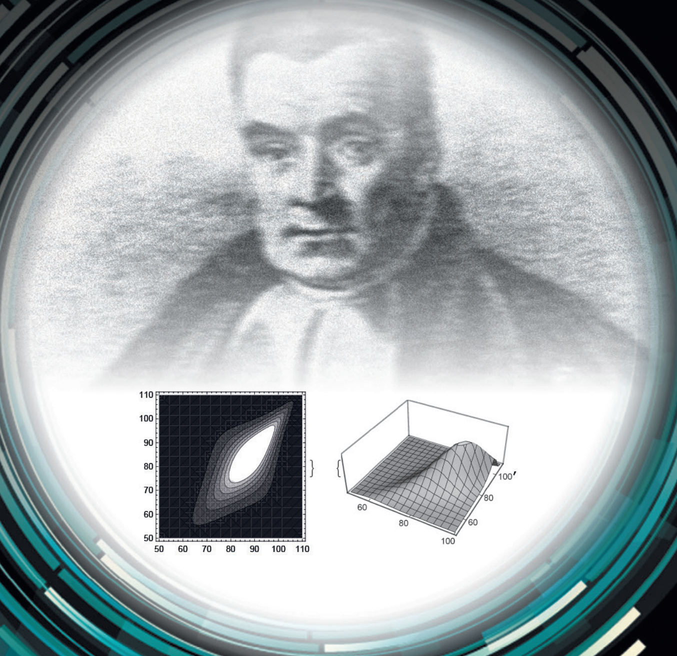
Introduction to
BAYESIAN ESTIMATION AND COPULA MODELS OF DEPENDENCE

Arkady Shemyakin
Alexander Kniazev



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To all our families:
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A.S. and A.K.

Acronyms

ABS	asset-backed security
AR	autoregressive
ARCH	autoregressive conditional heteroskedasticity
ARIMA	autoregressive integrated moving averages
BB1	Joe's BB1
BG	beta-geometric mixture model
BUGS	Bayes using Gibbs sampler (software)
CDO	collateralized debt obligation
CDS	credit default swap
CFTP	coupling from the past
CLT	the Central Limit Theorem
CMLE	canonical maximum likelihood estimation (estimates)
c.d.f.	cumulative distribution function
ENIAC	electronic numerical integrator and computer
FGM	Farlie–Gumbel–Morgenstern copulas
FTD	first-to-default (swaps)
GARCH	generalized autoregressive conditional heteroskedasticity
HAC	hierarchical Archimedean copula
HKC	hierarchical Kendall copula
IFM	inference from margins
i.i.d.	independent identically distributed
IMA	independent Metropolis algorithm
ISBA	International Society for Bayesian Analysis
JAGS	just another Gibbs sampler (software)
LLN	the Law of Large Numbers
MANIAC	mathematical analyzer, numerator, integrator, and computer
MCMC	Markov chain Monte Carlo
MHA	Metropolis–Hastings algorithm
MLE	maximum likelihood estimation (estimates)
MPLE	maximum pseudo-likelihood estimation (estimates)
PCC	pair copula construction

PD	probability of default
p.d.f	probability density function
RLUF	Rodriguez Lallena and Ubeda Flores copulas
RTO	regression through the origin
RWMHA	random walk Metropolis–Hastings algorithm
SACF	sample autocorrelation function
TTD	time-to-default
TTF	time-to-failure
WG	Weibull-gamma model
WGM	Weibull-gamma mixture model
WS	Weibull segmentation model

Glossary

$E(X)$	expected value (mean)
$Var(X)$	variance
$Bin(n, p)$	binomial distribution with n trials and success probability p
$Poiss(\lambda)$	Poisson distribution
$NB(p, r)$	negative binomial distribution
$N(\mu, \sigma^2)$	normal (Gaussian) distribution with mean μ and variance σ^2
$T(\eta)$	(Student) t-distribution with η degrees of freedom
$Gamma(z)$	gamma function
$Gamma(\alpha, \lambda)$	gamma distribution with shape α and rate λ
$Beta(\alpha, \beta)$	beta distribution
$Cov(X, Y)$	covariance
$\rho(X, Y)$	correlation
$MN(\mu, \mathbf{\Sigma})$	multivariate normal distribution
$L(x; \theta)$	likelihood function with data x and parameter θ
$\Lambda(x)$	likelihood ratio
$PG(1, r)$	polygamma function order r
$X \sim F$	random variable X has distribution F
$\propto$	proportional
$\preceq$	less or equal in sense of a special natural ordering

About the Companion Website

This book is accompanied by a companion website:

http://www.wiley.com/go/shemyakin/bayesian_estimation

The website includes:

- Solutions to selected exercises
- Excel dataset
- Excel simulation templates
- Appendices for Chapter 8
  - Datasets and results
  - Code in R

Introduction

Why does anyone need another book in Bayesian statistics? It seems that there already exist a lot of resources for those interested in the topic. There are many excellent books covering specific aspects of Bayesian analysis or providing a wide and comprehensive background of the entire field: Berger, Bernardo and Smith, Gamerman and Freitas Lopes, Gelman et al., Robert and Cassella, and many others. Most of these books, though, will assume a certain mathematical and statistical background and would rather fit a reader's profile of a graduate or advanced graduate level. Out of those aimed at a less sophisticated audiences, we would certainly recommend excellent books of William Bolstad, John Kruschke, and Peter Lee. There also exist some very good books on copulas: comprehensive coverage by Nelsen and Joe, and also more application-related Cherubini et al., Emrechts et al., and some others. However, instead of just referring to these works and returning to our extensive to-do lists, we decided to spend considerable amount of time and effort putting together another book—the book we presently offer to the reader.

The main reason for our endeavor is: we target a very specific audience, which as we believe is not sufficiently serviced yet with Bayesian literature. We communicate with members of this audience routinely in our day-to-day work, and we have not failed to register that just providing them with reading recommendations does not seem to satisfy their needs. Our perceived audience could be loosely divided into two groups. The first includes advanced undergraduate students of Statistics, who in all likelihood have already had some exposure to main probabilistic and statistical principles and concepts (most likely, in classical or "frequentist" setup), and may (as we probably all do) exhibit some traces of Bayesian philosophy as applicable to their everyday lives. But for them these two: statistical methods on one hand and Bayesian thinking on the other, belong to very different spheres and do not easily combine in their decision-making process.

The second group consists of practitioners of statistical methods, working in their everyday lives on specific problems requiring the use of advanced quantitative analysis. They may be aware of a Bayesian alternative to classical methods and find it vaguely attractive, but are not familiar enough with formal Bayesian analysis in order to put it to work. These practitioners populate analytical departments of banks, insurance companies, and other major businesses. In short, they might be involved in predictive modeling, quantitative forecasting, and statistical reporting which often directly call for Bayesian approach.

In the recent years, we have frequently encountered representatives of both groups described above as our collaborators, be it in undergraduate research or in applied consulting projects, or both at once (such things do happen). We have discovered a frequent need to provide to them a crash course in Bayesian methods: prior and posterior, Bayes estimation, prediction, MCMC, Bayesian regression and time series, Bayesian analysis of statistical dependence. From this environment we get the idea to concisely summarize the methodology we normally share with our collaborators in order to provide the framework for successful joint projects. Later on this idea transformed itself into a phantasy to write this book and hand it to these two segments of the audience as a potentially useful resource. This intention determines the content of the book and dictates the necessity to cover specific topics in specific order, trying also to avoid any unnecessary detail. That is why we do not include a serious introduction to probability and classical statistics (we believe that our audience has at least some formal knowledge of the main principles and facts in these fields). Instead, in Chapter 1 we just offer a review of the concepts we will eventually use. If this review happens to be insufficient to some readers, it will hopefully at least inspire them to hit the introductory books which will provide a more comprehensive coverage.

Chapter 2 deals with the basics of Bayesian statistics: prior information and experimental data, prior and posterior distributions, with emphasis on Bayesian parametric estimation, just barely touching Bayesian hypothesis testing. Some time is spend on addressing subjective versus objective Bayesian paradigms and brief discussion of noninformative priors. We spend just so much time with conjugate priors and analytical derivation of Bayes estimators that will give an idea of the scope and limitations of the analytical approach. It seems likely that most readers in their practical applications will require the use of MCMC—Markov chain Monte Carlo method—the most efficient tool in the hands of modern Bayesians. Therefore, Chapter 3 contains the basic mathematical background on both Markov chains and Monte Carlo integration and simulation. In our opinion, successful use of Markov chain Monte Carlo methods is heavily based on good understanding on these two components. Speaking of Monte Carlo methods, the central idea of variance reduction nicely transitions us to MCMC and its diagnostics. Equally important, both Markov chains and Monte Carlo methods have numerous important applications outside of Bayesian setting, and these applications will be discussed as examples.

Chapter 4 covers MCMC *per se*. It may look suspicious from traditional point of view that we do not particularly emphasize Gibbs sampling, rather deciding