PRINCIPLES AND MODERN APPLICATIONS OF MASS TRANSFER OPERATIONS
PRINCIPLES AND MODERN APPLICATIONS OF MASS TRANSFER OPERATIONS

Third Edition

Jaime Benitez

WILEY
A mis tres Teresas, Ivette, Humberto, y Jaime: mi hermosa familia.
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Preface to the Third Edition

The most significant difference between the first two editions and the third edition is the adoption in the latter of PTC Mathcad Prime most recent version (version 3.1 as of this writing). PTC Mathcad Prime—one of the world’s leading tools for technical computing in the context of engineering, science, and math applications—is a significant departure from the previous versions of Mathcad. There is a definite learning curve associated with making the switch from Mathcad to Mathcad Prime. However, the new features included in Mathcad Prime make switching from Mathcad worthwhile. Besides, programs written for the previous versions of Mathcad will not run in Mathcad Prime. Other differences in this edition are listed in the following paragraphs.

In Chapter 3 of the third edition, the material covered in Problems 3.14 and 3.15 of the second edition to determine analytically minimum flow rates in absorbers and strippers is incorporated in the theoretical presentation of Section 3.4 (Material Balances), and the corresponding Mathcad Prime code for solving these problems is given. In Section 3.5 of the third edition, Mathcad Prime code is given to determine analytically the number of ideal stages required for absorbers and strippers.

Section 4.2 of the third edition use, exclusively, the updated Billet and Schultes correlations for estimating the loading and flooding points in packed beds, and the corresponding gas-pressure drop for operation between these limits. The Generalized Pressure Drop Correlation (GPDC) is not included. The updated Billet and Schultes correlations are also used to estimate the volumetric mass-transfer coefficients in both liquid and gas phases. New end-of-chapter problems have been added throughout this third edition.

I want to acknowledge the extraordinarily thorough editing job that Katrina Maceda, Production Editor at Wiley, and Baljinder Kaur, Project Manager at Aptara did on this edition. The book is much better now because of them. It was a pleasure working with both of you. Ludo de Wolf, a physical therapist with gifted hands and a delightful sense of humor, literally removed from my shoulders the heavy load of completing this edition. Thanks to my wife Teresa for her unconditional love and support. I know it is not easy!

Jaime Benítez
Gainesville, Florida
Preface to the Second Edition

The idea for the first edition of this book was born out of my experience teaching a course on mass-transfer operations at the Chemical Engineering Department of the University of Puerto Rico during the previous 25 years. This course is the third in a three-course unit operations sequence. The first course covers momentum transfer (fluid mechanics), and the second course covers heat transfer. Besides these two courses, another prerequisite of the mass-transfer course is a two-semester sequence of chemical engineering thermodynamics.

I decided to write a textbook for a first course on mass-transfer operations with a level of presentation that was easy to follow by the reader, but with enough depth of coverage to guarantee that students using the book will, upon successful completion of the course, be able to specify preliminary designs of the most common mass-transfer equipment (such as absorbers, strippers, distillation columns, liquid extractors, etc.). I decided also to incorporate, from the very beginning of the book, the use of Mathcad, a computational tool that is, in my opinion, very helpful and friendly. The first edition of this book was the result of that effort.

Part of my objective was achieved, as evidenced by the following excerpt from a very thorough review of the first edition of my book, written by Professor Mark J. McCready, a well-known expert in chemical engineering education: “If the topics that are needed for a given course are included in this text, I would expect the educational experience to go smoothly for both student and instructor. I think that students will like this book, because the explanations are clear, the level of difficulty is appropriate, and the examples and included data give the book very much of a ‘handbook’ flavor. Instructors will find that, overall, the topics are presented in a logical order and the discussion makes sense; there are many examples and lots of homework problems” (McCready, M. J., AIChE J., Vol. 49, No. 1, January 2003).

“Each major section of the book has learning objectives which certainly benefit the students and perhaps the instructor. A key feature of the book, which separates it from the other texts mentioned above, is the incorporation of Mathcad for both example problems and homework questions. A library of Mathcad programs for solving the Maxwell-Stefan equations, packed column calculations, sieve-tray design, binary distillation problems by McCabe-Thiele method, and multistage counterflow extraction is given in the appendices. These programs enable students to obtain useful solutions with less effort, as well as allow them to explore the different variables or parameters. The wide availability, low cost, and ease of use of Mathcad allow it to be the modern equivalent of ‘back of the envelope’ calculations, which can be refined, if necessary, using full-scale process simulators” (McCready, 2003).
However, the same reviewer also points out some limitations of the book. One of the main objectives of this second edition is to remedy those shortcomings of the first edition to make it more attractive as a textbook to a broader audience. Another important objective of the second edition is to incorporate material related to mass-transfer phenomena in biological systems. Many chemical engineering departments all over the world are changing their names and curricula to include the area of biochemical engineering in their offerings. The second edition includes pertinent examples such as convection and diffusion of oxygen through the body's circulatory system, bio-artificial kidneys, separation of sugars by chromatography, and purification of monoclonal antibodies by affinity adsorption.

As with the first edition, the first four chapters of the book present a basic framework for analysis that is applicable to most mass-transfer operations. Chapters 5 to 7 apply this common methodology to the analysis and design of some of the most popular types of mass-transfer operations. Chapter 5 covers gas absorption and stripping; Chapter 6 covers distillation; and Chapter 7 covers liquid extraction. Chapter 8, new to the second edition, covers humidification operations in general, and detailed design of packed cooling towers specifically. These operations—in particular, cooling towers—are very common in industry. Also, from the didactic point of view, their analysis and design involve simultaneous mass- and heat-transfer considerations. Therefore, the reader is exposed in detail to the similarities and differences between these two transport phenomena. Chapter 9, also new, covers mass-transfer processes using barriers (membranes) and solid sorption agents (adsorption, ion exchange, and chromatography).

In response to suggestions by Professor McCready and other reviewers, some other revisions and additions to the second edition are:

- In Chapter 1, the Maxwell–Stefan equations (augmented by the steady-state continuity equation for each component) are solved numerically using a combination of a Runge-Kutta-based differential equation solver (Rkfixed) and an algebraic equation solver (Given-Find), both included in Mathcad. This methodology is much more flexible than the one presented in the first edition (orthogonal collocation), and its theoretical justification is well within the scope of the mathematical background required for a first course in mass-transfer operations.
- Chapter 1 includes a section on diffusion in solids.
- Chapter 2 includes a section on boundary-layer theory and an example on simultaneous mass and heat transfer during air humidification.
- Chapter 6 includes a section on multistage batch distillation.

I wish to acknowledge gratefully the contribution of the University of Puerto Rico at Mayagüez to this project. My students in the course INQU 4002 reviewed the material in the book, found quite a few errors, and gave excellent suggestions on ways to improve its content and presentation. My students are my source of motivation; they make all the effort to prepare this book worthwhile!

Jaime Benítez
Mayagüez, Puerto Rico
Preface to the First Edition

The importance of the mass-transfer operations in chemical processes is profound. There is scarcely any industrial process that does not require a preliminary purification of raw materials or final separation of products. This is the realm of mass-transfer operations. Frequently, the major part of the cost of a process is that for the separations accomplished in the mass-transfer operations, a good reason for process engineers and designers to master this subject. The mass-transfer operations are largely the responsibility of chemical engineers, but increasingly practitioners of other engineering disciplines are finding them necessary for their work. This is especially true for those engaged in environmental engineering, where separation processes predominate.

My objective in writing this book is to provide a means to teach undergraduate chemical engineering students the basic principles of mass transfer and to apply these principles, aided by modern computational tools, to the design of equipment used in separation processes. The idea for it was born out of my experiences during the last 25 years teaching mass-transfer operations courses at the University of Puerto Rico.

The material treated in the book can be covered in a one-semester course. Chapters are divided into sections with clearly stated objectives at the beginning. Numerous detailed examples follow each brief section of text. Abundant end-of-chapter problems are included, and problem degree of difficulty is clearly labeled for each. Most of the problems are accompanied by their answers. Computer solution is emphasized, both in the examples and in the end-of-chapter problems. The book uses mostly SI units, which virtually eliminates the tedious task of unit conversions and makes it “readable” to the international scientific and technical community.

Following the lead of other authors in the chemical engineering field and related technical disciplines, I decided to incorporate the use of Mathcad into this book. Most readers will probably have a working knowledge of Mathcad. (Even if they don’t, my experience is that the basic knowledge needed to begin using Mathcad effectively can be easily taught in a two-hour workshop.) The use of Mathcad simplifies mass-transfer calculations to a point that it allows the instructor and the student to readily try many different combinations of the design variables, a vital experience for the amateur designer.

The Mathcad environment can be used as a sophisticated scientific calculator, can be easily programmed to perform a complicated sequence of calculations (for example, to check the design of a sieve-plate column for flooding, pressure drop, entrainment, weeping, and calculating Murphree plate efficiencies), can be used to plot results, and as a word processor to neatly present homework
problems. Mathcad can perform calculations using a variety of unit systems, and will give a warning signal when calculations that are not dimensionally consistent are tried. This is a most powerful didactic tool, since dimensional consistency in calculations is one of the most fundamental concepts in chemical engineering education.

The first four chapters of the book present a basic framework of analysis that is applicable to any mass-transfer operation. Chapters 5 to 7 apply this common methodology to the analysis and design of the most popular types of mass-transfer operations. Chapter 5 covers gas absorption and stripping, chapter 6 distillation columns, and chapter 7 liquid extraction. This choice is somewhat arbitrary, and based on my own perception of the relevance of these operations. However, application of the general framework of analysis developed in the first four chapters should allow the reader to master, with relative ease, the peculiarities of any other type of mass-transfer operation.

I wish to acknowledge gratefully the contribution of the University of Puerto Rico at Mayagüez to this project. My students in the course INQU 4002 reviewed the material presented in the book, found quite a few errors, and gave excellent suggestions on ways to improve it. My special gratitude goes to Teresa, my wife, and my four children who were always around lifting my spirits during the long, arduous hours of work devoted to this volume. They make it all worthwhile!

Jaime Benítez
Mayagüez, Puerto Rico
**Nomenclature**

**LATIN LETTERS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>absorption factor; dimensionless.</td>
</tr>
<tr>
<td>A</td>
<td>mass flow rate of species A; kg/s.</td>
</tr>
<tr>
<td>A_a</td>
<td>active area of a sieve tray; m².</td>
</tr>
<tr>
<td>A_d</td>
<td>area taken by the downspout in a sieve tray; m².</td>
</tr>
<tr>
<td>A_h</td>
<td>area taken by the perforations on a sieve tray; m².</td>
</tr>
<tr>
<td>A_M</td>
<td>membrane area; m².</td>
</tr>
<tr>
<td>A_n</td>
<td>net cross-section area between trays inside a tray column; m².</td>
</tr>
<tr>
<td>A_t</td>
<td>total cross-section area, m².</td>
</tr>
<tr>
<td>a</td>
<td>mass-transfer surface area per unit volume; m⁻¹.</td>
</tr>
<tr>
<td>a_h</td>
<td>hydraulic, or effective, specific surface area of packing; m⁻¹.</td>
</tr>
<tr>
<td>B</td>
<td>mass flow rate of species B; kg/s.</td>
</tr>
<tr>
<td>B_0</td>
<td>viscous flow parameter; m².</td>
</tr>
<tr>
<td>c</td>
<td>total molar concentration; mol/m³.</td>
</tr>
<tr>
<td>c_p, C_i</td>
<td>molar concentration of species i; mol/m³.</td>
</tr>
<tr>
<td>C</td>
<td>total number of components in multicomponent distillation.</td>
</tr>
<tr>
<td>C_p</td>
<td>specific heat at constant pressure; J/kg·K.</td>
</tr>
<tr>
<td>C_S</td>
<td>humid heat; J/kg·K.</td>
</tr>
<tr>
<td>D</td>
<td>drag coefficient; dimensionless.</td>
</tr>
<tr>
<td>D_a</td>
<td>Damkoehler number for first-order reaction; dimensionless.</td>
</tr>
<tr>
<td>D_ij</td>
<td>Maxwell–Stefan diffusivity for pair i-j; m²/s.</td>
</tr>
<tr>
<td>D_ij</td>
<td>Fick diffusivity or diffusion coefficient for pair i-j; m²/s.</td>
</tr>
<tr>
<td>D_K,i</td>
<td>Knudsen diffusivity for component i; m²/s.</td>
</tr>
<tr>
<td>d_e</td>
<td>equivalent diameter; m.</td>
</tr>
<tr>
<td>d_i</td>
<td>driving force for mass diffusion of species i; m⁻¹.</td>
</tr>
<tr>
<td>d_o</td>
<td>outside diameter; m.</td>
</tr>
<tr>
<td>d_o</td>
<td>perforation diameter in a sieve plate; m.</td>
</tr>
<tr>
<td>d_p</td>
<td>particle size; m.</td>
</tr>
<tr>
<td>d_vs</td>
<td>Sauter mean drop diameter defined in equation (7-48); m.</td>
</tr>
<tr>
<td>DM</td>
<td>dimensional matrix.</td>
</tr>
<tr>
<td>D</td>
<td>tube diameter; m.</td>
</tr>
<tr>
<td>D</td>
<td>distillate flow rate; moles/s.</td>
</tr>
<tr>
<td>E</td>
<td>fractional entrainment; liquid mass flow rate/gas mass flow rate.</td>
</tr>
<tr>
<td>E</td>
<td>extract mass flow rate, kg/s.</td>
</tr>
<tr>
<td>E_m</td>
<td>mechanical efficiency of a motor-fan system; dimensionless.</td>
</tr>
<tr>
<td>E_o</td>
<td>Eotvos number defined in equation (7-53); dimensionless.</td>
</tr>
<tr>
<td>E_F</td>
<td>extraction factor defined in equation (7-19); dimensionless.</td>
</tr>
<tr>
<td>E_ME</td>
<td>Murphree stage efficiency in terms of extract composition.</td>
</tr>
<tr>
<td>E_MG</td>
<td>Murphree gas-phase tray efficiency; dimensionless.</td>
</tr>
<tr>
<td>E_MGE</td>
<td>Murphree gas-phase tray efficiency corrected for entrainment.</td>
</tr>
<tr>
<td>E_O</td>
<td>overall tray efficiency of a cascade; equilibrium trays/real trays.</td>
</tr>
<tr>
<td>E_OG</td>
<td>point gas-phase tray efficiency; dimensionless.</td>
</tr>
</tbody>
</table>
Nomenclature

\( f_{12} \) proportionality coefficient in equation (1-21).

\( f \) friction factor; dimensionless.

\( f \) fractional approach to flooding velocity; dimensionless.

\( f_{\text{ext}} \) fractional extraction; dimensionless.

\( F \) mass-transfer coefficient; mol/m²·s.

\( F \) molar flow rate of the feed to a distillation column; mol/s.

\( F_{\text{ext}} \) mass flow rate of the feed to a liquid extraction process; kg/s.

\( FR_{i,D} \) fractional recovery of component \( i \) in the distillate; dimensionless.

\( FR_{i,W} \) fractional recovery of component \( i \) in the residue; dimensionless.

\( Fr_{L} \) liquid Froude number; dimensionless.

\( Ga \) Galileo number; dimensionless.

\( G_{M} \) superficial molar velocity; mol/m²·s.

\( G_{Ma} \) superficial liquid-phase molar velocity; mol/m²·s.

\( G_{Mg} \) superficial gas-phase molar velocity; mol/m²·s.

\( G_{x} \) superficial liquid-phase mass velocity; kg/m²·s.

\( G_{y} \) superficial gas-phase mass velocity; kg/m²·s.

\( Gr_{D} \) Grashof number for mass transfer; dimensionless.

\( Gr_{H} \) Grashof number for heat transfer; dimensionless.

\( Gz \) Graetz number; dimensionless.

\( g \) acceleration due to gravity; 9.8 m/s².

\( g_{c} \) dimensional conversion factor; 1 kg·m/N·s².

\( H \) Henry’s law constant; atm, kPa, Pa.

\( H \) molar enthalpy; J/mol.

\( H \) height of mixing vessel; m.

\( H^{'} \) enthalpy of gas–vapor mixture; J/kg.

\( H_{\text{ETS}} \) height equivalent to a theoretical stage in staged liquid extraction columns; m.

\( HK \) heavy-key component in multicomponent distillation.

\( \Delta H_{S} \) heat of solution; J/mol of solution.

\( H_{\text{Lt}} \) height of a liquid-phase transfer unit; m.

\( H_{\text{Gt}} \) height of a gas-phase transfer unit; m.

\( H_{\text{OGt}} \) overall height of a gas-phase transfer unit; m.

\( H_{\text{OLt}} \) overall height of a liquid-phase transfer unit; m.

\( h \) convective heat-transfer coefficient, W/m²·K.

\( h_{d} \) dry-tray head loss; cm of liquid.

\( h_{l} \) equivalent head of clear liquid on tray; cm of liquid.

\( h_{L} \) specific liquid holdup; m³ holdup/m³ packed bed.

\( h_{t} \) total head loss/tray; cm of liquid.

\( h_{w} \) weir height; m.

\( h_{\sigma} \) head loss due to surface tension; cm of liquid.

\( h_{2\phi} \) height of two-phase region on a tray; m.

\( i \) number of dimensionless groups needed to describe a situation.

\( j_{D} \) Chilton–Colburn \( j \)-factor for mass transfer; dimensionless.

\( j_{H} \) Chilton–Colburn \( j \)-factor for heat transfer; dimensionless.

\( j_{i} \) mass diffusion flux of species \( i \) with respect to the mass-average velocity; kg/m²·s.

\( J_{i} \) molar diffusion flux of species \( i \) with respect to the molar-average velocity; mol/m²·s.
Nomenclature

\[ J_0 \]  Bessel function of the first kind and order zero; dimensionless.
\[ J_1 \]  Bessel function of the first kind and order one; dimensionless.
\[ K \]  distribution coefficient; dimensionless.
\[ K \]  Krogh diffusion coefficient; \( \text{cm}^3 \text{O}_2/\text{cm} \cdot \text{s} \cdot \text{torr} \).
\[ K \]  parameter in Langmuir adsorption isotherm; \( \text{Pa}^{-1} \).
\[ K_{AB} \]  molar selectivity parameter in ion exchange; dimensionless.
\[ K_W \]  wall factor in Billet–Schultes pressure-drop correlations; dimensionless.
\[ k \]  thermal conductivity, \( \text{W} / \text{m} \cdot \text{K} \).
\[ k_c \]  convective mass-transfer coefficient for diffusion of A through stagnant B in dilute gas-phase solution with driving force in terms of molar concentrations; m/s.
\[ k_c' \]  convective mass-transfer coefficient for equimolar counterdiffusion in gas-phase solution with driving force in molar concentrations; m/s.
\[ k_G \]  convective mass-transfer coefficient for diffusion of A through stagnant B in dilute gas-phase solution with driving force in terms of partial pressure; \( \text{mol} / \text{m}^2 \cdot \text{s} \cdot \text{Pa} \).
\[ K_G \]  overall convective mass-transfer coefficient for diffusion of A through stagnant B in dilute solutions with driving force in terms of partial pressures; \( \text{mol} / \text{m}^2 \cdot \text{s} \cdot \text{Pa} \).
\[ k_G' \]  convective mass-transfer coefficient for equimolar counterdiffusion in gas-phase solution with driving force in terms of partial pressure; \( \text{mol} / \text{m}^2 \cdot \text{s} \cdot \text{Pa} \).
\[ k_L \]  convective mass-transfer coefficient for diffusion of A through stagnant B in dilute liquid-phase solution with driving force in terms of molar concentrations; m/s.
\[ k_L' \]  convective mass-transfer coefficient for equimolar counterdiffusion in liquid-phase solution with driving force in terms of molar concentrations; m/s.
\[ K_n \]  Knudsen number, dimensionless.
\[ k_r \]  reaction rate constant; \( \text{mol} / \text{m}^2 \cdot \text{s} \cdot \text{mol} \) fraction.
\[ K_r \]  restrictive factor for diffusion of liquids in porous solids; dimensionless.
\[ k_x \]  convective mass-transfer coefficient for diffusion of A through stagnant B in dilute liquid-phase solution with driving force in terms of mol fractions; \( \text{mol} / \text{m}^2 \cdot \text{s} \).
\[ K_x \]  overall convective mass-transfer coefficient for diffusion of A through stagnant B in dilute solutions with driving force in terms of liquid-phase mol fractions; \( \text{mol} / \text{m}^2 \cdot \text{s} \).
\[ k_x' \]  convective mass-transfer coefficient for equimolar counterdiffusion in liquid-phase solution with driving force in terms of mol fractions; \( \text{mol} / \text{m}^2 \cdot \text{s} \).
\[ k_y \]  convective mass-transfer coefficient for diffusion of A through stagnant B in dilute gas-phase solution with driving force in terms of mol fractions; \( \text{mol} / \text{m}^2 \cdot \text{s} \).
\[ K_y \]  overall convective mass-transfer coefficient for diffusion of A through stagnant B in dilute solutions with driving force in terms of gas-phase mol fractions; \( \text{mol} / \text{m}^2 \cdot \text{s} \).
\[ k_y' \]  convective mass-transfer coefficient for equimolar counterdiffusion in gas-phase solution with driving force in terms of mol fractions; \( \text{mol} / \text{m}^2 \cdot \text{s} \).
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>characteristic length, m.</td>
</tr>
<tr>
<td>$L$</td>
<td>molar flow rate of the $L$-phase; mol/s.</td>
</tr>
<tr>
<td>$L$</td>
<td>length of settling vessel; m.</td>
</tr>
<tr>
<td>$L_K$</td>
<td>light-key component in multicomponent distillation.</td>
</tr>
<tr>
<td>$L_S$</td>
<td>molar flow rate of the nondiffusing solvent in the $L$-phase; mol/s.</td>
</tr>
<tr>
<td>$L'$</td>
<td>mass flow rate of the $L$-phase; kg/s.</td>
</tr>
<tr>
<td>$L'_S$</td>
<td>mass flow rate of the nondiffusing solvent in the $L$-phase; kg/s.</td>
</tr>
<tr>
<td>$L_e$</td>
<td>entrainment mass flow rate, kg/s.</td>
</tr>
<tr>
<td>$L_w$</td>
<td>weir length; m.</td>
</tr>
<tr>
<td>$l$</td>
<td>characteristic length, m.</td>
</tr>
<tr>
<td>$l$</td>
<td>tray thickness; m.</td>
</tr>
<tr>
<td>$l_M$</td>
<td>membrane thickness; m.</td>
</tr>
<tr>
<td>$L_e$</td>
<td>Lewis number; dimensionless.</td>
</tr>
<tr>
<td>$M_i$</td>
<td>molecular weight of species $i$.</td>
</tr>
<tr>
<td>$M_0$</td>
<td>oxygen demand; cm$^3$ O$_2$/cm$^3$·min.</td>
</tr>
<tr>
<td>$MTZ$</td>
<td>width of the mass-transfer zone in fixed-bed adsorption; m.</td>
</tr>
<tr>
<td>$m$</td>
<td>amount of mass; kg.</td>
</tr>
<tr>
<td>$m$</td>
<td>slope of the equilibrium distribution curve; dimensionless.</td>
</tr>
<tr>
<td>$n$</td>
<td>total mass flux with respect to fixed coordinates; kg/m$^2$·s.</td>
</tr>
<tr>
<td>$n_i$</td>
<td>mass flux of species $i$ with respect to fixed coordinates; kg/m$^2$·s.</td>
</tr>
<tr>
<td>$n$</td>
<td>number of variables significant to dimensional analysis of a given problem.</td>
</tr>
<tr>
<td>$n$</td>
<td>rate of mass transfer from the dispersed to the continuous phase in liquid extraction; kg/s.</td>
</tr>
<tr>
<td>$N$</td>
<td>number of species in a mixture.</td>
</tr>
<tr>
<td>$N_i$</td>
<td>total molar flux with respect to fixed coordinates; mol/m$^2$·s.</td>
</tr>
<tr>
<td>$N$</td>
<td>molar flux of species $i$ with respect to fixed coordinates; mol/m$^2$·s.</td>
</tr>
<tr>
<td>$N$</td>
<td>number of equilibrium stages in a cascade; dimensionless.</td>
</tr>
<tr>
<td>$N_E$</td>
<td>mass of B/(mass of A + mass of C) in the extract liquids.</td>
</tr>
<tr>
<td>$N_R$</td>
<td>number of stages in rectifying section; dimensionless.</td>
</tr>
<tr>
<td>$N_R$</td>
<td>mass of B/(mass of A + mass of C) in the raffinate liquids.</td>
</tr>
<tr>
<td>$N_S$</td>
<td>number of stages in stripping section; dimensionless.</td>
</tr>
<tr>
<td>$N_{OL}$</td>
<td>number of liquid-phase transfer units; dimensionless.</td>
</tr>
<tr>
<td>$N_{OG}$</td>
<td>number of gas-phase transfer units; dimensionless.</td>
</tr>
<tr>
<td>$N_{OD}$</td>
<td>overall number of dispersed-phase transfer units; dimensionless.</td>
</tr>
<tr>
<td>$N_{OG}$</td>
<td>overall number of gas-phase transfer units; dimensionless.</td>
</tr>
<tr>
<td>$N_{OL}$</td>
<td>overall number of liquid-phase transfer units; dimensionless.</td>
</tr>
<tr>
<td>$N_u$</td>
<td>Nusselt number; dimensionless.</td>
</tr>
<tr>
<td>$O_i$</td>
<td>molar oxygen concentration in the air leaving an aeration tank; percent.</td>
</tr>
<tr>
<td>$O_{eff}$</td>
<td>oxygen transfer efficiency; mass of oxygen absorbed by water/total mass of oxygen supplied.</td>
</tr>
<tr>
<td>$p'$</td>
<td>pitch, distance between centers of perforations in a sieve plate; m.</td>
</tr>
<tr>
<td>$p_i$</td>
<td>partial pressure of species $i$; atm, Pa, kPa, bar.</td>
</tr>
<tr>
<td>$p_{B,M}$</td>
<td>logarithmic mean partial pressure of component B; atm, Pa, kPa, bar.</td>
</tr>
<tr>
<td>$P$</td>
<td>total pressure; atm, Pa, kPa, bar.</td>
</tr>
<tr>
<td>$P$</td>
<td>permeate flow through a membrane; mol/s.</td>
</tr>
<tr>
<td>$P_m$</td>
<td>mpeller power; kW.</td>
</tr>
<tr>
<td>$P_c$</td>
<td>critical pressure, Pa, kPa, bar.</td>
</tr>
<tr>
<td>$Pe_D$</td>
<td>Peclet number for mass transfer.</td>
</tr>
</tbody>
</table>
### Nomenclature

- **Pe**\(_H\) Peclet number for heat transfer.
- **P**\(_i\) vapor pressure of species \(i\); atm, Pa, kPa, bar.
- **P**\(_o\) power number defined in equation (7-37); dimensionless.
- **Pr** Prandtl number; dimensionless.
- **Q** volumetric flow rate; m\(^3\)/s.
- **Q**\(_n\) net rate of heating; J/s.
- **Q**\(_m\) membrane permeance; m/s.
- **R** membrane permeability; barrer, m\(^2\)/s.
- **q** parameter defined by equation (6-27); dimensionless.
- **q**\(_m\) parameter in Langmuir adsorption isotherm; g/g.
- **r** rank of the dimensional matrix, DM; dimensionless.
- **r**\(_A\) solute particle radius; m.
- **R** radius; m.
- **R** ideal gas constant; J/mol·K.
- **R** reflux ratio; mol of reflux/mol of distillate.
- **R** raffinate mass flow rate; kg/s.
- **R**\(_A\) volumetric rate of formation of A; mol per unit volume per unit time.
- **R**\(_m\) retentate flow in a membrane; mol/s.
- **Re** Reynolds number; dimensionless.
- **R**\(_i\) volumetric rate of formation of component \(i\); mol/m\(^3\)-s.
- **S** surface area, cross-sectional area; m\(^2\).
- **S** stripping factor, reciprocal of absorption factor (A); dimensionless.
- **S** mass flow rate of the solvent entering a liquid extraction process; kg/s.
- **Sc** Schmidt number; dimensionless.
- **Sh** Sherwood number; dimensionless.
- **SR** salt rejection; dimensionless.
- **St**\(_D\) Stanton number for mass transfer; dimensionless.
- **St**\(_H\) Stanton number for heat transfer; dimensionless.
- **t** tray spacing; m.
- **t** time; s, h.
- **t**\(_b\) breakthrough time in fixed-bed adsorption; s.
- **t**\(_res\) residence time; min.
- **T** temperature; K.
- **T**\(_as\) adiabatic saturation temperature; K.
- **T**\(_b\) normal boiling point temperature; K.
- **T**\(_c\) critical temperature, K.
- **T**\(_w\) wet-bulb temperature; K.
- **u** fluid velocity past a stationary flat plate, parallel to the surface; m/s.
- **v** mass-average velocity for multicomponent mixture; m/s.
- **v**\(_i\) velocity of species \(i\); m/s.
- **v**\(_t\) terminal velocity of a particle; m/s.
- **V** molar-average velocity for multicomponent mixture; m/s.
- **V** volume; m\(^3\).
- **V** molar flow rate of the V-phase; mol/s.
- **V**\(_S\) molar flow rate of the nondiffusing solvent in the V-phase; mol/s.
- **V**\(_V\) mass flow rate of the V-phase; kg/s.
- **V**\(_S\) mass flow rate of the nondiffusing solvent in the V-phase; kg/s.
- **V**\(_A\) molar volume of a solute as liquid at its normal boiling point; cm\(^3\)/mol.
- **V**\(_B\) boilup ratio; mol of boilup/mol of residue.
Nomenclature

$V_b$  molar volume of a substance as liquid at its normal boiling point; \( \text{cm}^3/\text{mol} \).
$V_c$  critical volume; \( \text{cm}^3/\text{mol} \).
$w$  mass-flow rate; \( \text{kg/s} \).
$W$  work per unit mass; \( \text{J/kg} \).
$W_r$  molar flow rate of the residue from a distillation column; \( \text{mol/s} \).
$W_e$  Weber number defined in equation (7-49); dimensionless.
$x_i$  mol fraction of species \( i \) in either liquid or solid phase.
$x_i$  mass fraction of species \( i \) in raffinate (liquid extraction).
$x_{B,M}$  logarithmic mean mol fraction of component \( B \) in liquid or solid phase.
$x'$  rectangular coordinate.
$x'_{L}$  mass of \( C \)/mass of \( A \) in raffinate liquids.
$X$  mol ratio in phase \( L \); mol of \( A \)/mol of \( A \)-free \( L \).
$X$  flow parameter; dimensionless.
$X$  parameter in Gilliland’s correlation, see equation (6-87); dimensionless.
$X$  mass of \( C \)/(mass of \( A \) + mass of \( C \)) in the raffinate liquids.
$X_r$  mass ratio in phase \( L \); kg of \( A \)/kg of \( A \)-free \( L \).
$y$  rectangular coordinate.
$y'$  mass of \( C \)/mass of \( B \) in extract liquids.
$y_{B,M}$  logarithmic mean mol fraction of component \( B \) in gas phase.
$y_i$  mol fraction of species \( i \) in the gas phase.
$y_i$  mass fraction of species \( i \) in extract (liquid extraction).
$Y$  mol ratio in phase \( V \); mol of \( A \)/mol of \( A \)-free \( V \).
$Y$  parameter in Gilliland’s correlation, see equation (6-86); dimensionless.
$Y$  mass of \( C \)/(mass of \( A \) + mass of \( C \)) in the extract liquids.
$Y$  molal absolute humidity; mol \( A \)/mol \( B \).
$Y'$  absolute humidity; kg \( A \)/kg \( B \).
$Y_r$  mass ratio in phase \( V \); kg of \( A \)/kg of \( A \)-free \( V \).
$z$  rectangular coordinate.
$z_i$  average mol fraction of component \( i \) in a solution or multiphase mixture.
$Z$  total height; m.
$Z_c$  compressibility factor at critical conditions; dimensionless.
$Z_R$  total height of the rectifying section of a packed fractionator; m.
$Z_S$  total height of the stripping section of a packed fractionator; m.

GREEK LETTERS

$\alpha$  thermal diffusivity; \( \text{m}^2/\text{s} \).
$\alpha$  relative volatility; dimensionless.
$\alpha_m, \alpha_{AB}$  membrane separation factor; dimensionless.
$\beta$  volume coefficient of thermal expansion; \( \text{K}^{-1} \).
$\Gamma$  matrix of thermodynamic factors defined by equation (1-32).
$\Gamma$  concentration polarization factor; dimensionless.
$\gamma_i$  activity coefficient of species \( i \) in solution.
$\delta$  length of the diffusion path; m.
$\delta$  velocity boundary-layer thickness; m.
$\delta_{ij}$  Kronecker delta; 1 if \( i = k \), 0 otherwise.