Understanding Symmetrical Components for Power System Modeling

J.C. Das

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UNDERSTANDING SYMMETRICAL COMPONENTS FOR POWER SYSTEM MODELING
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J. C. DAS
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He received MSEE degree from Tulsa University, Tulsa, Oklahoma; and BA (advanced mathematics) and BEE degrees from Punjab University, India.
T HIS BOOK BY J. C. DAS OFFERS AN IN-DEPTH, practical, yet intellectually appealing treatment of symmetrical components not seen since the late Paul M. Anderson’s classic, *Analysis of Faulted Power Systems*, which was first published in 1995 by the Wiley-IEEE Press in the Power Engineering Series. The present book leverages the author’s well over 30 years of experience in power system studies, and continues in his same tradition of attention to details, which should appeal to those professionals who benefitted from his writing style demonstrated in his four earlier books. The subject is taught at the undergraduate and graduate courses in most universities with a power systems option.

The advent of the symmetrical components concept is due to the Westinghouse electrical engineer Charles LeGeyt Fortescue, who was born in 1876 at York Factory in Manitoba, Canada, who became the first electrical engineer to graduate from Queen’s University at Kingston in Ontario, Canada, in 1898. In 1918, Fortescue contributed an 88 page, now classic, remarkable paper by the title “Method of Symmetrical Coordinates Applied to the Solution of Polyphase Networks” in the Transactions of the American Institute of Electrical Engineers (AIEE), one of the two predecessors of present day IEEE. This breakthrough is due to Fortescue’s investigations of railway electrification problems which began in 1913. Following the paper’s publication, the earlier name “Symmetrical Coordinates” was changed to “Symmetrical Components” and the approach gained in popularity ever since it was disclosed as an indispensable method of dealing with unbalanced three-phase operation problems of electric power systems. A thorough understanding of the application of symmetrical components is required for proper design of electric power protection systems.

Chapter 1 uses matrix algebra to demonstrate the non-uniqueness of symmetrical component transformations. Chapter 2 treats sequence impedances, their networks, and their reduction. Chapters 3 and 4 discuss symmetrical component applications in generating models for transmission lines, cables, synchronous generators, and induction motors. Chapter 3 notes that much of the theoretical underpinnings of the area discussed should be reviewed elsewhere. Prior to discussing three-phase models of two-winding three-phase transformers and conductors, Chapter 5 begins by advising the reader to study this chapter along with Chapter 7. Chapter 6 covers unsymmetrical shunt and series faults and also calculations of overvoltages at the fault plane.

M. E. El-Hawary
THIS SHORT BOOK consisting of seven chapters attempts to provide a clear understanding of the theory of the symmetrical component transformation and its applications in power system modeling.

Chapter 1 takes a mathematical approach to document that the symmetrical component eigenvectors are not unique and one can choose arbitrary vectors meeting the constraints, but these will not be very meaningful in the transformation—thus selection of vectors as they are forms a sound base of the transformation. This is followed by Chapter 2, which details the concepts of sequence impedances, their models, formation of sequence impedance networks and their reduction. Chapters 3 and 4 are devoted to symmetrical component applications in generating the models for transmission lines, cables, synchronous generators, and induction motors. Chapters 5 and 7 are meant to be read together and describe three-phase models and phase-coordinate method of solution where the phase-unbalance in the power system cannot be ignored and symmetrical components cannot be applied. Chapter 6 covers unsymmetrical shunt and series faults and also calculations of overvoltages at the fault point (COG). It has a worked out longhand example to illustrate the complexity of calculations even in a simple electrical distribution system. This is followed with the matrix methods of solution which have been adopted for calculations on digital computers. The author is thankful and appreciates all the cooperation and help received from Ms. Mary Hatcher, Wiley-IEEE and her staff in completing this publication. She rendered similar help and cooperation for the publications of author’s other two books by IEEE Press (see Author’s profile). An author cannot expect anything better than the help and cooperation rendered by Ms. Mary Hatcher.

The authors special thanks go to Dr. M.E. El-Hawary, Professor of Electrical and Computer Engineering, Dalhousie University, Canada for writing the Foreword to this book. He is a renowned authority on Electrical Power System; the author is grateful to him, and believes that this Foreword adds to the value and the marketability of the book.

J. C. Das
THE METHOD of symmetrical components was originally proposed by Fortescue in 1918 [1]. We study three-phase balanced systems, by considering these as single-phase system. The current or voltage vectors in a three-phase balanced system are all displaced by 120 electrical degrees from each other. The fundamental texts on electrical circuits [2] derive the equations governing the behavior of three-phase balanced systems. This simplicity of representing a three-phase as a single-phase system is lost for unbalanced systems. The method of symmetrical components has been an important tool for the study of unbalanced three-phase systems, unsymmetrical short-circuit currents, models of rotating machines and transmission lines, etc.

There have been two approaches for the study of symmetrical components:

- A physical description, without going into much mathematical matrix algebra equations.
- A mathematical approach using matrix theory.

This book will cover each of these two approaches to provide a comprehensive understanding. The mathematical approach is adopted in this chapter followed by Chapter 2, which provides some practical concepts and physical significance of symmetrical components. Some publications on symmetrical components are in References [3–6].

It can be mentioned that in the modern age of digital computers, the long-hand calculations using symmetrical components is outdated. See an example of short-circuit calculations in Chapter 6, which is a tedious and lengthy hand calculation for a simple system consisting of five components connected to two buses. In practical power systems, the number of buses can exceed 1000. Today, the systems are modeled with raw input data, and the programs will calculate the sequence components and apply these to derive a result demanded by the problem. Yet, an understanding of symmetrical components is necessary to understand the results from system simulation programs. The reader should understand the limitations of simulation models which are discussed in the chapters to follow.
1.1 TRANSFORMATIONS

Symmetrical component method is a transform. There are number of transformations in electrical engineering, for example, Laplace transform, Fast Fourier transform, Park’s transform, Clarke component transform, and the like. There are three steps that are applicable in any transform for the solution of a problem:

- The parameters of the original problem are transformed by the application of the transform to entirely new parameters.
- The solution with the altered parameters is arrived at. The fundamental concept is that the transformed parameters are much easier to solve than the parameters of the original problem.
- Inverse transform is applied to the solved parameters to get to the solution of the original problem.

1.2 CHARACTERISTIC ROOTS, EIGENVALUES, AND EIGENVECTORS

The matrix theory can be applied to understand some fundamental aspects of symmetrical components. The reader must have some knowledge of the matrices as applied to electrical engineering [7, 8], though enough material is provided for continuity of reading.

1.2.1 Definitions

1.2.1.1 Characteristic Matrix
For a square matrix \( \tilde{A} \), the matrix formed as \( |\tilde{A} - \lambda \tilde{I}| \) is called the characteristic matrix. Here \( \lambda \) is a scalar and \( \tilde{I} \) is a unity matrix.

1.2.1.2 Characteristic Polynomial
The determinant \( |\tilde{A} - \lambda \tilde{I}| \) when expanded gives a polynomial is called the characteristic polynomial of matrix \( \tilde{A} \).

1.2.1.3 Characteristic Equation
The equation \( |\tilde{A} - \lambda \tilde{I}| = 0 \) is called the characteristic equation of matrix \( \tilde{A} \).

1.2.1.4 Eigenvalues
The roots of the characteristic equation are called the characteristic roots or eigenvalues.

1.2.1.5 Eigenvectors, Characteristic Vectors
Each characteristic root \( \lambda \) has a corresponding non-zero vector \( \tilde{x} \) that satisfies the equation

\[
|\tilde{A} - \lambda \tilde{I}| \tilde{x} = 0
\]  

(1.1)
The non-zero vector $\vec{x}$ is called the characteristic vector or eigenvector.

Some properties of the eigenvalues are:

- Any square matrix $\bar{A}$ and its transpose $\bar{A}'$ have the same eigenvalues.
- The sum of the eigenvalues of a matrix is equal to the trace of the matrix (the sum of the elements on the principal diagonal is called the trace of the matrix).
- The product of the eigenvalues of the matrix is equal to the determinant of the matrix. If

$$\lambda_1, \lambda_2, \ldots, \lambda_n$$

are the eigenvalues of $\bar{A}$, then the eigenvalues of

$$k\bar{A}$$

are $k\lambda_1, k\lambda_2, \ldots, k\lambda_n$

$$\bar{A}^m$$

are $\lambda_1^m, \lambda_2^m, \ldots, \lambda_n^m$

$$\bar{A}^{-1}$$

are $1/\lambda_1, 1/\lambda_2, \ldots, 1/\lambda_n$ (1.2)

- Zero is a characteristic root of a matrix, only if the matrix is singular.
- The characteristic roots of a triangular matrix are diagonal elements of the matrix.
- The characteristic roots of a Hermitian matrix are all real.
- The characteristic roots of a real symmetric matrix are all real, as the real symmetric matrix will be Hermitian. A square matrix $\bar{A}$ is called a Hermitian matrix if every $i$-$j$th element of the matrix is equal to the conjugate complex $j$-$i$th element, that is, the matrix

$$\begin{vmatrix}
1 & 2 + j3 & 3 + j \\
2 - j3 & 2 & 1 - j2 \\
3 - j & 1 + j2 & 5
\end{vmatrix}$$

is a Hermitian matrix.

**Example 1.1** Find eigenvalues and eigenvectors of matrix

$$\bar{A} = \begin{vmatrix}
-2 & 2 & -3 \\
2 & 1 & -6 \\
-1 & 2 & 0
\end{vmatrix}$$

Write the characteristic equation

$$\bar{A} = \begin{vmatrix}
-2 - \lambda & 2 & -3 \\
2 & 1 - \lambda & -6 \\
-1 & 2 & 0 - \lambda
\end{vmatrix} = 0$$

Its solution can be shown to be

$$(\lambda + 3)(\lambda + 3)(\lambda - 5) = 0$$

Therefore, the eigenvalues are

$$\lambda = -3, -3, 5$$