Theory of Elasticity and Stress Concentration

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WILEY
THEORY OF ELASTICITY
AND STRESS
CONCENTRATION
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Preface

The theory of elasticity is not applied mathematics. Solving differential equations and integral equations is not the objective of the theory of elasticity. Students and young researchers, who can use the modern commercial finite element method (FEM) software, are not attracted by the classical approach of applied mathematics. This situation is not good. Students, young researchers and young engineers skip directly from the elementary theory of the strength of materials to FEM without understanding the basic principles of the theory of elasticity. The author has seen many mistakes and judgement errors made by students, young researchers and young engineers in their applications of FEM to practical problems. These mistakes and judgement errors mostly come from a lack of basic knowledge of the theory of elasticity. Firstly, this book provides the basic but very important essence of the theory of elasticity. Second, many useful and interesting applications of the basic way of thinking are presented and explained. Readers do not need special mathematical knowledge to study this book. They will be able to understand the new approach of the theory of elasticity which is different from the classical mathematical theory of elasticity and will enjoy solving many interesting problems without using FEM.

The basic knowledge and engineering judgement acquired in Part I will encourage the readers to enter smoothly into Part II in which various important new ways of thinking and simple solution methods for stress concentration problems are presented. Approximate estimation methods for stress concentration will be very useful from the viewpoint of correct boundary conditions as well as the magnitude and relative importance of numerical variables. Thus, readers will be able to quickly find approximate solutions with practically sufficient accuracy and to avoid fatal mistakes produced by FEM calculations, performed without basic knowledge of the theory of elasticity and stress concentration.

The author believes with confidence that readers of this book will be able to develop themselves to a higher level of research and structural design.
Preface for Part I: Theory of Elasticity

Part I of this book presents a new way of thinking for the theory of elasticity. Several good quality textbooks on this topic have already been published, but they tend to be too mathematically based. Students can become confused by the very different approaches taken towards the elementary theory of strength of materials (ETSM) and the theory of elasticity and, therefore, believe that these two cannot be easily used cooperatively.

To study this book, readers do not need special mathematical knowledge such as differential equations, integral equations and tensor analysis. The concepts of stress field and strain are the most important themes in the study of the theory of elasticity. However, these concepts are not explored in sufficient depth within ETSM in order to teach engineers how to apply simple solutions using the theory of elasticity to solve practical problems. As various examples included in this book demonstrate, this book will help readers to understand not only the difference between ETSM and the theory of elasticity but also the essential relationship between them.

In addition to the concepts of field, the concepts of infinity and infinitesimal are also important. It is natural that everyone experiences difficulties in imagining infinity or infinitesimal. As a result, we must use caution when using unbounded or very small values, as the results are sometimes unexpected. We should be aware that infinity and infinitesimal are relative quantities.

Once the concepts of field and those of infinity and infinitesimal are mastered, the reader will become a true engineer having true engineering judgement, even if they cannot solve the problems using lengthy and troublesome differential or integral equations. However, the existing solutions must be used fully and care must be taken at times, very large values being treated as infinitesimal and very small values as infinite values depending on the specific problem. It will be seen in many cases treated in this book that small and large are only our impressions and that approximation is not only reasonable but very important.
Stresses and strains in an orthogonal coordinate system 
\((x, y, z)\)

Normal stress \((\sigma_x, \sigma_y, \sigma_z)\)
Normal strain \((\epsilon_x, \epsilon_y, \epsilon_z)\)
Shear stress \((\tau_{xy}, \tau_{yz}, \tau_{zx})\)
Shear strain \((\gamma_{xy}, \gamma_{yz}, \gamma_{zx})\)

Stresses and strains in a cylindrical coordinate system 
\((r, \theta, z)\)

Normal stress \((\sigma_r, \sigma_\theta, \sigma_z)\)
Normal strain \((\epsilon_r, \epsilon_\theta, \epsilon_z)\)
Shear stress \((\tau_{r\theta}, \tau_{\theta z}, \tau_{zr})\)
Shear strain \((\gamma_{r\theta}, \gamma_{\theta z}, \gamma_{zr})\)

Rotation
Normal stress and shear stress in a \(\xi-\eta-\zeta\) coordinate system

Remote stress
Principal stresses
Principal strains
Direction cosines
Pressure
Concentrated force
Body force
Bending moment per unit length
Twisting moment per unit length
Twisting moment (torsional moment) or temperature
Torsional angle per unit length or crack propagation angle
Surface tension
Airy’s stress function or stress function in torsion
Stress concentration factor
Stress intensity factor of Mode I
Stress intensity factor of Mode II
Stress intensity factor of Mode III

Normal stress \((\sigma_x, \sigma_y, \sigma_z)\)
Normal strain \((\epsilon_x, \epsilon_y, \epsilon_z)\)
Shear stress \((\tau_{xy}, \tau_{yz}, \tau_{zx})\)
Shear strain \((\gamma_{xy}, \gamma_{yz}, \gamma_{zx})\)
Normal stress \((\sigma_r, \sigma_\theta, \sigma_z)\)
Normal strain \((\epsilon_r, \epsilon_\theta, \epsilon_z)\)
Shear stress \((\tau_{r\theta}, \tau_{\theta z}, \tau_{zr})\)
Shear strain \((\gamma_{r\theta}, \gamma_{\theta z}, \gamma_{zr})\)
Rotation
Normal stress and shear stress in a \(\xi-\eta-\zeta\) coordinate system
Remote stress
Principal stresses
Principal strains
Direction cosines
Pressure
Concentrated force
Body force
Bending moment per unit length
Twisting moment per unit length
Twisting moment (torsional moment) or temperature
Torsional angle per unit length or crack propagation angle
Surface tension
Airy’s stress function or stress function in torsion
Stress concentration factor
Stress intensity factor of Mode I
Stress intensity factor of Mode II
Stress intensity factor of Mode III

\(\omega\)
Normal stress \((\sigma_{\xi}, \sigma_{\eta}, \sigma_{\zeta})\)
Shear stress \((\tau_{\xi\eta}, \tau_{\eta\zeta}, \tau_{\zeta\xi})\)
\(\sigma_0, \tau_0\) or \(\sigma_{x00}, \sigma_{y00}, \tau_{xy00}\)
\(\sigma_1, \sigma_2, \sigma_3\)
\(\epsilon_1, \epsilon_2, \epsilon_3\)
l, m, n, (i = 1, 2, 3)
p or q
P, Q
\(X, Y, Z\) or \(F_x, F_y\)
\(M_x, M_y\)
\(M_{xy}\) or \(M_{yx}\)
\(T\)
\(\theta_0\)
\(S\)
\(\phi\)
\(K_I\)
\(K_{II}\)
\(K_{III}\)
| **Radius of circle or major radius of ellipse or crack** | $a$ |
| **Minor radius of ellipse** | $b$ |
| **Notch root radius or radius of curvature in membrane** | $\rho$ |
| **Notch depth** | $t$ |
| **Young’s modulus** | $E$ |
| **Poisson’s ratio** | $\nu$ |
| **Shear modulus** | $G$ |
| **Displacement in $x, y, z$ coordinate system** | $u, v, w$ |
| **Displacement of membrane** | $z$ |
| **Width of plate** | $W$ |

(Note: $v$ looks the same as Poisson’s ratio but is different.)
Preface for Part II: Stress Concentration

Part II of this book is a compilation of the ideas on stress concentration which the author has developed over many years of teaching and research. This is not a handbook of stress concentration factors. This book guides a fundamental way of thinking for stress concentration. Fundamentals, typical misconceptions and new ways of thinking about stress concentration are presented. One of the motivations for writing this book is the concern about a decreasing basic knowledge of recent engineers about the nature of stress concentration.

It was reported in the United States and Europe [1–3] that the economic loss of fracture accidents reaches about 4% of GDP. Fracture accidents occur repeatedly regardless of the progress of science and technology. It seems that the number and severity of serious accidents is increasing. The author was involved in teaching strength of materials and theory of elasticity for many years in universities and industry and a recent impression based on the author’s experience is that many engineers do not understand the fundamentals of the theory of elasticity.

How many engineers can give the correct answers to basic problems such as those in Figures 1 and 2?

The theory of elasticity lectures are likely to be abstract and mathematical. This trend is evident in the topics and emphasis of many text books. Such textbooks may be useful for some researchers but are almost useless for most practicing engineers. The author has been aware of this problem for many years and has changed the pedagogy of teaching the theory of elasticity by introducing various useful ways of thinking (see Part I). Engineers specializing in strength design and quality control are especially requested to acquire the fundamentals of theory of elasticity and afterwards to develop a sense about stress concentration. The subject is not difficult. Rather, as readers become familiar with the problems contained in this book, they will understand that the problems of stress concentration are full of interesting paradoxes.

Few accidents occur because of a numerical mistake or lack of precision in a stress analysis. A common attitude that analysis by FEM software will guarantee the correct answer and safety is the root cause of many failures. Most mistakes in the process of FEM analysis are made at the
beginning stage of determining boundary conditions regarding forces and displacements. Even worse, many users of FEM software are often not aware of such mistakes even after looking at strange results because they do not have a fundamental understanding of theory of elasticity and stress concentration.

The origin of fracture related accidents are mostly at the stress concentrations in a structure. As machine components and structures have various shapes for functional reasons, stress concentration cannot be avoided. Therefore, strength designers are required to evaluate stress concentration correctly and to design the shape of structures so that the stress concentration does not exceed the safety limits.

In this book, various elastic stress concentration problems are the main topic. The strains in an elastic state can be determined by Hooke’s law in terms of stresses. In elastic–plastic conditions, the relationship between stresses and strains deviates from Hooke’s law. Once plastic

**Figure 1** Stress concentration at a circular hole in a wide plate. How large is the maximum stress? (See Figure 1.2 in Example problem 1.1 in Part II, Chapter 1.)

**Figure 2** A cylindrical specimen for comparison of the fracture strengths at a smooth part and a notched part under tension (material is 0.13% annealed carbon steel, dimension unit is mm). Where does this specimen fracture from by tensile test? (See Figure 14.7 in the Example problem 14.1 in Part II, Chapter 14.)
yielding occurs at a notch root, the stress concentration factor decreases compared to the elastic value and approaches one. However, the strain concentration factor increases and approaches the elastic value squared. Therefore, in elastic-plastic conditions, fatigue behavior is described in terms of strain concentration. However, if the stress and strain relationship at the notch root does not deviate much from Hooke’s law or work hardening of material occurs after yielding, the description based on elastic stress concentration is valid. In general, in the case of high cycle fatigue, it is reasonable and effective for the solution of practical problems to consider only the elastic stress concentration. Thus, it is crucially important for strength design engineers to understand the nature of elastic stress concentration.

References


Part II Nomenclature

Stresses and strains in orthogonal coordinate system \((x, y, z)\)
- Normal stress \((\sigma_x, \sigma_y, \sigma_z)\)
- Normal strain \((\varepsilon_x, \varepsilon_y, \varepsilon_z)\)
- Shear stress \((\tau_{xy}, \tau_{yz}, \tau_{zx})\)
- Shear strain \((\gamma_{xy}, \gamma_{yz}, \gamma_{zx})\)

Stresses and strains in cylindrical coordinate system \((r, \theta, z)\)
- Normal stress \((\sigma_r, \sigma_\theta, \sigma_z)\)
- Normal strain \((\varepsilon_r, \varepsilon_\theta, \varepsilon_z)\)
- Shear stress \((\tau_{r\theta}, \tau_{\theta z}, \tau_{rz})\)
- Shear strain \((\gamma_{r\theta}, \gamma_{\theta z}, \gamma_{rz})\)

Normal stress and shear stress in \(\xi-\eta\) coordinate system
- Normal stress \((\sigma_\xi, \sigma_\eta)\)
- Shear stress \(\tau_{\xi\eta}\)

Remote stress \(\sigma_0, \tau_0\) or \(\sigma_\infty, \sigma_{xy}^\infty, \tau_{xy}^\infty\)
Principal stresses \(\sigma_1, \sigma_2\)
Pressure \(p\) or \(q\)
Concentrated force \(P, Q\)
Stress concentration factor \(K_t\)
Stress concentration factor in elastic plastic state \(K_\sigma\)
Strain concentration factor in elastic plastic state \(K_e\)
Stress intensity factor of Mode I \(K_1\)
Stress intensity factor of Mode II \(K_{II}\)
Stress intensity factor of Mode III \(K_{III}\)
Radius of circle or major radius of ellipse \(a\)
Minor radius of ellipse (or Burger’s vector of dislocation) \(b\)
Notch root radius \(\rho\)
Notch depth \(t\)
Young’s modulus \(E\)
Poisson’s ratio \(\nu\)
Shear modulus \(G\)
Displacement in \(x, y, z\) coordinate system \(u, v, w\)
Shape parameter of ellipse \(R = \sqrt{(a + b)/(a - b)}\)
Plastic zone size \(R\)
Acknowledgments

This book is a joint version of two books originally published separately in Japanese by Yokendo, Tokyo: one is Theory of Elasticity and the other is New Way of Thinking for Stress Concentration. The Japanese version of Theory of Elasticity has been used in many universities since its publication in 1984 and it is regarded as one of the best elasticity textbooks in Japan. For the publication of the Japanese versions, the author was indebted especially to the late Prof. Tatsuo Endo and the late Prof. Makoto Isida for their invaluable comments and moreover to Mr. Kiyoshi Oikawa, the president of Yokendo Publishing Co. Ltd. for publishing the original Japanese versions of these two books and kindly approving the publication of this English version.

In advance of publication of the English version, the author gave lectures on the theory of elasticity and stress concentration at several universities in Europe and the United States. Especially at Aalto University in Finland, the author taught this subject for one complete semester and realized the importance of the new way of teaching the theory of elasticity. Throughout the author’s long experience of teaching the theory of elasticity and its relationship with fatigue design, the author got useful comments and encouragements from Prof. Gary Marquis, Dean of the School of Engineering at Aalto University, Prof. Darrell Socie, University of Illinois, Prof. Stefano Beretta, Politecnico di Milano, Prof. Masahiro Endo, Fukuoka University and Prof. Hisao Matsunaga, Kyushu University. The author would like to express his sincere thanks to them.

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Part I

Theory of Elasticity