrial activity in creating a product, system, or service is always associated with certain environmental impacts that follow at each phase of development. These considerations have become all the more necessary in the 21st century as the world resources continue to become scarce and the cost of materials and energy keep rising. It is not difficult to visualize that by employing the strategy of dematerialization, minimum energy and minimum waste, while maximizing the yield and developing economically viable and safe processes (clean production and clean technologies), we will create minimal adverse effect on the environment during production and disposal at the end of the life. This is basically the goal of performability engineering.

It may be observed that the above-mentioned performance attributes are interrelated and should not be considered in isolation for optimization of performance. Each book in the series should endeavor to include most, if not all, of the attributes of this web of interrelationship and have the objective to help create optimal and sustainable products, systems, and services.
(Saint Kabir wrote this verse to sing the glory of Guru, without whose help, one cannot cross this ocean of worldly life. He asks, “If both, Guru and God in form of Govind were to appear at the door, whose feet will I worship first?” He answers, “It has to be the Guru’s feet first, because without him, how would I have recognized (known) God?”)

Fondly dedicated to
Prof. Krishna B. Misra (My Guru)- Veena (Guru Maa)
And my very own small world
Sanjekta(Daughter), Shaurya(Son), Ekta(Wife) and Uma-(late) Aditya (parents)
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Preface

The growth of reliability has assumed a new dimension in the recent years primarily because of the consequential impact(s) of failures of present day’s complex systems that may lead to day-to-day annoyance to the operational efficiency and uneconomical maintenance, and even to the extent of endangering life to our planet where a compromise with quality and reliability might be disastrous.

Although several books have been published in the area of reliability theory and practice, no book has been published on the topics covered in this book as the information presented in this book has either been confined to journals or given some space as a part of a chapter in a book. The topics covered in this book will interest not only the reliability community but also the teachers/educators and students of electrical, computer science, electronics, communication engineering with their allied areas. The text of this book is envisioned to be useful and can also serve as a one-semester course to senior undergraduate, graduate or postgraduate students in engineering. For researchers, practising engineers, managers, and designers, it would serve as a valuable reference and primer in the area of network reliability.

A very concerted effort has been made to keep the book ideally suitable for first course or even for a novice stepping into the area of network reliability. The mathematical treatment is kept as minimally as possible with an assumption on the readers’ side that they have basic knowledge in graph theory, probabilities laws, Boolean laws and set theory. A number of solved examples have been provided to make the topics pellucid with some exercises given at the end of chapters for readers to voluntarily test themselves and to have a better command of the material. The references provided at the end of each chapter are no way complete as no book of this size can claim to give a comprehensive survey of the subject spanning over a several decades. But they indeed serve as a platform and guiding factor for further research in this area.
In engineering theory and applications, we think and operate in terms of logics and models with some acceptable and reasonable assumptions. Reliability theory is not an exception where a rather popular model for studying and analysing computer/communication/transportation/water/electrical networks is as a probabilistic graph with a characteristic of edges and/or nodes subject to failures. The network reliability modelling and analysis is an important issue in system design, manufacture and maintenance, wherein the performance of a network depends upon the probability of a specified set of nodes being communicable or not being communicable. The popular measures of network reliability in vogue are 2-terminal reliability with or without capacity constraint on links, k-terminal and all-terminal reliability. The publications of hundreds of research papers in the last few decades on the assessment of such measures indicate the importance of this area.

Among the several approaches of network reliability evaluation, the multiple-variable-inversion sum-of-disjoint product (MVI-SDP) approach finds a well-deserved niche as it provides the reliability expression in a most efficient and compact manner. However, it does require an efficiently enumerated minimal inputs (minimal path, spanning tree, minimal k-trees, minimal cut, minimal global-cut, minimal k-cut) depending on the desired reliability. The present book is a maiden endeavour by the author to cover these two aspects in detail through the application of various techniques devised by the ‘reliability fraternity’ and could be its USP.

The author does not claim to be an ace programmer, and has provided very efficient and user friendly Matlab® programs which can be downloaded at www.scrivenerpublishing.com However, they are amenable to such modifications for the readers who love to do programming. The book is organized as follows.

Chapter 1 introduces the basic definitions, terminology, common assumptions with a broad category of techniques to tackle and evaluate network reliability problems. Chapter 2 succinctly provides the commonly employed hazard models and basic building blocks of a reliability block diagram. It describes a flexible Misra Matrix Method to solve a General series-parallel system reliability model consisting of various types of redundancies.

Chapter 3 pertains to the notion of network connectivity with respect to a specified set of nodes of the network graph termed as Minimal Path Sets, in general or 2-terminal, k-terminal and all-terminal minimal path sets, in particular. It describes several methods of enumeration to such requirement for measuring network reliability metrics. The chosen methods are
simple enough for classroom teaching but become powerful once implemented on a computer using a suitable programming language.

Chapter 4 deals with the dis-connectivity criteria of a network reliability graph under a specified set of nodes termed as Minimal Cut Sets, in general or 2-terminal, k-terminal and all-terminal minimal cut sets, in particular. It also provides a general algorithm developed by the author to enumerate them. It also explains various sub-problems encountered in enumerations and their solutions thereof.

Chapter 5 discusses and describes Sum-of-Disjoint-Product based MVI approaches such as KDH88, CAREL, HM-1 and HM-2 to obtain and evaluate 2-terminal, k-terminal and all-terminal network reliability/unreliability measures.

Chapter 6 puts network reliability methodology and measures discussed in earlier chapters under a unified framework and extend 2-terminal reliability measure to link’s capacity-based reliability measure-CRR and describe a methodology to obtain the measure under such scenario.

In the last Chapter 7, the author has provided two case studies to show the approaches in action.

The author has tried his level best to make the text complete with logical flow and free of omissions. Nevertheless, as a student of reliability engineering, the author realises that ‘failures are inevitable and can never ever be predicted in advance, and cannot be eliminated’ but they and their consequences can definitely be minimized and mitigated. The author takes full responsibility for all those that still remain and shall be grateful if any such shortcomings or suggestions be brought to his notice.

Comments and suggestions regarding the book are most welcome and can be sent to skcerc@hijli.iitkgp.ernet.in.

Kharagpur, India

Sanjay K. Chaturvedi

March, 2016
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The author would like to express his gratitude to the people who have motivated or helped at various stages of writing this book. Although, the skeleton of the text was drawn in the author’s mind long back, the impetus to bring it to fruition was provided by Professor Krishna B. Misra - a ‘Guru’ of eminence who had shown the research path to the author and to many others as well.

Admittedly, it would not have been possible to complete this task without painstaking secretarial assistance provided by my student-friend, Lt Col Rajvinder Singh for proof-reading, incorporating numerous corrections on several drafts, and formatting. My research scholars, notably Mr. Gaurav Khanna (for collating the references and executing the software codes on several examples); Mr. Rajarajan, Mr. Tauheed Ahamed and Ms. Shabnam Samima deserve special thanks for providing assistance as and when it was needed.

The author also owes special thanks to his colleagues Prof. V. N. A. Naikan (Head of our Centre) and Prof. Neeraj Goyal for their helpful comments and suggestions at the early stages of this task.

Finally, the author would like to express his sincere thanks to the authority of his institution, Indian Institute of Technology, Kharagpur, India, for providing the necessary facility, aura and environment to undertake this project.
As the systems grow in size and complexity, they become more prone to failures and it becomes essential to ensure their performance by carrying out reliability analysis. Here, the word system connotes any assemblage of functional units and may be used to denote a complete installation or equipment. A system may be quite gigantic such as computer communication networks or it could be as small as an integrated circuitry.

The problem of determining the reliability of systems, whose components can have one or more failure modes, often arises in variety of applications, ranging from telecommunication, transportation, power systems, and mechanical systems to integrated circuits and computer communication systems or large software structure. Therefore, all such systems can naturally be expressed as in the form of a network, arising from the interconnections of various system subdivisions. For instance, a telecommunication or a computer communication network may have vertices representing the physical locations of computers or transmitters/receivers and may have several edges representing the communication links between different sites. Depending on whether vertices or edges work or fail, the network itself can be considered to be either working or failed.
For the applications cited above, continued availability of communication between specified vertices of a network is an important requirement. With the widespread use of and dependence upon such networks, it becomes imperative for these networks to be highly reliable. Hence the networks are often designed with the criteria of having several communication paths between any two vertices.

Ideally, if completely diverse path between every pair of vertices were available, the probability of existing at least one communication path between any two vertices at a given time would be very high. However, cost of designing and maintaining such networks inhibit this solution. As a compromise, networks are designed in such a way that any two vertices connected through a few disjoint path sets; additional path sets that have common links are also made available.

A major problem in this area lies with the task of determining reliability of such a network and it is desirable to have some quantitative measure of a given network's performance.

1.1 Graph Theory: A Tool for Reliability Evaluation

Graph theory has drawn increased interest of scientists and engineers in the last several decades. The main reason for this accelerated interest in graph theory is in its demonstrated ability to solve problems from a wide variety of areas. Because of their intuitive diagrammatic representation, graphs have been found extremely useful in modelling systems arising in physical science, engineering, social sciences and economic problems and reliability engineering has not been an exception.

The application of graph theory to reliability studies received little attention till 1970. Ever since the application of the graph theory for network reliability evaluation was suggested by (Misra & Rao, 1970), a large number of studies have appeared in the literature. To quote (Singh & Proctor, 1976): “Until 1970, the subject received little attention with the exception of (Shooman, 1968) popular text Probabilistic Reliability, published in 1968. Nevertheless, he did little more than mention the topic. However (Misra & Rao, 1970), developed signal flow graphs- a development recognized as a significant step forward in the evaluation of network reliability”. After this a number of algorithms, techniques and approaches have been suggested in the literature. In fact today, the use of graph theory has become inseparable from network reliability evaluation.

In performing the reliability analysis of large and complex systems, it is almost impossible to treat system in its entirety. The logical approach
is to decompose the system into its smaller functional entities composed of units, subsystems or components. Even a unit can be quite a sizeable subsystem. A unit can further be broken into elements each of which can only be a circuit or a part. In general, the hierarchical order is: system, subsystems, units, equipment, parts and components. The operational relationship amongst its constituent entities is provided through the logical relationship of system failure (or success) with the failure (or success) of its parts. These relationships are depicted through what is commonly known as the reliability logic diagram (RLD). Based on the functional interaction that subsystems or elements of a subsystem can have, the entities may fall in either of these categories viz., series, parallel, series-parallel (SP) or parallel-series (PS). However, certain design considerations or complex failure mode may produce a system in which its representation by pure parallel or series or their combination may not be possible or appropriate. In general, almost all practical systems fall in this category and are better known as non-series-parallel systems (NSP).

The reliability analysis and evaluation of NSP system are quite complicated, memory intensive and time consuming as well. However, any technique, which computes reliability of NSP systems, can easily be applied to series/parallel systems as well. Many of the series-parallel (or parallel-series) system are represented through Reliability Logic or Block Diagram. However, particularly for NSP systems, simpler ways to represent the system through a graph like structure.

A network graph $G= (V, E)$ consists of a set of vertices (or nodes) $|V|$ or $n$ and a set of edges (or links) $|E|$ or $e$. If an edge connects two vertices $i$ and $j$, $j$ is said to be adjacent to $i$. The $n$ number of nodes in the graph is assigned number 1, 2, 3…$n$ sequentially. The $e$ number of links of the network can be arbitrarily and sequentially assigned numbers. One of the earliest DARPA (Defence Advance Research and Project Agency, USA) communication network graph model is shown in Figure 1.1, Figure 1.2, and Figure 1.5. Here, $|V| = n = 5$ and $|E| = e = 7$ and source node ‘$s$’ can be number ‘1’ while the destination node ‘f’ could be represented by ‘5’. With this graph model, depending on the state (working or failed) of vertices (or nodes) and / or edges (or links) with specified probability, the network can be considered either working or failed with estimated probability.

On the basis of reliability, networks/systems modelled through graphs have been classified as:

- Undirected network
- Directed network
- Mixed network.
Network Reliability

1.1.1 Undirected Network

It is a connected graph $G$ for a system wherein nodes are connected by undirected edges. An undirected edge is an edge with no head or tail (no arrow shown). Undirected edges in a graph are used to indicate two-way communication links between nodes. They are represented as unordered node pairs $(i, j)$ joined by the communication link or edge. An edge is said to be incident upon two nodes if the two nodes are joined by the edge.

Example 1.1: The graph in Figure 1.1 is an example of an undirected network where, node-set and edge-set are:

$$V = \{1, 2, 3, 4, 5\}, \text{ and } E = \{(1, 2), (1, 3), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (3, 5), (4, 2), (4, 3), (4, 5), (5, 3), (5, 4)\}$$

$$= \{1, 2, 3, 4, 5, 6, 7\}$$

1.1.2 Directed Network

It is another form of a system representation through connected graph $G$ wherein each edge has an orientation. Obviously, a source node would not have any edge incidents on it whereas a destination node would not have any edge emerging out of it. Some text also refers directed edges as arcs representing a unidirectional communication links between two nodes depicting the information flow in the direction that an arc points. An arc from node $i$ to node $j$ is represented as an ordered pair $(i, j)$, where $i$ is called the tail and $j$ is called the head of the arc. Figure 1.2 is an example of a directed network, where node-set and edge set are:

$$V = \{1, 2, 3, 4, 5\}, \text{ and }$$

$$E = \{(1, 2), (1, 3), (2, 3), (2, 4), (3, 4), (3, 5), (4, 5)\}$$

$$= \{1, 2, 3, 4, 5, 6, 7\}$$
In a directed graph, a strongly connected component is a maximal set of nodes for which there exists a directed path between every ordered pair of nodes in the component, such that the paths pass only through nodes that are also in the component. Figure 1.3 shows two examples of strongly connected components and Figure 1.4 shows two examples of components that are not strongly connected.

In a directed graph, a strongly connected component is a maximal set of nodes for which there exists a directed path between every ordered pair of nodes in the component, such that the paths pass only through nodes that are also in the component. Figure 1.3 shows two examples of strongly connected components and Figure 1.4 shows two examples of components that are not strongly connected.

1.1.3 Mixed Network

A mixed network $G$ is a graph in which some edges may be directed and some may be undirected. It is determined by the triplet $(V, E, D)$ where $V$ is the set of nodes, $E$ is the set of undirected edges and $D$ is the set of directed edges.
The underlying undirected graph is obtained by deleting the orientation of the arcs in \( D \). An orientation of a mixed graph means, that we orient the undirected edges (and leave the directed ones). Figure 1.5 shows such depiction.

Summarily, each item (component/part/subdivision etc...) of a system can be represented by a two terminal graph. Then the logical interconnection of various items form a network like structure and is better known as a \textit{probabilistic graph} of the system due to the associated probability of success/failure of its each items, and this structure can also be designated as a \textit{system} or a \textit{network} (Misra, 1992). To analyse such networks is an extremely difficult, time consuming and laborious task, almost impossible to do manually. Thus, the use of computer becomes absolutely necessary for which one would need a computer-coding scheme representing the network that can easily and suitably be manipulated by the algorithms in addition to computer-programs to provide a solution to the problem.

The commonly used schemes to code these networks have been: \textit{incidence matrix}, \textit{adjacency matrix} and \textit{adjacency list} representation. However, the most popular, simplest and easily manipulative coding scheme for a moderate size network has been the adjacency matrix or connection matrix scheme. The connection matrices for the some cases are as shown in Table 1.1.

![Figure 1.5 A mixed reliability graph of a network.](image)

<table>
<thead>
<tr>
<th>Connection matrix Figure 1.1</th>
<th>Connection Matrix Figure 1.2</th>
<th>Connection matrix Figure 1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node 1 2 3 4 5</td>
<td>Node 1 2 3 4 5</td>
<td>Node 1 2 3 4 5</td>
</tr>
<tr>
<td>1 0 1 1 0 0</td>
<td>0 1 1 0 0</td>
<td>0 1 1 0 0</td>
</tr>
<tr>
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<td>2 0 0 1 1</td>
<td>2 0 0 1 1</td>
</tr>
<tr>
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<td>0 0 0 0 0</td>
<td>5 0 0 1 0</td>
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