HARMONIC BALANCE
FINITE ELEMENT
METHOD
This book is dedicated to my wife Michelle, without her support I would never complete this book, and in memory to my parents.

– Junwei Lu

This book is dedicated to my wife Weichun Cui, since she has helped me a lot during the writing of this book. I also would like to express my gratitude to my beloved parents, who have always supported me.

– Xiaojun Zhao
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Preface

In writing this book on the *Harmonic Balance Finite Element Method (HBFEM): Applications in Nonlinear Electromagnetics and Power Systems*, two major objectives were borne in my mind. Firstly, the book intends to teach postgraduate students and design engineers how to define quasi-static nonlinear electromagnetic (EM) field and harmonic problems, build EM simulation models, and solve EM problems by using the HBFEM. Secondly, this book will delve into a field of challenging innovations pertinent to a large readership, ranging from students and academics to engineers and seasoned professionals.

The art of HBFEM is to use Computational Electromagnetics (CEMs) with harmonic balance theories, and CEM technologies (with IEEE Standard 1597.1 and IEEE Standard 1597.2) to analyze or investigate nonlinear EM field and harmonic problems in electrical and electronic engineering and electrical power systems. CEM technologies have been significantly developed in the last three decades, and many commercially available software packages are widely used by students, academics and professional engineers for research and product design. However, it takes untrained engineers or users several months to understand how to use those packages properly, due to a lack of knowledge on CEMs and EM modeling, and computer simulation techniques. This is particularly true for the harmonic analysis technique, which has not been fully presented in any CEM textbook or used in any commercially available packages. Although a number of CEM-related books are available, these books are normally written for experts rather than students and design engineers. Some of these books only cover one or a few areas of CEMs, and many common CEM techniques and real-world harmonic problems are not introduced. This book attempts to combine the fundamental elements of nonlinear EM, harmonic balance theories, CEM techniques and HBFEM approaches, rather than providing a comprehensive treatment of each area.
This book covers broad areas of harmonic problems in electrical and electronic engineering and power systems, and includes the basic concepts of CEMs, nonlinear EM field and harmonic problems, IEEE Standards 1597.1 and 1597.2, and various numerical analysis methods. In particular, it covers some of the methods that are very useful in solving harmonic-related problems – such as the HBFEM – that are not mentioned in any other numerical calculation books or commercial software packages. In relation to computational technology, this book introduces high-performance parallel computation, cloud computing, and visualization techniques. It covers application problems from component level to system level, from low-frequency to high-frequency, and from electronics to power systems.

This book is divided into six chapters and three appendices. Chapter 1 provides a short introduction to the HBFEM used for solving various harmonic problems in nonlinear electromagnetic field and power systems. This chapter will also discuss definitions of CEM techniques and the various methods used for nonlinear EM problem solving. It also describes high-performance computation, visualization and optimization techniques for EMs, and CEM standards and validation (IEEE Standard 1597.1 and IEEE Standard 1597.2, 2010).

Chapter 2 highlights some fundamental EM theory used in nonlinear EM fields, harmonic problems in transformer power supplies, DC-biased phenomenon in High Voltage Direct Current (HVDC) power transformers, harmonic problems in geomagnetic disturbances (GMDS), geomagnetic induced current (GIC), harmonic problems in distributed energy resource (DER) systems and microgrids, and future smart grids with electric vehicles (EV) and vehicle to grid (V2G).

Chapter 3 covers: the fundamental theory of harmonic balance methods used in nonlinear circuit problems; CEM for nonlinear EM field and harmonic problems; basic concepts of HBFEM used in nonlinear magnetic field analysis; HBFEM for electric circuits and magnetic field coupled problems; HBFEM for three-phase electric circuits coupled with magnetic field; and HBFEM for DC-biased HVDC power transformers.

Chapter 4 investigates HBFEM and its applications in nonlinear magnetic fields and harmonic problems. Several case study problems are presented, such as: HBFEM for a nonlinear magnetic field with current driven (inductor and single phase transformer); HBFEM for a nonlinear magnetic field with voltage-driven (switch mode power supply transformer); three-phase magnetic tripler transformer (electric circuit and magnetic field coupled problems); three-phase high speed motor based on frequency tripler using HBFEM; DC-biased 3D asymmetrical magnetic structure transformer using HBFEM.

Chapter 5 is devoted to the advanced numerical approaches of HBFEM. These include: the decomposed algorithm of HBFEM; HBFEM with a fixed-point technique; hysteresis model based on a neural network and consuming function; and analysis of hysteretic characteristics under sinusoidal and DC bias excitation, parallel computing techniques for multi-frequency domain problem.

Chapter 6 discusses: three-phase power supply transformer model; magnetically controlled shunt reactors (MCSR); computation taking account of hysteresis effects based
on fixed-point reluctance; harmonics analysis in HVDC transformers (three phase model) with geo-magnetics and geomagnetic induced current (GIC); HBFEM used for low-voltage network transformers in renewable energy and microgrid grid systems with distributed energy resource (DER); and electric vehicle (EV) charging systems and vehicle to grid (V2G).

There are three appendices included in this book: MATLAB Program 1 (magnetic circuit analysis of a single phase transformer) and MATLAB Program 2 (main program for 2D magnetic field analysis in current driven); and Fortran program 3 (3D Asymmetrical magnetic structure transformer using HBFEM).

Junwei Lu
About the Companion Website

Don’t forget to visit the companion website for this book:

www.wiley.com/go/lu/HBFEM

There you will find valuable material designed to enhance your learning, including:

- HBFEM program codes
- Explanations

Scan this QR code to visit the companion website
Introduction to Harmonic Balance Finite Element Method (HBFEM)

1.1 Harmonic Problems in Power Systems

The harmonics problem in power systems is not a new problem. It has existed since the early 1900s – as long as AC power itself has been available. The earliest harmonic distortion issues were associated with third harmonic currents produced by saturated iron in machines and transformers, or so-called ferromagnetic loads. Later, arcing loads, like lighting and electric arc furnaces, were also shown to produce harmonic distortion. The final type, electronic loads, burst onto the power scene in the 1970s and 1980s, and has represented the fastest growing category ever since [1].

Since power system harmonic distortion is mainly caused by non-linear loads and power electronics used in the electrical power system [2, 3], the presence of non-linear loads and the increasing number of distributed generation power systems in electrical grids contributes to changing the characteristics of voltage and current waveforms in power systems (which differ from pure sinusoidal constant amplitude signals). The impact of non-linear loads and power electronics used in electrical power systems has been increasing during the last decade.

Such electrical loads, which introduce non-sinusoidal current consumption patterns (current harmonics), can be found in power electronics [4], such as: DC/AC inverters; switch mode power supplies; rectification front-ends in motor drives; electronic ballasts for discharge lamps; personal computers or electrical appliances; high-voltage DC (HVDC) power systems; impulse transformers; magnetic induction devices; and various
electric machines. In addition, the harmonics can be generated in distributed renewable energy systems, geomagnetic disturbances (GMDs) and geomagnetic induced currents (GICs) [5, 6].

Harmonics in power systems means the existence of signals, superimposed on the fundamental signal, whose frequencies are integer numbers of the fundamental frequency. The presence of harmonics in the voltage or current waveform leads to a distorted signal for the voltage or current, and the signal becomes non-sinusoidal. Thus, the study of power system harmonics is an important subject for electrical engineers. Electricity supply authorities normally abrogate responsibility on harmonic matters by introducing standards or recommendations for the limitation of voltage harmonic levels at the points of common coupling between consumers.

1.1.1 Harmonic Phenomena in Power Systems

A better understanding of power system harmonic phenomena can be achieved by consideration of some fundamental concepts, especially the nature of non-linear loads, and the interaction of harmonic currents and voltages within the power system. By definition, harmonic (or non-linear) loads are those devices that naturally produce a non-sinusoidal current when energized by a sinusoidal voltage source. As shown in Figure 1.1, each “waveform” represents the variation in instantaneous current over time for two different loads each energized from a sinusoidal voltage source. This pattern is repeated continuously, as long as the device is energized, creating a set of largely-identical waveforms that adhere to a common time period. Both current waveforms were produced by turning on some type of load device. In the case of the current on the left, this device was probably an electric motor or resistance heater. The current on the right could have been produced by an electronic variable-speed drive, for example. The devices could be single- or three-phase, but only one phase current waveform is shown for illustration. The other phases would be similar.

A French mathematician, Jean Fourier, discovered a special characteristic of periodic waveforms in the early 19th century. The method describing the non-sinusoidal

![Figure 1.1](a) Sine wave. (b) Distorted waveform or non-sinusoidal
A waveform is called its Fourier Series. The Fourier theorem breaks down a periodic wave into its component frequencies. Periodic waveforms are those waveforms comprised of identical values that repeat in the same time interval, as shown in Figure 1.2. Fourier discovered that periodic waveforms can be represented by a series of sinusoids summed together. The frequency of these sinusoids is an integer multiple of the frequency represented by the fundamental periodic waveform.

The distorted (non-linear) waveform, however, deserves further scrutiny. This waveform meets the continuous, periodic requirement established by Fourier. It can be described, therefore, by a series of sinusoids. This example waveform is represented by only three harmonic components, but some real-world waveforms (square wave, for example) require hundreds of sinusoidal components to describe them fully. The magnitude of these sinusoids decreases with increasing frequency, often allowing the power engineer to ignore the effect of components above the 50th harmonic.

1.1.2 Sources and Problems of Harmonics in Power Systems

Harmonic sources generated in power systems can be divided into two categories: established and known; and new and future. Table 1.1 presents sources and problems of harmonics. Harmonic problems in power systems can be traced to a number of factors [3], such as: (a) the substantial increase of non-linear loads resulting from new technologies such as silicon-controlled rectifiers (SCRs), power transistors, and microprocessor controls, which create load-generated harmonics throughout the system; and (b) a change in equipment design philosophy.

In the past, equipment designs tended to be under-rated or over-designed. Nowadays, in order to be competitive, power devices and equipment are more critically designed and, in the case of iron-core devices, their operating points are more focused on non-linear regions. Operation in these regions results in a sharp rise in harmonics.
1.1.3 Total Harmonic Distortion (THD)

The reduced impedance at the peak voltage results in a large, sudden rise in current flow until the impedance is suddenly increased, resulting in a sudden drop in current. Because the voltage and current waveforms are no longer related, they are said to be “non-linear”. These non-sinusoidal current pulses introduce unanticipated reflective currents back into the power distribution system, and the currents operate at frequencies other than the fundamental 50/60 Hz. Ideally, voltage and current waveforms are perfect sinusoids. However, because of the increased non-linear load and power electronic devices based on switch mode power supplies and motor drives, these waveforms quite often become distorted. This deviation from a perfect sine wave can be represented by harmonics – sinusoidal components having a frequency that is an integral multiple of the fundamental frequency, as shown in Figure 1.3. Thus, a non-sinusoidal wave has distortion and harmonics. To quantify the distortion, the term total harmonic distortion (THD) is used, and this expresses the distortion as a percentage of the fundamental voltage and current waveforms.

Table 1.1 Sources and problems of harmonics

<table>
<thead>
<tr>
<th>Established and known</th>
<th>New and future</th>
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<tr>
<td>Tooth ripple or ripples in the voltage waveform of rotating machines.</td>
<td>Energy conservation measures, such as those for improved motor efficiency and load-matching, which employ power semiconductor devices and switching for their operation. These devices often produce irregular voltage and current waveforms that are rich in harmonics.</td>
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<tr>
<td>Variations in air-gap reluctance over synchronous machine pole pitch.</td>
<td>Motor control devices, such as speed controls for traction.</td>
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<td>Flux distortion in the synchronous machine from sudden load changes.</td>
<td>High-voltage direct current power conversion and transmission.</td>
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<td>Non-sinusoidal distribution of the flux in the air gap of synchronous machines.</td>
<td>Interconnection of wind and solar power converters with distribution systems.</td>
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<td>Transformer magnetizing currents.</td>
<td>Static var compensators which have largely replaced synchronous condensers as continuously variable-var sources.</td>
</tr>
<tr>
<td>Network non-linearities from loads such as rectifiers, inverters, welders, arc furnaces, voltage controllers, frequency converters, etc.</td>
<td>The development and potential use of electric vehicles that require a significant amount of power rectification for battery charging.</td>
</tr>
<tr>
<td>N/A</td>
<td>The potential use of direct energy conversion devices, such as magneto-hydrodynamics, storage batteries, and fuel cells that require DC/AC power converters.</td>
</tr>
<tr>
<td>N/A</td>
<td>Cyclo-converters used for low-speed high-torque machines.</td>
</tr>
<tr>
<td>N/A</td>
<td>Pulse-burst-modulated heating elements for large furnaces.</td>
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Harmonics have frequencies that are integer multiples of the waveform’s fundamental frequency. For example, given a 60 Hz fundamental waveform, the 2nd, 3rd, 4th and 5th harmonic components will be at 120 Hz, 180 Hz, 240 Hz and 300 Hz, respectively. Thus, harmonic distortion is the degree to which a waveform deviates from its pure sinusoidal values as a result of the summation of all these harmonic elements. The ideal sine wave has zero harmonic components. In that case, there is nothing to distort this perfect wave. Total harmonic distortion, or THD, is the summation of all harmonic components of the voltage or current waveform, compared against the fundamental component of the voltage or current wave:

$$\text{THD} = \sqrt{\left( \frac{V_2^2}{V_1^2} + \frac{V_3^2}{V_1^2} + \frac{V_4^2}{V_1^2} + \cdots + \frac{V_n^2}{V_1^2} \right)}$$ (1-1)

The formula above (Equation 1-1) shows the calculation for THD on a voltage signal. The end result is a percentage comparing the harmonic components to the fundamental component of a signal. The higher the percentage, the more distortion that is present on the mains signal. The concept that a distorted waveform (including a square wave) can be represented by a series of sinusoids is difficult for many engineers, but it is absolutely essential for understanding the harmonic analysis and mitigation to follow. It is important for the power engineer to keep the following facts in mind:

- The equivalent harmonic components are just a representation – the instantaneous current as described by the distorted waveform is what is actually flowing on the wire.
• This representation is necessary, because it facilitates analysis of the power system. The effect of sinusoids on typical power system components (transformers, conductors, capacitors) is much easier to analyze than distorted signals.
• Power engineers comfortable with the concept of harmonics often refer to individual harmonic components as if each really exists as a separate entity. For example, a load might be described as producing “30 A of 5th harmonic.” What is intended is not that the load under consideration produced 30 A of current at 300 Hz, but rather that the load produced a distorted (but largely 60 Hz) current, one sinusoidal component of which has a frequency of 300 Hz with an rms magnitude of 30 A.
• The equivalent harmonic components, while imaginary, fully and accurately represent the distorted current. As one test, try summing the instantaneous current of the harmonic components at any point in time. Compare this value to the value of the distorted waveform at the same time (see Figure 1.3). These values are equal.

The current drawn by non-linear loads passes through all of the impedance between the system source and load. This current produces harmonic voltages for each harmonic as it flows through the system impedance. The sum of these harmonic voltages produces a distorted voltage when combined with the fundamental. The voltage distortion magnitude is dependent on the source impedance and the harmonic voltages produced. Figure 1.4 illustrates how the distorted voltage is created. As illustrated, non-linear loads are typically modeled as a source of harmonic current.

With low source impedance, the voltage distortion will be low for a given level of harmonic current. If the harmonic current increases, however, system impedance changes due to the harmonic resonance (discussed below) can significantly increase voltage distortion.

IEEE Std. 519-1992, which is titled IEEE Recommended Practices and Requirements for Harmonic Control in Electric Power Systems, is the main document for harmonics in North America. This standard serves as an excellent tutorial on harmonics. The most

![Figure 1.4 Creation of distorted current](image-url)
important part of this document to the industrial user is Chapter 10 (“Recommended Practices for Individual Consumers” [7]).

The electric consumption is a significant part of the total energy consumption and, consequently, the complete chain of generation, transportation and usage of electricity should be optimized. The usage of electrical energy is often optimized by controlling the output of electrical equipment towards the desired value. Advances in power electronic (PE) energy conversion have led to an optimization of electrical equipment. Practical examples of PE-controlled energy conversion are dimmable halogen lighting, low- and high-pressurized discharge lights, AC drives for induction machines (IM), and so on. In addition to the advantages of PE in terms of energy optimization, a lot of PE is also used for DC power supply, such as IT equipment, DC arcing or electrolysis [8, 9].

Harmonics are a distortion of the normal electrical current waveform, generally transmitted by non-linear loads. Switch-mode power supplies (SMPS), variable speed motors and drives, photocopiers, personal computers, laser printers, fax machines, battery chargers and UPSs are examples of non-linear loads. Single-phase non-linear loads are prevalent in modern office buildings, while three-phase, non-linear loads are widespread in factories and industrial plants, and in DC-biased power transformers in HVDC power systems [10].

The study of these harmonics problems is normally focused on the electrical circuit level. A large number of articles and reports have been published in this area. However, the harmonics problem in the component level (or electromagnetic fields) has not been fully investigated, due to a lack of understanding of the characteristics of non-linear electromagnetic fields and a lack of theory and methodology dealing with harmonics generated from non-linear electromagnetic fields. Only a very limited number of papers and reports related to HBFEM used in solving the harmonic problems in electromagnetic field [11, 12]. Detailed HBFEM theory development and various application problem-solving examples are presented in later chapters.

1.2 Definitions of Computational Electromagnetics and IEEE Standards 1597.1 and 1597.2

1.2.1 “The Building Block” of the Computational Electromagnetics Model [13, 14]

The objective of computational electromagnetics (CEM) is to create a representation of real-life problems that can be examined and analyzed by computer resources, as an alternative to building a system, exciting it, and measuring the generated fields. Once the problem has been defined, the important physical characteristics must be identified. All CEM models can be broken into three parts: the source of EM energy, the geometry
of the model components, and the remaining problem space. The following elements of a physical CEM model should be taken into account during the simulation:

1.2.1.1 The Sources of EM Energy

- **Source** – Sources include both intended and unintended sources that electromagnetically couple to and drive conductors (such that energy is conducted into areas that can energize and drive the electric machine to make a correct operation, or can cause problems with the correct operation of the victim devices).
- **Physical Source Modeling** – Sources may be characterized by their electrical size, the distance from materials with which they interact, their geometry, and the excitation applied to them.
- **Source Excitation** – Like fully specified circuit model sources, field sources must also be defined by their amplitude and impedance.

1.2.2 The Geometry of the Model and the Problem Space

The major concern of every CEM model is the geometry of the problem to be solved. A less complex representation must be created, which includes all the important details while avoiding unnecessary details. In addition to the fixed portions of the geometry, it is often necessary to include variables such as the range of positions in which a nearby wire – or any other conductor – could be placed. Together with the geometry of a problem, the properties of all materials used must also be included in the model. If the computational domain were of infinite extent, the simulation of free space would be involved. This can be achieved by using mesh truncation techniques or absorbing boundary conditions. These techniques require that extra free space is added around the model components.

1.2.3 Numerical Computation Methods

Substantial advancements have been made in enhancing the important numerical techniques – for example: the method of moments (MoM); the finite-difference time-domain (FDTD) method; the finite-element method (FEM); the proposed harmonic balance method (HBFEM) in this book; and the transmission line matrix (TLM) method. Many numerical methods were invented decades ago but, in all cases, additional novel ideas were required to make them applicable to today’s real-world electromagnetic problems.

- **The quasi-static field** can be expressed by several different partial differential equations (PDEs). Although existing computational electromagnetic solvers provide preliminary insight, a multi-physics simulation system is needed to model coupled problems in their entity. **Multi-physics problems** are often related to more than
two fields, such as thermal and $\mathbf{E}$ fields, or the $\mathbf{H}$ field, thermal dynamic field, and so forth. In the quasi-static field, the following methods are often used in FEM based EM computation:

- **Time-domain techniques** use a band-limited impulse to excite the simulation across a wide frequency range. The result obtained from a time-domain code is the model’s response to this impulse. Where frequency-domain information is required, a Fourier transform is applied to the time-domain data.

- **Frequency-domain codes** solve for one frequency at a time. This is usually adequate for antenna work or electric machine simulation, and for examining specific issues. Frequency-domain codes are, in general, faster than their time-domain cousins. Therefore, several frequency-domain simulations can usually be run in the time it would take for a single time-domain simulation. However, in nonlinear EM field problems, there is a coalition between each frequency domain, particularly for solving harmonic problems in nonlinear time periodic problems. This can be called the multi-frequency-domain or HBFEM.

### 1.2.4 High-Performance Computation and Visualization (HPCV) in CEM

With the rapid growth of microelectronics and computer technologies, cluster-based high-performance parallel computers are becoming more and more powerful and cost-effective. This provides a new opportunity to apply computational electromagnetics technologies to challenging problems in EM computer modeling and simulation. Since the computational technique extends from numeric analysis to visualization analysis, the demand for innovative visualization techniques becomes higher and higher. Visualization is closely related to high-performance computation using visualization techniques to deal with the complex dynamic electromagnetic system problems.

Visualization techniques for computational electromagnetics in 2D and 3D promise to radically change the way data is analyzed. To minimize eddy current loss and other problems in nonlinear EM fields, the optimization algorithms have been considered in current computational electromagnetic (CEM) modeling approaches. In fact, the action of EM computer modeling and simulation involves several physical effects. Detailed knowledge of all these effects is a prerequisite for effective and efficient design. The first step in reducing the design time and allowing for aggressive design strategies is to use EM computer modeling techniques that will let designers try “what if” experiments in hours instead of months.

### 1.2.5 IEEE Standards 1597.1 and 1597.2 for Validation of CEM Computer Modeling and Simulations

IEEE P1597.1 and P1597.2 Standards, developed by the EMC community, were released in 2008 and 2010 respectively. IEEE Standard 1597.1-2008 is related to the
IEEE Standard for validation of computational electromagnetics computer modeling and simulations. IEEE Standard 1597.2-2010 was released for IEEE recommended practice for validation of computational electromagnetics computer modeling and simulations. The following highlighted descriptions are based on IEEE Standards 1597.1 and 1597.2 [15, 16].

The development of IEEE standards, and recommended practices for computational electromagnetics (CEM) computer modeling and simulation and code validation, has been a topic of much interest within the EMC community particularly since the mid-1980s [17]. This has been due to advances in computer hardware and software technologies, as well as the arrival of new CEM codes and applications. The areas of concern include, but are not limited to, high-frequency areas such as analyzing printed circuit boards (PCBs), radiated and conducted emissions/immunity, system-level electromagnetic compatibility (EMC), radar cross-section (RCS) of complex structures, and the simulation of various EM environment effects problems. In particular, there are concerns regarding the lack of well-defined methodologies to achieve code-to-code or even simulation-to-measurement validations with a consistent level of accuracy.

Since the mid-1960s, a number of CEM techniques have been developed, and numerical codes have been generated, to analyze various related electromagnetic problems, including electromagnetic compatibility (EMC). While each is based on classical electromagnetic theory and implements Maxwell’s equations in one form or another, these techniques, and the manner in which they are used to analyze a given problem, can produce quite different results. A well-defined, mature, and robust methodology for validating computational electromagnetic techniques, with a consistent level of accuracy, is lacking. Indeed, this has eluded the EMC community for many years, and methods have been sought to address this deficiency. The EMC community has persisted regarding the validity, accuracy and applicability of existing numerical techniques to the general class of EMC problems of interest.

The IEEE P1597.1 Standard defines a method to validate computational electromagnetics (CEM) computer modeling and simulation (M&S) techniques, codes and models. It is applicable to a wide variety of electromagnetic (EM) applications, including (but not limited to) the fields of electromagnetic compatibility (EMC), radar cross-section (RCS), signal integrity (SI), and antennas. Validation of a particular solution data set can be achieved by comparing the data set obtained by measurements, alternate codes, canonical, or analytic methods.

IEEE P1597.2™, recommended practice for validation of computational electromagnetics computer modeling and simulation, has been developed to provide examples and problem sets for use in the validation of CEM computer modeling and simulation techniques, codes and models. It is applicable to a wide variety of electromagnetic applications. The recommended practice, in conjunction with this standard, shows how to validate a particular solution data set by comparing it to the data set obtained by measurements, alternate codes, canonical, or analytic methods. The key areas addressed
include model accuracy, convergence, and techniques or code validity for a given set of canonical, benchmark, and standard validation models.

In fact, computer predictions have been compared to measurements to provide a first-order validation, but there is also much interest in how the techniques, when applied to a given problem or a class of problems, compare to each other and the fundamental theory upon which they are based. Hence, additional efforts are needed to establish a standardized method for validating these techniques, and to instill confidence in them. Therefore, the purpose of this first-of-its-kind standard is to define the specific process and steps that will be used to validate CEM techniques and to significantly reduce uncertainty (as it pertains to their implementation and application to practical EMC problem-solving tasks). The standardized process, based on the Feature Selective Validation (FSV) method, is used to validate various techniques against each other, as well as against measurement baselines (in order to determine the degree of agreement or convergence, and to identify the potential error sources that would lead to divergent trends).

In general, CEM techniques and codes, and the manner in which they are used to analyze a given problem, can produce quite different results. These results are affected by the way in which the underlying physic formalisms have been implemented within the codes, including the mathematical basis functions, numerical solution methods, numerical precision, and the use of building blocks (primitives) to generate computational models. Despite all CEM codes having their basis in Maxwell’s equations in one form or another, their accuracy and convergence rate depends on how the physics equations are cast (e.g., integral or differential form, frequency or time domain), what numerical solver approach is used (full or partial wave, banded or partitioned matrix, non-matrix), inherent modeling limitations, approximations, and so forth. The physics formalism, available modeling primitives (canonical surface or volumetric objects, wires, patches, facets), analysis frequency, and time or mesh discretization further combine to affect accuracy, solution convergence and overall validity of the computer model.

The critical areas that must be addressed include model accuracy, convergence, and techniques or code validity for a given set of canonical, benchmark and standard validation models. For instance, uncertainties may arise when the predicted results using one type of CEM technique do not agree favorably or consistently with the results of other techniques or codes of comparable type, or even against measured data on benchmark models. Furthermore, it can be difficult to compare the results between certain techniques or codes, despite their common basis in Maxwell’s equations. Exceptions can be cited, in particular, when comparing the results of “similar” codes grouped according to their physics, solution methods, and modeling element domains. Nevertheless, disparities even among codes in a certain “class” have been observed. Many examples can be cited where fairly significant deviations have been observed between analytical or computational techniques and empirical-based methods. Differences are not unexpected, but the degree of disparity in certain cases cannot be readily