

Zurich Lectures on Multiphase Flow

*Series Editors:*

George Yadigaroglu · Sanjoy Banerjee · Geoffrey F. Hewitt

George Yadigaroglu  
Geoffrey F. Hewitt *Editors*

# Introduction to Multiphase Flow

Basic Concepts,  
Applications and Modelling



 Springer

# **Zurich Lectures on Multiphase Flow**

## **Series editors**

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The series tracks critical research and engineering practice imparted through the Zurich Short Courses on Multiphase Flows. Multiphase flows occur in very many situations of industrial importance, including conventional or nuclear power plants, the oil and gas industry, chemical processing plants; as well as in biomedical applications. Understanding the basic laws governing the behavior of multiphase flows—such as flows of air and water, steam and water, oil and natural gas—, and producing methods, tools, codes, and best practices capable of predicting their behavior are crucial to engineering in these areas. The Zurich Lectures on Multiphase Flows brings together concise volumes that cover the field starting from the very fundamentals, such as void fraction, pressure drop, and heat transfer in multiphase flows; governing equations, closure laws, phenomenological modeling, correlations, experimental techniques, to the more advanced and specialized topics including non-equilibrium flows, instabilities, numerical methods for multiphase flows, applications to multiphase flows in nuclear power plants, in the oil-and-gas industry, and related technologies. The monographs appearing in the Zurich Lecture Series comprise a critical knowledge base particularly relevant to a range of engineering and scientific problems in industrial/commercial R&D, facility operation/optimization, product quality assurance, as well as in academic/biomedical research settings.

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George Yadigaroglu · Geoffrey F. Hewitt  
Editors

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Basic Concepts, Applications and Modelling

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# Preface

Sometime in 1983 four of us, Sanjoy Banerjee, Gad Hetsroni, Geoff Hewitt and George Yadigaroglu met and decided to organize a Short Course on multiphase flows, following the model that had been successfully tested at Stanford University previously (1979–1983). George Yadigaroglu was appointed the local organizer in Zurich. This was the beginning of a great “enterprise” that is still going on. The first Zurich Short Course took place in March 1984 and an unexpectedly large number of persons participated. Zurich turned out to be an excellent venue and we are grateful to ETH Zurich for hosting the course in its excellent facilities. The audiences kept growing and over 2000 participants took part in the Zurich courses until now. The courses that are still offered—obviously with new material and some new lecturers—became an initiation rite for the junior staff of the research and engineering departments of large companies, national laboratories, university laboratories, etc. Beginners, doctoral students, as well as their professors, young and older scientists, and engineers attended.

In March 2015, Gad Hetsroni died after a short illness. He had a very protective love for the Short Courses and had been the main organizer of the earlier sessions in California. He was involved as a Course Director and Lecturer up to 2015 and material from his lectures is included in these published Zurich notes. We wish to place on record our sincere appreciation of the crucial role he had played.

The Zurich Short Courses not only offered the opportunity to the participants to meet and interact with outstanding lecturers, but also with colleagues working worldwide on similar topics but in different industries. An aim of the courses was to promote interdisciplinary information exchanges between various industries and areas where multiphase flows are important but communications poor. For the lecturers also, the annual meetings became excellent opportunities to meet and interact.

The courses started with the four founders mentioned above, but soon the number of lecturers was expanded: we tried to attract not only the best specialists in their areas but also good communicators and teachers. In 1989, two parallel sessions were offered and the course became modular. The first three days were devoted to Basics while on Thursday and Friday the participants could choose

between two options. Part A was always the “nuclear” one, as many participants came from nuclear industries, even when their particular research interests were not necessarily in the narrower nuclear area. Part B, in response to the increasing interest in computational modelling and computational multi-fluid dynamics, was fully devoted to these topics. In fact, the computational aspects became gradually more important *throughout* the course, reflecting on-going changes and progress. From 2005 on, the course was enriched by bringing in the practical experience of the commercial code developers. By then the number of lecturers was expanded from the initial four to over a dozen.

Today the courses are organized in this modular form as an intensive introduction for persons having basic knowledge of fluid mechanics, heat transfer and numerical techniques and also serve as advanced courses for specialists wishing to obtain the latest information in the field; this series of books has the same goal. In 2007 introductory tutorials were used for the first time; they were mailed to the participants before the course to introduce the very basic concepts, fill any gaps in their basic background and prepare them for the tough week of lectures to come: four to five hour-and-a-half lectures per day. Tutorials are also appended to the volumes now.

The four principals met after each course to organize the next one: Geoff Hewitt always wrote the next outline. Obviously the lectures have evolved over the years, older material had to be shed to make place for new knowledge. The emphasis on the various topics also changed. After some 35 years of Short Courses, an impressive amount of material had accumulated. We finally decided to print the notes in book form, also allowing use of the new electronic means of disseminating the information. We are grateful to Springer that was interested in publishing this material as a series. We are assembling the lecture notes in thematic volumes: basics, conservation equations and closures, phenomenological modelling, boiling heat transfer, two-phase flow dynamics, multiphase flows in the nuclear industry, computational multi-fluid dynamics, etc. The present volume is the first of the series. We expect our volumes to be of interest to scientists and engineers working in the great variety of industries, thermal, chemical, nuclear, petrochemical, food, pharmaceutical, oil-and-gas, etc. where multiphase flows are ubiquitous.

The chapters of the volumes, although initially reflecting the state of the art at the time they were originally written, have been continuously updated over the years and fully updated again for this publication. Although most of the material can be found in the notes distributed to the participants over the years, it has been rearranged to better fit the format of a book. The final result approaches the form of a handbook or a series of textbooks; the pedagogical aspects remain very present. The best parts of lectures (often given by different persons) were re-assembled in the newly updated chapters that continue representing the state of the art.

The reader should not be surprised if a lot of original, old, seminal work is referenced: we prefer to cite the original author and work rather than its latest mention or presentation in a recent paper or textbook; this may make some of the references look “old” but they have been included only if their value has passed the test of time. Throughout the chapters, we insist on understanding of the physics and

on mechanistic modelling while not ignoring the empirical, well-established methods as well as the numerical applications. Our approach is to start from the basic principles and ideas and to progress gradually, ending up with the state of the art and occasionally looking beyond. A special effort is made to remain as educational as possible.

A few words about the present volume on *Introduction to Multiphase Flow* and Basics: After introducing the reader to multiphase and in particular two-phase flows, the basic concepts, variables, notions and tools necessary for the following chapters are introduced in Chap. 1. Chapter 2 discusses the various alternatives available for *modelling* and studying analytically or numerically two-phase flows. The more advanced alternatives are simply surveyed in this chapter as they need more in-depth and formal treatment in other volumes. Chapter 3 is an introduction to the *interfacial instabilities* that govern a multitude of two-phase flow phenomena; the early introduction of this material will help the following presentations. A long Chap. 4 covers in depth the *flow regimes*, a special characteristic of two-phase flows that governs their mechanistic modelling; consequently this chapter gives an excellent opportunity to introduce the reader to phenomenological modelling. Finally, Chaps. 5 and 6 cover *void fraction* and *pressure drop* in two-phase flow, two topics that are of particular concern to any designer or analyst of two-phase equipment and systems.

Zurich, Switzerland  
London, UK  
January 2017

George Yadigaroglu  
Geoffrey F. Hewitt



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# Chapter 1

## Nature of Multiphase Flows and Basic Concepts

George Yadigaroglu and Gad Hetsroni

### 1.1 The Nature of Multiphase Flows

Multiphase flows and heat transfer with phase change are one specialized discipline among many others in the much more general area combining thermodynamics, fluid mechanics and heat transfer; often this area is referred to as *thermal-hydraulics*. The study of multiphase flows requires basic knowledge in these three areas. Appendix I of this volume, a tutorial, provides some of the indispensable minimal background from these three areas for the reader that may need it.

The adjective *multiphase* characterizes situations where several different phases—liquids, gases, solids—are flowing simultaneously. In the case of liquids, the two flowing media can also be two chemically different ones, e.g. oil and water. *Two-phase flows* take place when only two phases are present, most often liquid and gas.

Another categorization of flows is according to the presence or absence of heat transfer: we speak of *adiabatic* or *diabatic* multiphase flows. In diabatic multiphase flows, we can have phase change: *vaporization* or *condensation*. Phase change can also take place, however, in adiabatic flows due to pressure changes; such vaporization is referred as *flashing*.

Another important distinction is according to the flow direction: *vertical*, *horizontal*, or *inclined*. Flow direction is more important in multiphase flows when gravity plays a more important role in presence of large differences in the density of

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Gad Hetsroni—Deceased Author.

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the phases; such differences can easily be of the order of 1000 (e.g. air–water flow at atmospheric pressure).

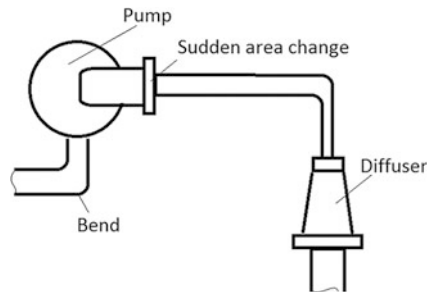
Finally, we speak of *parallel or co-current flow* when the two-phases flow in the same direction, and of *counter-current flow* when they flow in opposite directions (e.g. falling liquid and rising gas).

Multiphase flows are present in everyday life, in nature, in industrial processes, in power plants, in the oil and gas industry, etc. All phase-change processes such as boiling and condensation produce two-phase flows; these heat and mass transfer processes are “core” considerations in the multiphase flow area where many applications involve phase change or at least interactions between phases. Water boils in power plants to produce steam. In processing plants, different phases are mixed to react or are the products of chemical reactions. Thus, heat transfer with phase change (e.g. boiling or condensation) is intimately linked to two-phase flows.

After some reminders and formal definitions in Sect. 1.2, we will consider in Sect. 1.3 a few examples where the nature of multiphase flows is dominant, and controls the behaviour, the response and the operation of the whole system. In particular, we note that some peculiarities of multiphase flow produce unique responses that may have a profound effect on the dynamics and safe operation of the system; we touch here only two such phenomena: the Critical Heat Flux (CHF) phenomenon and a particular type of flow instability.

We will deal mainly with *one-dimensional* flows in ducts. In fluid mechanics, the notion of one-dimensional flow is applied in a broader sense. For example, the flow in a complex system of pipes that may have bends, enlargements, curved sections, etc., such as the one shown in Fig. 1.1 is, strictly speaking, certainly not one dimensional. However, as long as the three-dimensional details of the velocity and temperature distributions do not interest us and we deal only with the average velocity and the mass flux-weighted average fluid temperature, such systems can still be treated as “one dimensional”. Indeed, we are dealing in this volume mainly with one-dimensional two-phase flow, very much like in the excellent book by G.B. Wallis that has exactly this title (Wallis 1969).

**Fig. 1.1** A complex circuit with bends, enlargements, fittings, a pump, etc. that can be still considered as one dimensional when only cross-sectional-average variables are of interest



## 1.2 Phases, Components, Fields

A *phase* is a thermodynamic definition for the state of the matter, which can be either solid, liquid or gas; these can co-exist in a conduit. Examples of multiphase flows are abundant, e.g. when oil is produced, one normally gets oil, water, gas and sand flowing in the pipelines (three-phase flow). Normally, by two-phase (or by extension multiphase) flow we mean a mixture of two (or more) phases which is not extremely dilute and the phases have a distinguishable size. Thus, though clean city tap water is, strictly speaking, a two- (water–air) or three- (water–air–microscopic solids) phase mixture, for practical purposes, it is considered a single phase. However, city water carrying sand should, for filtration purposes at least, be considered a two-phase mixture.

The term *mixture* is most of the time used to denote the two (or more) phases flowing together and does not necessarily imply that these are intimately *mixed*. For example, in the case of annular flow that we will introduce below, we may still refer to the flow as *the two-phase mixture* in spite of the fact that the liquid film on the wall and the gaseous core are not at all “mixed”. The term “separated flow” is often used loosely to denote two-phase flows where the two phases have different average velocities. This distinguishes such flows from the *homogeneous* ones, where the phases have the same average velocity; again, such flows may strictly speaking not be homogeneous at all. For example, bubbly flow with fairly large bubbles can be considered as *homogeneous*.

A *component*, is a chemical species. So, the term two-component is used to describe the flow of two chemical species. A water–steam mixture is two-phase, one-component, while a water–air mixture is two-phase, two-component flow; a water–oil mixture is one-phase, two-component, etc. The approach in modelling of the two alternative two-phase configurations—with one or two components—is often the same or very similar, though the physical behaviour of different mixtures may be quite different.

The term *field* is used to denote a topologically distinct or clearly identifiable fraction of a phase. For example, in the so-called annular flow, the liquid can be present as either a film on the wall or as droplets in the core where the gas flows; the droplets and the film can be considered as different fields. In a closed vessel such as a pressure cooker containing boiling water, we may define a field of steam bubbles in the liquid and a field of steam in the space above the liquid surface as separate fields.

For simplicity of presentation, the approaches and various methods that will be covered here will be presented mainly for two-phase flows. However, their extension to multiphase and multi-field flows is most often possible, although such applications are less frequently encountered in practice.

In this volume, we mainly deal with liquid-gas mixtures. There are many other two-phase flows, e.g. gas-solid flows such as fluidized beds, conveying of granular materials by gas; liquid-solid flows where sedimentation and filtration may be phenomena of interest, etc. Such flows will not be treated in this volume and in this

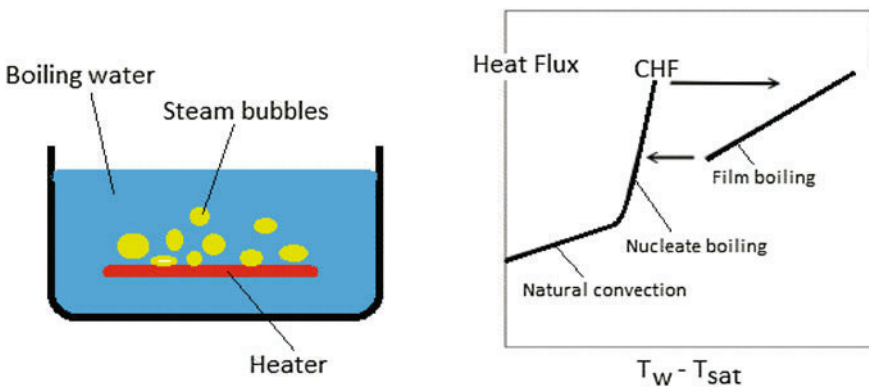
series of books which are mostly directed to two-phase gas-liquid flows of industrial interest: steam-water flows, oil and gas flows, two- or multiphase flows in processing plants, etc. There are other texts which deal with the other types of multiphase flows, namely the Handbook by Hetsroni (1982) and monographs such as Fan and Zhu (1998) for gas-solid flows and Brown and Heywood (1991) for liquid-solid flows (slurries).

### 1.3 Multiphase Flow Phenomena

We give examples in the following section of situations where peculiarities of multiphase flow produce unique responses not found in single-phase flows. The Critical Heat Flux (CHF) phenomenon and a particular type of flow instability will be briefly introduced. In Sect. 1.3.2, we discuss phenomena that are not unique to multiphase flows but become much more complex in the presence of several phases.

#### 1.3.1 Phenomena Unique to Multiphase Flows

The *critical heat flux (CHF)* phenomenon is a situation that may take place in heat transfer with boiling. To illustrate the situation, we consider a very simple experiment conducted with a heated tube immersed in a pool of liquid, Fig. 1.2. This is the situation referred to as *pool boiling*. The heat flux from the wall is plotted in the graph of Fig. 1.2 against the excess wall temperature (i.e. the wall temperature  $T_w$  minus the saturation temperature  $T_{sat}$  of the fluid in the pool). The heat flux is gradually increased; one can observe that once a certain heat flux is reached, the



**Fig. 1.2** A pool boiling experiment (left). The boiling curve (right)

CHF, a dramatic change takes place and a further increase in heat flux will cause a very large, sudden rise in wall temperature, resulting most likely in the failure of the tube. When the CHF is reached, the liquid can no longer wet the heater surface and cooling takes place through a vapour film surrounding the heater rather than by boiling of the liquid in contact with the wall. As the vapour has much reduced thermal conductivity and density, heat transfer deteriorates markedly and leads to the temperature excursion. This subject is of utmost practical importance and will be further treated in other volumes. It is a phenomenon taking place only in heat transfer with phase change.

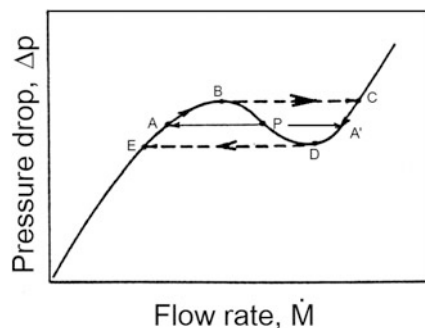
We also note here that as the heat flux is *reduced*, the operating point stays on the film boiling characteristic curve rather than going back at CHF to the nucleate-boiling curve. It appears that the film surrounding the heater is stable, keeping film boiling going on; it will take a further reduction of the heat flux to produce another jump back to the nucleate-boiling curve. This particular hysteresis behaviour during pool boiling was studied by Nukiyama (1934) who published the *boiling curve*.

Another example which is typical to two-phase flow is a flow excursion which can occur in a two-phase loop having certain flow characteristics. The situation is illustrated in Fig. 1.3. If the two-phase section of the loop has a negative slope in its pressure drop—flow-rate characteristic ( $\dot{M}$ ,  $\Delta p$ ), i.e. if  $\partial p / \partial \dot{M} < 0$  something that happens often in two-phase flow (and cannot happen in single-phase flow), a flow excursion will occur. This situation was described first by Ledinegg (1938). Ledinegg instabilities can be of great practical importance regarding the stability of two-phase equipment such as steam generators. Flow instabilities will be dealt with in another volume; some additional information is included in the following section.

### 1.3.2 Phenomena Complicated by the Presence of Many Phases

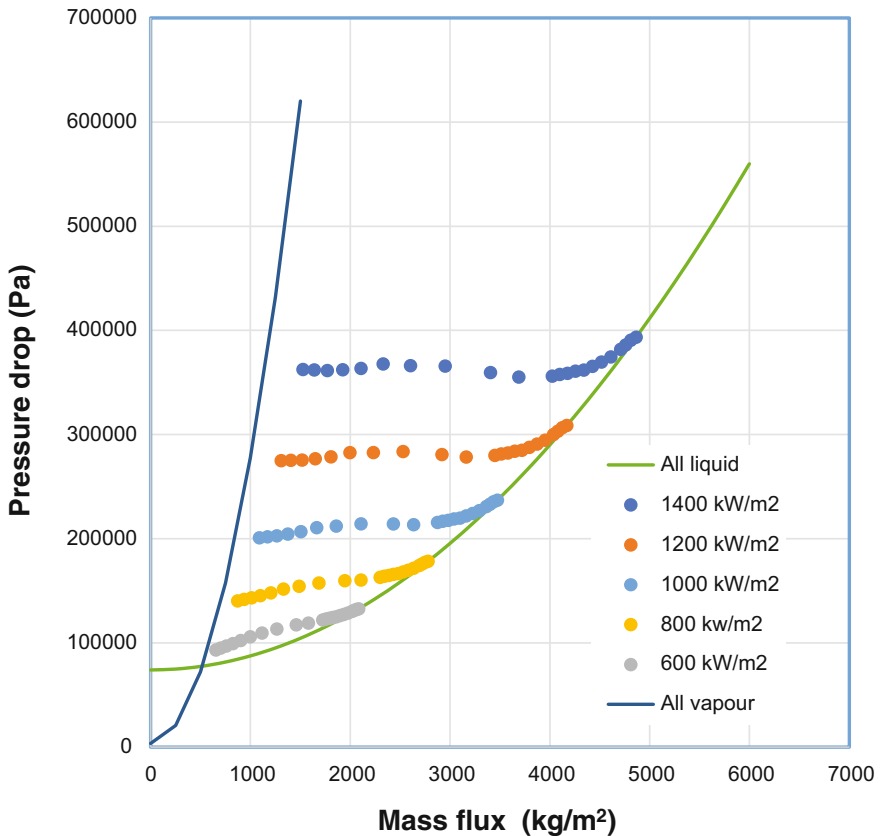
A number of single-phase flow phenomena take also place in single-phase flows, but the presence of two or more phases complicates greatly the situation.

**Fig. 1.3** The flow-rate versus pressure-drop characteristic of a boiling channel. The Ledinegg instability occurs if the slope of the characteristic is locally negative, as in the segment BD. Point P is unstable and the operating point will migrate to either A or A'.



For example, the *pressure drop* is a factor of importance in the design of any hydraulic system. Its estimation is rather straightforward in single-phase systems, but it becomes much more difficult—and inaccurate in fact—in two-phase flows. Figure 1.4 shows the pressure drop—flow rate characteristic of a heated pipe. The all-liquid and all-vapour characteristics are nearly parabolic, but, in between the heated pipe exhibits a much more complex behaviour in the presence of two-phase flow. As the heat flux increases, a negative-slope part of the characteristic emerges; it is such behaviour that is responsible for the Ledinegg instability just mentioned.

*Critical flows* are another case where the multiphase nature of the problem greatly complicates the situation. We will use this phenomenon to show the importance of non-equilibria in two- or multiphase flows. In the simpler case of



**Fig. 1.4** The flow rate—pressure drop characteristics of a heated pipe at different heat fluxes and with all-liquid and all-vapour flow (with no heat addition). Pipe length  $L = 10$  m, diameter  $D = 0.02$  m, friction factor  $f = 0.005$ . Pressure  $p = 69$  bar. Inlet non-dimensional subcooling  $\Delta h_{in}/h_{LG} = 0.45$ . The computations were performed with Thom's charts given as an Appendix to Chap. 6

two-phase flow, the gas and the liquid may have different average velocities and temperatures; this is what we denote as *non-equilibrium*.

In the case of single-phase flow, the flow becomes choked as the critical flow condition is reached when, essentially, the flow velocity reaches the sonic velocity of the fluid; the latter is determined as a thermodynamic fluid property depending only on temperature and pressure (e.g. Shapiro 1953). The assumption is made that the properties of the fluid are given along its thermodynamic path by its state equation. The same approach can be taken for two-fluid mixtures but difficulties appear: the state equation of the mixture depends now on whether the gas and liquid velocities and temperatures are equal or not. As such equilibria are subject to all sorts of flow conditions and on the history of the fluid upstream of the choking point, there is no simple answer to this problem and the critical flow of a two-phase mixture depends on assumptions made about the state of the mixture.

## 1.4 Flow Regimes

One of the major difficulties in multiphase or two-phase flows is that the phases are distributed in the duct in particular ways; the various typical (topological) configurations that result are called *flow regimes*.

The boundaries between phases are called *interfaces*. The topology of the flow, i.e. the geometry of the interfaces, is not known and cannot be determined a priori, but is rather a part of the solution. In contrast, for example, in single-phase flow of a fluid in a tube, knowing the geometry we can determine, either experimentally or analytically, the velocity distributions, the shear stress distribution, the pressure drop, etc. When two phases flow in a conduit, we cannot tell a priori how the phases are going to distribute themselves: whether bubbles will be distributed uniformly throughout the liquid (we call this situation *bubbly flow*) or whether the bubbles will coalesce and gas will flow in the centre of the pipe, while the liquid will form a film on the wall (this is called *annular flow*). *Flow regime determination* is needed to answer such questions; this is treated in Chap. 4.

The cross-sectional distribution of the phases in the pipe determines other parameters such as heat transfer, pressure drop and, without knowing this phase distribution, we cannot calculate these. Furthermore, it is not likely that the two phases will flow with the same average velocity. On the contrary, most likely, the gas will flow at a higher average axial velocity than the liquid, which will cause a change of the *void fraction* or *volume fraction* of the gas phase in the channel (i.e. the fraction of area occupied by gas in the cross section). Often, until we have a way to determine the flow regime, there is really no way we can accurately model and



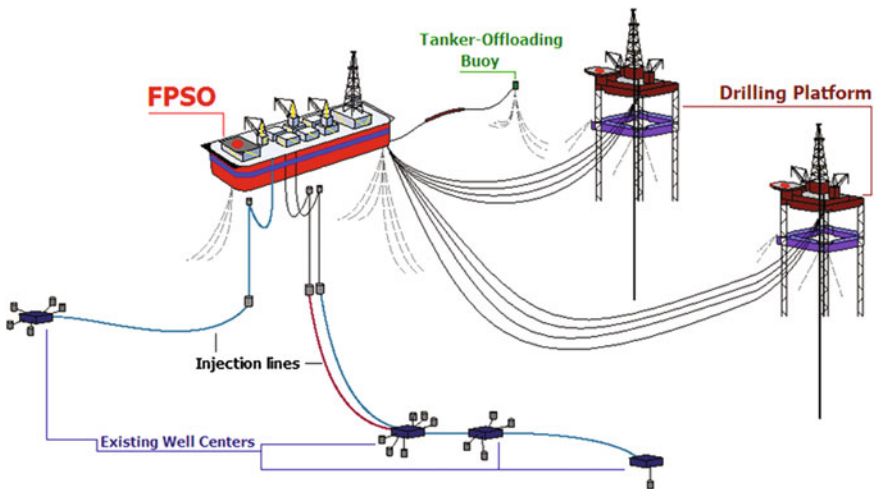
calculate other parameters of engineering significance. Therefore, one of the first problems we will address in this volume is the study and determination of *flow patterns* or *flow regimes*. The reader should be aware that flow patterns depend on many parameters, such as the inclination of the conduit, the geometry, pressure, type of fluid, etc.

## 1.5 Some Important Multiphase Flow Systems

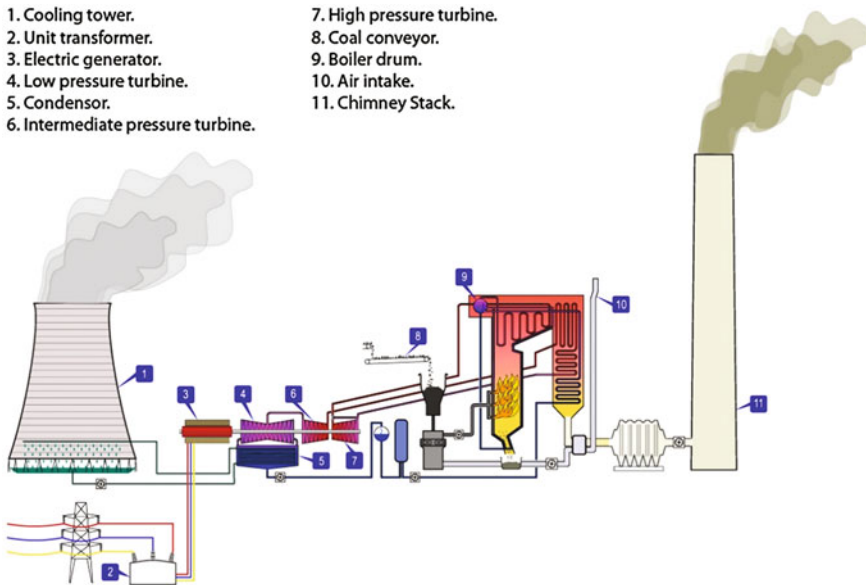
We site now some examples where multiphase flows play a dominant effect in the design and operation of very large engineering systems.

*Offshore production* has been important for the extraction of hydrocarbons since the 1950s. It usually involves a drilling platform which operates a number of wells to produce crude oil. The oil comes out of the ground usually as a mixture of crude oil, sea water, sand and gas—a multi-component, multiphase mixture which needs to be handled carefully because of various hazards and because of the extremely costly equipment involved. Figure 1.5 shows production wells, drilling platforms, and a floating production, storage and offloading vessel.

*Electric power production from oil, coal or gas:* A majority of power plants use a steam cycle where water is the working fluid. Figure 1.6 shows the steam cycle of a modern plant; water gets vaporized in the high-pressure steam generator section and the low-pressure steam exiting from the turbine gets condensed in the condenser. Again, two-phase flows and phase change processes are present throughout the steam plant.

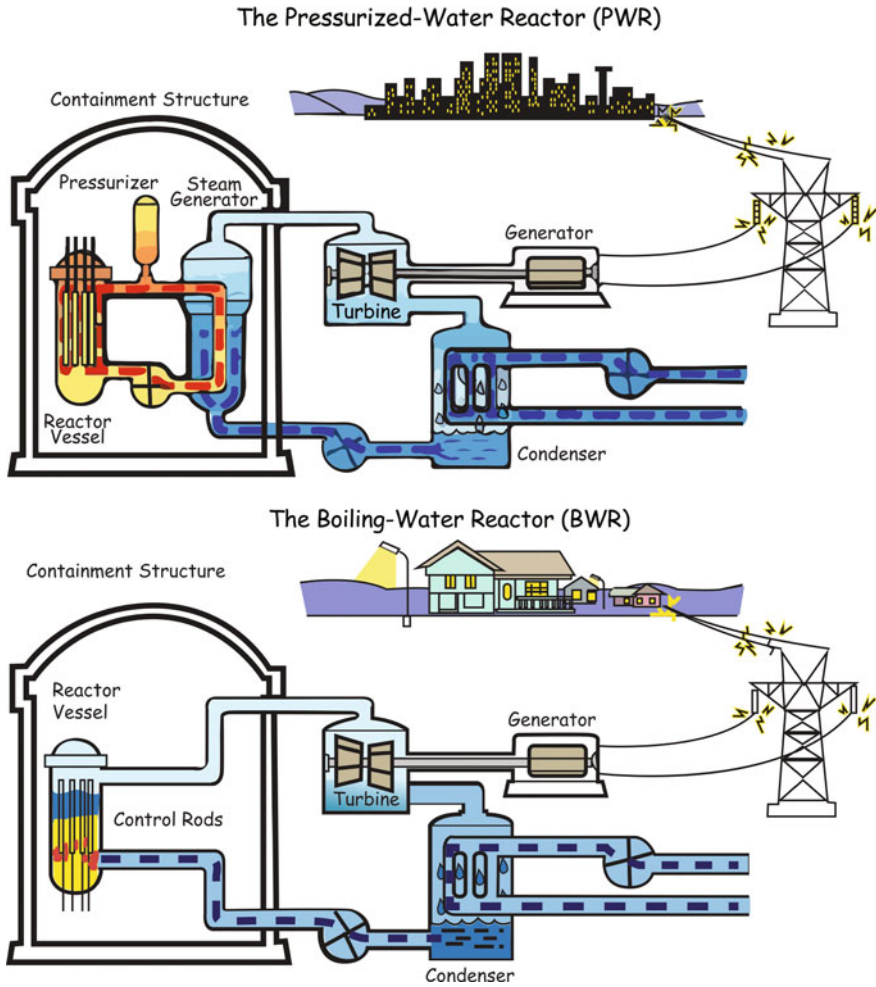


**Fig. 1.5** FPSO, floating, production, storage and offloading for offshore production of oil



**Fig. 1.6** A modern steam power plant. Boiling and two-phase flows take place in the steam generator while evaporation of droplets in the cooling tower (from <http://www.zeroco2.no>)

In a nuclear power plant (NPP), the *nuclear steam supply system* (NSSS or “en-triple-ess”) is the part of the plant where water at high pressure is heated in the reactor vessel by the nuclear chain reaction taking place in the fuel rods. Steam is then produced (either in the reactor vessel, as in a boiling water reactor, Fig. 1.7 (bottom), or in a steam generator, as in a pressurized water reactor, Fig. 1.7 (top), and used in the turbine-generator to produce electric power. The multiphase flow of water and steam takes place in very costly equipment. To operate the system safely and efficiently for the lifetime of the equipment, one needs detailed knowledge of multiphase flow and heat transfer. Understanding two-phase flow situations and phenomena and being able to predict the outcomes becomes even more important in case of an accident in a NPP when unusual situations are encountered. In fact, the disciplines of two-phase flow and heat transfer with phase change progressed tremendously the last three or four decades of the twentieth century driven by the need to fully understand, model and simulate the complex phenomena taking place during thermal-hydraulic accidents in NPPs. Other books in this series will be devoted to two-phase flows in nuclear power plants.



**Fig. 1.7** Pressurized Water Reactor (*top*) and Boiling Water Reactor (*bottom*). Figures from [www.nrc.gov](http://www.nrc.gov). The conventional power production systems of these two main types on NPPs are quite similar and operate under very similar thermodynamic conditions. In the BWR, direct steam generation takes place in the reactor core while, in the PWR, it takes place in the intermediate steam generator

## 1.6 Averaging in Two-Phase Flows

It is often necessary to perform averaging in multiphase flows and in particular averaging over the flow cross section in one-dimensional, two-phase flows. This is in particular necessary when dealing with the conservation equations where one starts with instantaneous local equations that must be integrated in time and space to arrive at usable forms.

Averaging will be applied either to two-phase *mixture* variables (variables for both the liquid and the gas flowing together) or to each phase separately. We will use averages of *mixture* properties such as the mixture density as well as averages of *phase* variables such as the channel-cross-sectional-average liquid or gas velocity. This section briefly introduces the concepts.

Generally, we will add the subscript  $k$ , ( $k = L, G$ ), where  $L$  denotes the liquid and  $G$  the gas, to the phase variables to differentiate them from mixture variables that are usually written without a subscript. Thus,  $f_k$  is a local instantaneous variable pertaining to phase  $k$ , e.g. the local, instantaneous velocity or enthalpy,  $u_k$  or  $h_k$ .

### 1.6.1 Space Averaging

We consider space averages of generally instantaneous values. The following averages can be defined:

Cross-sectional average of any variable  $f$

$$\langle f \rangle = \frac{1}{A} \int_A f dA, \quad (1.6.1)$$

where the angle brackets operator  $\langle \cdot \rangle$  denotes the cross-sectional averaging over the flow area  $A$ .

*Phase* cross-sectional average of a *phase* variable  $f_k$

$$\langle f_k \rangle_k = \frac{1}{A_k} \int_{A_k} f_k dA, \quad k = L, G \quad (1.6.2)$$

where the integration is now performed only over the area of the channel  $A_k$  occupied by phase  $k$  at a given moment. The second subscript outside the angle brackets in  $\langle f_k \rangle_k$  reminds us that the cross-sectional average of the local phase property  $f_k$  was performed over the area of the channel occupied by phase  $k$  only:  $\langle \cdot \rangle_k$ .

### 1.6.2 Time Averaging

The time or statistical average of any variable  $f$  is

$$\bar{f} = \frac{1}{T} \int_{[T]} f dt. \quad (1.6.3)$$

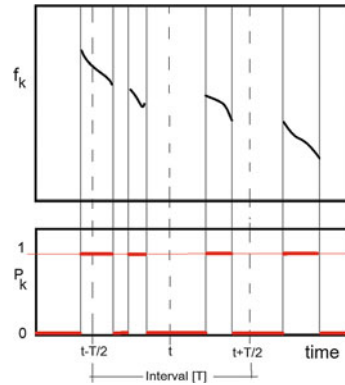
The integration interval  $[T]$  must be chosen such that, for example, high frequency perturbations such as those created by turbulence are averaged out while the transient nature of the process is still well represented.

The *phase* time or statistical average of a *phase* variable  $f_k$  is

$$\bar{f}_k^k = \frac{1}{T_k} \int_{[T_k]} f_k dt, \quad (1.6.4)$$

where  $[T_k]$  is the subset of residence time intervals where phase  $k$  is present at a given point and  $T_k$  is the sum of the presence times of phase  $k$  at that given point.  $T$  is the total averaging time interval. The rather heavy notation for the interval  $[T_k]$  is necessary to remind us that it is discontinuous, Fig. 1.8.

**Fig. 1.8** Measurements of a piecewise continuous variable  $f_k$ , such as phase velocity, pertaining to phase  $k$  (*above*) and phase indicator function  $P_k$  (*below*). Such signals could be actually measured, e.g. by a hot-wire anemometer (phase velocity); the signal of the anemometer could be treated to detected the phase present



## 1.7 Void Fractions and Their Measurement

The term *void fraction* (denoted in general by  $\varepsilon_G$ , where the subscript  $G$  denotes the gas, dimensionless) generically describes in space or time the fraction occupied by the gas phase. The notion is specialized as needed below. For each definition of the void fraction, an experimental method that may produce such a measurement is indicated; some additional information on void fraction measurements is given in Chap. 5.

### 1.7.1 The Local Void Fraction

The *local void fraction* is defined as the fraction of time in which the gas phase occupies a given point in space  $\mathbf{r}$ . We may characterize the presence (or absence) of

phase  $k$  ( $k = G, L$ ) at a given point  $\mathbf{r}$  and at a given time  $t$  by the unit or zero value of the *phase density* or *phase presence* function  $P_k(\mathbf{r}, t)$

$$P_k(\mathbf{r}, t) = \left\{ \begin{array}{l} 1 \text{ if } \mathbf{r} \text{ is in phase } k \\ 0 \text{ if } \mathbf{r} \text{ is in the other phase} \end{array} \right\}.$$

The instantaneous value is usually integrated over a time period  $T$  to give a time-averaged value. Thus the *time-averaged local  $k$ -phase fraction* is the time-averaged phase density function

$$\bar{\varepsilon}_k(\mathbf{r}, t) = \frac{1}{T} \int_T P_k(\mathbf{r}, t) dt = \overline{P_k(\mathbf{r}, t)} \quad (1.7.1)$$

according to the definition of the time-averaging operator. In the case of gas,  $k = G$ , this becomes the *local statistical or time-average void fraction*:

$$\bar{\varepsilon}_G(\mathbf{r}, t) = \frac{1}{T} \int_T P_G(\mathbf{r}, t) dt = \overline{P_G(\mathbf{r}, t)}.$$

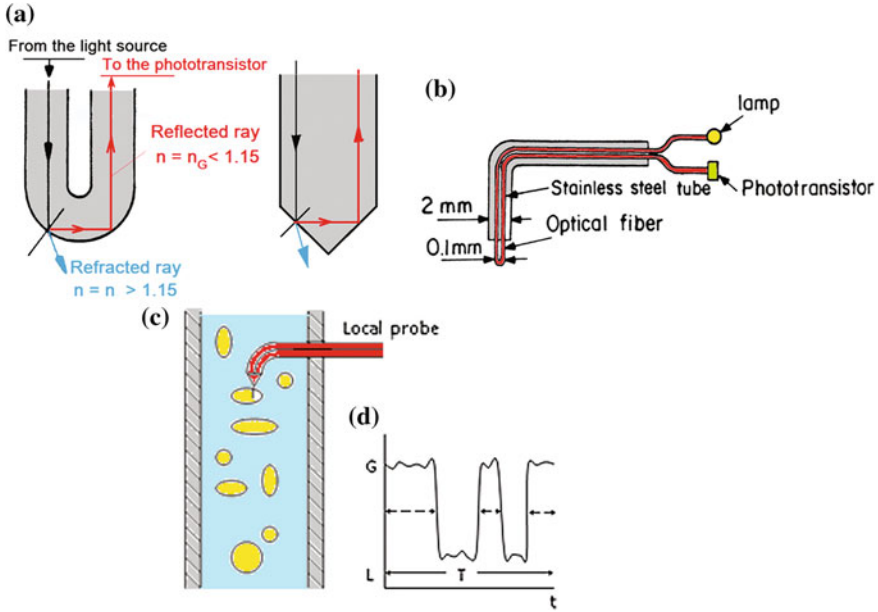
Alternatively, we can define the local statistical or time-average void fraction as the fractional presence time of the gas phase at a given point:

$$\bar{\varepsilon}_G \equiv \frac{T_G}{T} \quad \text{and also} \quad 1 - \bar{\varepsilon}_G \equiv \bar{\varepsilon}_L \equiv \frac{T_L}{T} \quad (1.7.2)$$

where  $T_k$  ( $k = L, G$ ) is the summation of all the times during which phase  $k$  was present at the measuring point, and  $T$  is the total period of observation. The bar over  $\varepsilon$  denotes time or statistical averaging. The overbar will be dropped for simplicity of notation when it is implicit that we are dealing with time-average values. Also, if no subscript is added to  $\varepsilon$ , it is assumed that the meaning is  $\varepsilon_G$  and the liquid local time-averaged fraction becomes  $1 - \varepsilon = \varepsilon_L$ . Clearly,  $\varepsilon_G + \varepsilon_L = 1$ .

The local void fraction can be measured by a miniature resistive probe, a U-shaped fibre-optical sensor or a hot-wire anemometer. All these devices detect the presence of a phase at their sensing tip.

An optical probe is sensitive to the change in the refractive index of the surrounding medium enabling measurements of local void fraction. It can also record interfacial passages and interface passage frequencies. Optical probes can operate in conducting or non-conducting liquids. The tiny optical probe, which is frequently used, Fig. 1.9, consists, e.g. of a single optical fibre, 40  $\mu\text{m}$  in diameter, which may be bent in a U shape. A light shines from a source through one arm of the U bend. This light, according to Snell's law, is either refracted (if the surrounding medium is liquid) or reflected to the other side of the U bend (if the surrounding fluid is gas), Fig. 1.9a. By analysing the signal of the phototransistor, one can infer the *local* void fraction, Fig. 1.9d.



**Fig. 1.9** a Active parts of fibre-optical sensors showing the refraction or reflection of the light. b U-shaped fibre-optical sensor assembly (Danel and Delhaye 1971). c Optical probe positioned inside the channel in bubbly flow. d Probe signal indicating the presence of gas or liquid

### 1.7.2 The Chordal Void Fraction

The *chordal void fraction* is defined as the fraction of a chord or ray occupied at a given instant by gas:

$$\varepsilon_{G1} = \frac{L_G}{L_L + L_G},$$

where  $L_k$ ,  $k = L, G$  is the instantaneous cumulative length on the chord occupied by phase  $k$ . This is best illustrated in Fig. 1.10 where  $L_G = L_1 + L_2$ .

The chordal-average void fraction is typically measured by means of radiation absorption methods. These methods use  $\gamma$  or X-ray beams which are attenuated by the material through which they pass. The intensity after absorption of a collimated beam of initial intensity  $I_0$  (photons/m<sup>2</sup>s) is given by

$$I = I_0 e^{-\mu z},$$

where  $\mu$  is the linear absorption coefficient, which is a property of the absorbing material and the type of radiation and  $z$  is the distance travelled through a