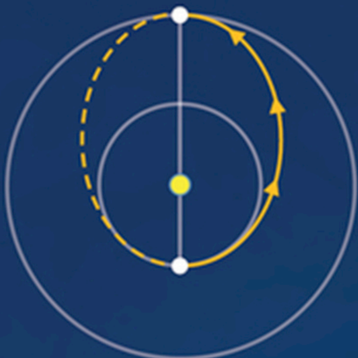

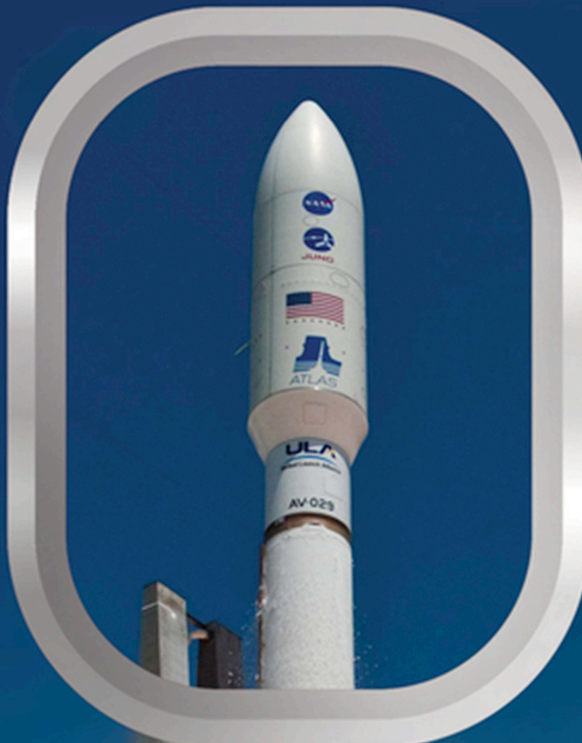

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Space Flight Dynamics

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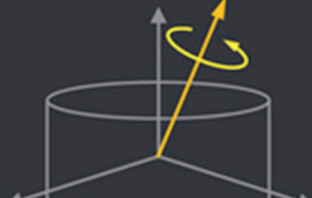

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Space Flight Dynamics

Craig A. Kluever
University of Missouri-Columbia, USA

WILEY

This edition first published 2018

Registered Offices
John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, USA
John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, UK

Editorial Office
The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, UK

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Library of Congress Cataloging-in-Publication Data

Names: Kluever, Craig A. (Craig Allan), author.
Title: Space flight dynamics / by Craig A. Kluever.
Description: First edition. | Hoboken, NJ : John Wiley & Sons, 2018. |
Includes bibliographical references and index. |
Identifiers: LCCN 2017042818 (print) | LCCN 2017054455 (ebook) | ISBN 9781119157908 (pdf) | ISBN 9781119157847 (epub) | ISBN 9781119157823 (cloth)
Subjects: LCSH: Astrodynamics. | Space flight.
Classification: LCC TL1050 (ebook) | LCC TL1050 .K555 2018 (print) | DDC 629.4/1–dc23
LC record available at https://lccn.loc.gov/2017042818

Cover design by Wiley
Cover image: An Atlas V rocket with NASA's Juno spacecraft lifts off from Space Launch Complex 41 of the Cape Canaveral Air Force Station in Florida. Photo credit: Pat Corkery, United Launch Alliance

Set in 10/12pt Warnock by SPi Global, Pondicherry, India

Printed and bound by CPI Group (UK) Ltd, Croydon, CR0 4YY

10 9 8 7 6 5 4 3 2 1

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Series Preface

The field of aerospace is multi-disciplinary and wide-ranging, covering a large variety of products, disciplines and domains, not solely in engineering but also in many related supporting activities. These combine to enable the aerospace industry to produce innovative and technologically advanced vehicles. The wealth of knowledge and experience that has been gained by expert practitioners in the various aerospace fields needs to be passed onto others working in the industry and also researchers, teachers and the student body in universities.

The *Aerospace Series* aims to provide a practical, topical and relevant series of books aimed at people working within the aerospace industry, including engineering professionals and operators, engineers in academia, and allied professions such commercial and legal executives. The range of topics is intended to be wide-ranging, covering design and development, manufacture, operation and support of aircraft, as well as topics such as infrastructure operations and current advances in research and technology.

There is currently a renewed interest world-wide in space, both in terms of interplanetary exploration, and its commercialisation via a range of different opportunities including: communications, asteroid mining, space research and space tourism. Several new companies have been set up with the aim of exploiting the commercial opportunities. A fundamental issue for any space mission is how to get the system into space and then how to control its trajectory and attitude to complete the mission objectives.

This book, *Space Flight Dynamics,* provides a comprehensive coverage of the topics required to enable space vehicles to achieve their design goals whilst maintaining the desired performance, stability and control. It is a very welcome addition to the Wiley Aerospace Series.

Peter Belobaba, Jonathan Cooper and Allan Seabridge

Preface

This textbook is intended for an introductory course in space flight dynamics. Such a course is typically required for undergraduates majoring in aerospace engineering. It is also frequently offered as an elective in mechanical and aerospace engineering curricula. Whether taken for required or elective credit, this course is usually taken in the junior or senior year, after the student has completed work in university physics, rigid-body dynamics, and differential equations. A brief survey of university catalogs shows that titles for these courses include *Orbital Mechanics*, *Astrodynamics*, *Astronautics*, and *Space Flight Dynamics*. The principal topic covered in essentially all courses is two-body orbital motion, which involves orbit determination, orbital flight time, and orbital maneuvers. A secondary topic that appears in many of these courses is spacecraft attitude dynamics and attitude control, which involves analyzing and controlling a satellite's rotational motion about its center of mass. A number of space flight courses also cover topics such as orbital rendezvous, launch trajectories, rocket propulsion, low-thrust transfers, and atmospheric entry flight mechanics. The primary goal of this textbook is to provide a comprehensive yet concise treatment of all of the topics that can comprise a space flight dynamics course. To my knowledge, a single space flight textbook that covers all the topics mentioned above does not exist.

A secondary goal of this textbook is to demonstrate concepts using real engineering examples derived from actual space missions. It has been my experience that undergraduate students remain engaged in a course when they solve "real-world" problems instead of academic "textbook" examples. A third goal is to produce a readable textbook with a conversational style inspired by my textbook-author role model, John D. Anderson, Jr. *Space Flight Dynamics* is a distillation of 20 years of course notes and strategies for teaching space flight in the Mechanical and Aerospace Engineering Department at the University of Missouri-Columbia.

Chapter 1 is a brief historical overview of the important figures and events that have shaped space flight. Chapter 2 provides the foundation of this textbook with a treatment of orbital mechanics. Here we are able to obtain analytical expressions for the orbital motion of a small body (such as a satellite) relative to a large gravitational body (such as a planet). Chapter 3 extends these concepts with a discussion of orbit determination, that is, the process of completely characterizing a satellite's orbit. In Chapter 4 we present Kepler's time-of-flight equations which allow us to predict a satellite's orbital position at a future (or past) time. We also discuss Lambert's problem: the process of determining an orbit that passes through two points in space separated by a particular flight time.

Chapter 5 introduces orbital perturbations that arise from the non-spherical shape of the attracting body, third-body gravity forces, and atmospheric drag. Perturbations cause the satellite's motion to deviate from the analytical solutions we obtained for the two-body motion studied in Chapters 2–4. We also introduce the restricted three-body problem where gravitational forces from two primary bodies (such as the Earth and moon) simultaneously influence the satellite's motion.

Chapter 6 presents fundamentals of rocket propulsion and launch trajectories. This chapter serves as a key transitional link to subsequent chapters that involve orbital maneuvers. Chapter 6 shows that burning a given quantity of rocket propellant corresponds to a change in orbital velocity, or Δv. The next four chapters involve orbital maneuvers, where the performance metric is typically the Δv increment. Chapter 7 discusses orbital changes achieved by so-called impulsive maneuvers where a rocket thrust force produces a velocity change in a relatively short time. Chapter 8 treats relative motion and orbital rendezvous, where a satellite moves in proximity to a desired orbital location or another orbiting satellite. In Chapter 9, we discuss low-thrust orbit transfers where an electric propulsion system provides a continuous but small perturbing thrust force that slowly changes the orbit over time. Interplanetary trajectories are treated in Chapter 10. Here we analyze a space mission by piecing together three flight segments: a planetary departure phase, an interplanetary cruise phase between planets, and a planetary arrival phase.

Chapter 11 introduces atmospheric entry or the flight mechanics of a spacecraft as it moves from orbital motion to flight through a planetary atmosphere. Here we develop analytical solutions for entry flight both with and without an aerodynamic lift force.

Chapters 2–11 involve particle dynamics, where we treat the satellite as a point mass. The last two chapters involve analyzing and controlling the rotational motion of a satellite about its center of mass. Chapter 12 presents attitude dynamics, or the analysis of a satellite's rotational motion. Topics in Chapter 12 include rotational motion in the absence of external torques, spin stability, and the effect of disturbance torques on rotational motion. Chapter 13 presents an introduction to attitude control. Here we primarily focus on controlling a satellite's angular orientation by using feedback and attitude control mechanisms such as reaction wheels and thruster jets.

Numerous examples are provided at key locations throughout Chapters 2–13 in order to illustrate the topic discussed by the particular section. Chapters 2–13 also contain end-of-chapter problems that are grouped into three categories: (1) conceptual problems; (2) MATLAB problems; and (3) mission applications. Many of the example and end-of-chapter problems illustrate concepts in space flight by presenting scenarios involving contemporary and historical space missions.

Appendix A presents the physical constants for celestial bodies. Appendix B provides a brief review of vectors and their operations and Appendix C is a review of particle kinematics with respect to inertial and rotating coordinate frames.

My intent was to write a comprehensive yet concise textbook on space flight dynamics. A survey of 35 space flight courses offered by US aerospace engineering programs shows that nearly half (17/35) are "orbits only" courses that focus on orbital mechanics, orbit determination, and orbital transfers. The remaining (18/35) courses include a mix of orbital motion and attitude dynamics and control. In addition, more than one-third (13/35) of the surveyed courses cover rocket performance and atmospheric entry. Few existing space flight textbooks adequately cover all of these topics. I believe that this textbook has the breadth and depth so that it can serve all of these diverse space flight courses.

Several people have contributed to the production of this textbook. Many reviewers provided valuable suggestions for improving this textbook and they are listed here:

Jonathan Black, Virginia Polytechnic Institute and State University
Craig McLaughlin, University of Kansas
Eric Monda, United Launch Alliance
Erwin Mooij, Delft University of Technology
Henry Pernicka, Missouri University of Science and Technology
David Spencer, The Pennsylvania State University
Srinivas Rao Vadali, Texas A&M University
Ming Xin, University of Missouri-Columbia

I am grateful for Jonathan Jennings' help with figures and illustrations. Finally, I would like to thank my wife Nancy M. West for her patience, encouragement, and skilled editorial work throughout this project. This book is dedicated to her.

University of Missouri-Columbia, May 2017 *Craig A. Kluever*

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1

Historical Overview

1.1 Introduction

Before we begin our technical discussion of space flight dynamics, this first chapter will provide a condensed historical overview of the principle contributors and events associated with the development of what we now commonly refer to as *space flight*. We may define space flight as sending a human-made satellite or spacecraft to an Earth orbit or to another celestial body such as the moon, an asteroid, or a planet. Of course, our present ability to launch and operate satellites in orbit depends on knowledge of the physical laws that govern orbital motion. This brief chapter presents the major developments in astronomy, celestial mechanics, and space flight in chronological order so that we can gain some historical perspective.

1.2 Early Modern Period

The fields of astronomy and celestial mechanics (the study of the motion of planets and their moons) have attracted the attention of the great scientific and mathematical minds. We may define the *early modern period* by the years spanning roughly 1500–1800. This time frame begins with the late Middle Ages and includes the Renaissance and Age of Discovery. Figure 1.1 shows a timeline of the important figures in the development of celestial mechanics during the early modern period. The astute reader will, of course, recognize these illuminous figures for their contributions to mathematics (Newton, Euler, Lagrange, Laplace, Gauss), physics (Newton, Galileo), dynamics (Kepler, Newton, Euler, Lagrange), and statistics (Gauss). We will briefly describe each figure's contribution to astronomy and celestial mechanics.

The first major figure is Nicolaus Copernicus (1473–1543), a Polish astronomer and mathematician who developed a solar-system model with the sun as the central body. Galileo Galilei (1546–1642) was an Italian astronomer and mathematician who defended Copernicus' sun-centered (or "heliocentric") solar system. Because of his heliocentric view, Galileo was put on trial by the Roman Inquisition for heresy and spent the remainder of his life under house arrest.

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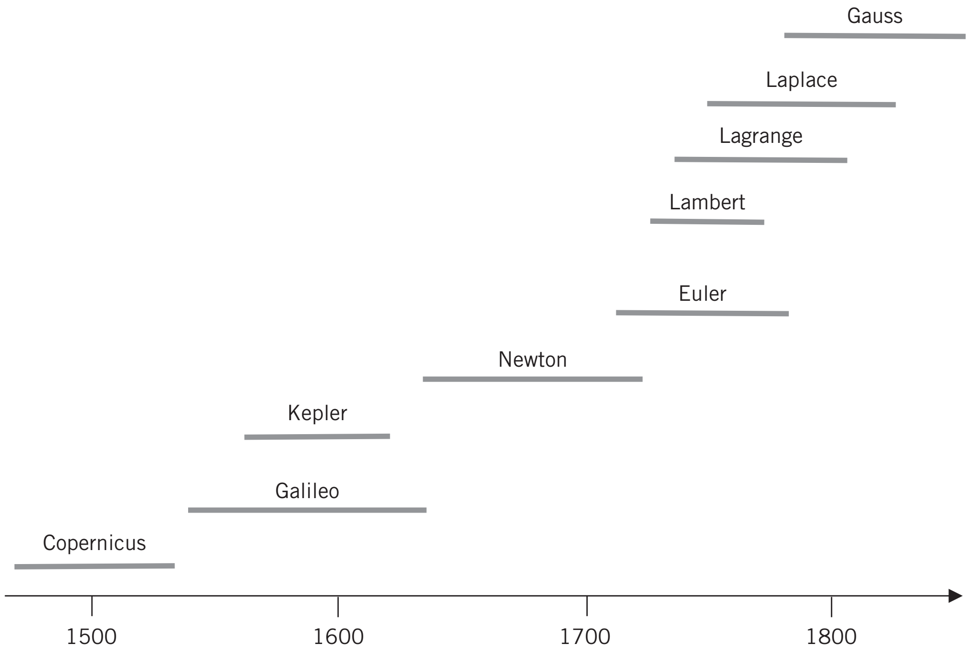

Figure 1.1 Timeline of significant figures in the Early Modern Period.

Johann Kepler (1571–1630) developed the fundamental laws for planetary motion based on astronomical observations of the planet Mars compiled by the Danish nobleman Tycho Brahe (1546–1601). Kepler's three laws are:

1) The orbit of a planet is an ellipse, with the sun located at a focus.
2) The radial line from the sun to the planet sweeps out equal areas during equal time intervals.
3) The square of a planet's orbital period for one revolution is proportional to the cube of the planet's "mean distance" from the sun.

The third law notes the planet's "mean distance" from the sun. In Chapter 2 we will define this "mean distance" as one-half of the length of the major axis of an ellipse. Kepler published his first two laws of planetary motion in 1609 and his third law in 1619. Kepler developed an expression for the time-of-flight between two points in an orbit; this expression is now known as *Kepler's equation*.

Isaac Newton (1642–1727) was an English astronomer, mathematician, and physicist who developed calculus and formulated the laws of motion and universal gravitation. Newton's three laws of motion are:

1) A body remains at rest or moves with a constant velocity unless acted upon by a force.
2) The vector sum of the forces acting on a body is equal to the mass of the body multiplied by its absolute acceleration vector (i.e., $\sum \mathbf{F} = m\mathbf{a}$).
3) When a body exerts a force on a second body, the second body exerts an equal-and-opposite force on the first body.

The first and second laws hold relative to a fixed or inertial reference frame. Newton published the three laws of motion in *Principia* in 1687. Newton's universal law of gravitation states that any two bodies attract one another with a force that is proportional to the product of their masses and inversely proportional to the square of their separation distance. Newton's laws of motion and gravitation explain the planetary motion that Kepler described by geometrical means.

Leonhard Euler (1707–1783), a Swiss mathematician, made many mathematical and scientific contributions to the fields of calculus, mathematical analysis, analytical mechanics, fluid dynamics, and optics. Euler also developed equations that govern the motion of a rotating body; these equations serve as the foundation for analyzing the rotational motion of satellites in orbit. Johann Heinrich Lambert (1728–1777), also a Swiss mathematician, formulated and solved the problem of determining the orbit that passes through two known position vectors with a prescribed transit time. Known today as *Lambert's problem*, its solution provides a method for the orbit-determination process as well as planning orbital maneuvers. Joseph-Louis Lagrange (1736–1813) was an Italian-born mathematician who made significant contributions in analytical mechanics and celestial mechanics, including the determination of equilibrium orbits for a problem with three bodies and the formulation of *Lagrange's planetary equations* for orbital motion. Pierre-Simon Laplace (1749–1827) was a French mathematician who, among his many mathematical contributions, formulated the first orbit-determination method based solely on angular measurements. Carl Friedrich Gauss (1777–1855), a German mathematician of great influence, made significant contributions to the field of orbit determination. In mid-1801 he predicted the orbit of the dwarf planet Ceres using a limited amount of observational data taken before Ceres became obscured by the sun. In late 1801, astronomers rediscovered Ceres just as predicted by Gauss.

1.3 Early Twentieth Century

Let us next briefly describe the important figures in the early twentieth century. It is during this period when mathematical theories are augmented by experimentation, most notably in the field of rocket propulsion. It is interesting to note that the important figures of this period were inspired by the nineteenth century science fiction literature of H.G. Wells and Jules Verne and consequently were tantalized by the prospect of interplanetary space travel.

Konstantin Tsiolkovsky (1857–1935) was a Russian mathematician and village school teacher who worked in relative obscurity. He theorized the use of oxygen and hydrogen as the optimal combination for a liquid-propellant rocket in 1903 (the same year as the Wright brothers' first powered airplane flight). Tsiolkovsky also developed theories regarding rocket propulsion and a vehicle's velocity change – the so-called "rocket equation."

Robert H. Goddard (1882–1945), a US physicist, greatly advanced rocket technology by combining theory and experimentation. On March 16, 1926, Goddard successfully launched the first liquid-propellant rocket. In 1930, Goddard moved his laboratory to New Mexico and continued to develop larger and more powerful rocket engines.

Hermann J. Oberth (1894–1989) was born in Transylvania and later became a German citizen. A physicist by training, he independently developed theories regarding human space flight through rocket propulsion. Oberth was a key figure in the German Society for Space Travel, which was formed in 1927, and whose membership included the young student Wernher von Braun. Von Braun (1912–1977) led the Nazi rocket program at Peenemünde during World War II. Von Braun's team developed the V-2 rocket, the first long-range rocket and the first vehicle to achieve space flight above the sensible atmosphere.

At the end of World War II, von Braun and members of his team immigrated to the US and began a rocket program at the US Army's Redstone Arsenal at Huntsville, Alabama. It was during this time that the US and the Soviet Union were rapidly developing long-range intercontinental ballistic missiles (ICBMs) for delivering nuclear weapons.

1.4 Space Age

On October 4, 1957, the Soviet Union successfully launched the first artificial satellite (Sputnik 1) into an Earth orbit and thus ushered in the *space age*. Sputnik 1 was a polished 84 kg metal sphere and it completed an orbital revolution every 96 min. The US successfully launched its first satellite (Explorer 1) almost 4 months after Sputnik on January 31, 1958. Unlike Sputnik 1, Explorer 1 was a long, tube-shaped satellite, and because of its shape, it unexpectedly entered into an end-over-end tumbling spin after achieving orbit.

Our abridged historical overview of the first half of the twentieth century illustrates the very rapid progress achieved in rocket propulsion and space flight. For example, in less than 20 years after Goddard's 184 ft flight of the first liquid-propellant rocket, Nazi Germany was bombarding London with long-range V-2 missiles. Twelve years after the end of World War II, the USSR successfully launched a satellite into orbit. Another point of interest is that in this short period, rocket propulsion and space flight transitioned from the realm of the singular individual figure to large team structures funded by governments. For example, the US established the National Aeronautics and Space Administration (NASA) on July 29, 1958.

The US and USSR space programs launched and operated many successful missions after the space age began in late 1957. Table 1.1 summarizes notable robotic space missions (i.e., no human crew). A complete list of successful space missions would be quite long; Table 1.1 is not an exhaustive list and instead presents a list of mission "firsts." It is truly astounding that 15 months after Sputnik 1, the USSR sent a space probe (Luna 1) to the vicinity of the moon. Equally impressive is the first successful interplanetary mission (Mariner 2), which NASA launched less than 5 years after Explorer 1. Table 1.1 shows that spacecraft have visited all planets in our solar system and other celestial bodies such as comets and asteroids.

On April 12, 1961, the USSR successfully sent the first human into space when Yuri Gagarin orbited the Earth in the Vostok 1 spacecraft. Less than 1 month later, the US launched its first human into space when Alan Shepard flew a suborbital mission in a Mercury spacecraft. Table 1.2 presents notable space missions with human crews (as with Table 1.1, Table 1.2 focuses on first-time achievements). Tables 1.1 and 1.2 clearly illustrate the accelerated pace of accomplishments in space flight. Table 1.2 shows

Table 1.1 Notable robotic space missions.

Mission	Date	Achievement	Country
Sputnik 1	October 4, 1957	First artificial satellite to achieve Earth orbit	USSR
Luna 1	January 2, 1959	First satellite to reach the vicinity of the moon	USSR
Mariner 2	December 14, 1962	First spacecraft to encounter (fly by) another planet (Venus)	US
Mariner 4	July 14, 1965	First spacecraft to fly by Mars	US
Luna 9	February 3, 1966	First spacecraft to land on another body (moon)	USSR
Luna 10	April 3, 1966	First spacecraft to orbit the moon	USSR
Venera 7	December 15, 1970	First spacecraft to land on another planet (Venus)	USSR
Mariner 9	November 14, 1971	First spacecraft to orbit another planet (Mars)	US
Pioneer 10	December 3, 1973	First spacecraft to fly by Jupiter	US
Mariner 10	March 29, 1974	First spacecraft to fly by Mercury	US
Viking 1	July 20, 1976	First spacecraft to land on Mars	US
Voyager 1	March 1979, November 1980	Fly by encounters with Jupiter, Saturn, and Saturn's moon Titan	US
Voyager 2	January 1986, August 1989	First spacecraft to fly by Uranus and Neptune	US
Galileo	December 8, 1995	First spacecraft to orbit Jupiter	US
Mars Pathfinder	July 4, 1997	First rover on the planet Mars	US
NEAR Shoemaker	February 12, 2001	First spacecraft to land on an asteroid (433 Eros)	US
Cassini-Huygens	July 2004, January 2005	First spacecraft to orbit Saturn (Cassini) and first spacecraft to land on the moon Titan (Huygens)	US and Europe
Stardust	January 16, 2006	First spacecraft to return samples from a comet	US
MESSENGER	March 18, 2011	First spacecraft to orbit Mercury	US
New Horizons	July 14, 2015	First spacecraft to fly by Pluto	US

Table 1.2 Notable space missions with human crews.

Mission	Date	Achievement	Country
Vostok 1	April 12, 1961	First human to reach space and orbit the Earth	USSR
Vostok 6	June 16, 1963	First woman in space	USSR
Voskhod 2	March 18, 1965	First human "spacewalk" outside of orbiting spacecraft	USSR
Gemini 6A	December 15, 1965	First orbital rendezvous	US
Apollo 8	December 24, 1968	First humans to orbit the moon	US
Apollo 11	July 20, 1969	First humans to land and walk on the moon	US
Salyut 1	April 19, 1971	First orbiting space station with crew	USSR
STS-1	April 12, 1981	First flight of a reusable spacecraft (Space Shuttle)	US
International Space Station	November 20, 1998	First multinational space station and largest satellite placed in Earth orbit	Russia, US, Europe, Japan, Canada

Table 1.3 Significant advances in space flight dynamics in the twentieth century.

Researcher(s)	Achievement
Dirk Brouwer Yoshihide Kozai	Developed pioneering work in the field of analytical satellite theory, including the perturbing effects of a non-spherical Earth
Theodore Edelbaum	Obtained analytical optimal trajectory solutions for spacecraft propelled by low-thrust electric propulsion engines
Richard Battin	Developed guidance and navigation theories for lunar and interplanetary spacecraft
Rudolf Kalman	Developed an optimal recursive estimation method (the *Kalman filter*) that has been applied to orbit determination and satellite navigation
W.H. Clohessy and R.S. Wiltshire	Developed closed-form solutions for the motion of a satellite relative to an orbiting target satellite (i.e., orbital rendezvous)
Derek Lawden	Developed theories for optimal rocket trajectories
A.J. Eggers and H.J. Allen Dean Chapman	Obtained analytical solutions for the entry flight phase of a ballistic capsule or lifting spacecraft returning to Earth from space
Robert Farquhar	Conceived of and managed space missions that targeted orbits where the satellite is balanced by the gravitational attracting of two celestial bodies
Ronald Bracewell Vernon Landon	Developed theories regarding the stability of a spinning satellite in orbit
Paul Cefola	Developed the Draper Semianalytical Satellite Theory (DSST) for rapid orbital calculations over a long time period

the very rapid progress in space missions with human crews in the 1960s, culminating with the first Apollo lunar landing on July 20, 1969. To date, three countries have developed human space flight programs: USSR/Russia (1961); US (1961); and China (2003).

We end this chapter with a brief summary of the significant twentieth century figures in the field of space flight dynamics. Table 1.3 presents these figures and their accomplishments. This list is certainly not exhaustive; furthermore, it is difficult to identify single individuals when the tremendous achievements in space flight over the past 60 years involve a large team effort.

2

Two-Body Orbital Mechanics

2.1 Introduction

In this chapter, we will develop the fundamental relationships that govern the orbital motion of a satellite relative to a gravitational body. These relationships will be derived from principles that should be already familiar to a reader who has completed a course in university physics or particle dynamics. It should be no surprise that we will use Newton's laws to develop the basic differential equation relating the satellite's acceleration to the attracting gravitational force from a celestial body. We will obtain analytical (or closed-form) solutions through the conservation of energy and angular momentum, which lead to "constants of motion." By the end of the chapter the reader should be able to analyze a satellite's orbital motion by considering characteristics such as energy and angular momentum and the associated geometric dimensions that define the size and shape of its orbital path. Understanding the concepts presented in this chapter is paramount to successfully grasping the subsequent chapter topics in orbit determination, orbital maneuvers, and interplanetary trajectories.

2.2 Two-Body Problem

At any given instant, the gravitational forces from celestial bodies such as the Earth, sun, moon, and the planets simultaneously influence the motion of a space vehicle. The magnitude of the gravitational force of *any* celestial body acting on a satellite with mass m can be computed using Newton's law of universal gravitation

$$F_{\text{grav}} = \frac{GMm}{r^2} \tag{2.1}$$

where M is the mass of the celestial body (Earth, sun, moon, etc.), G is the universal constant of gravitation, and r is the separation distance between the gravitational body and the satellite. It is not difficult to see that Eq. (2.1) is an *inverse-square* gravity law. The gravitational force acts along the line connecting the centers of the two masses. Figure 2.1 illustrates Newton's gravitational law with a two-body system comprising the Earth and a satellite. The Earth attracts the satellite with gravitational force vector

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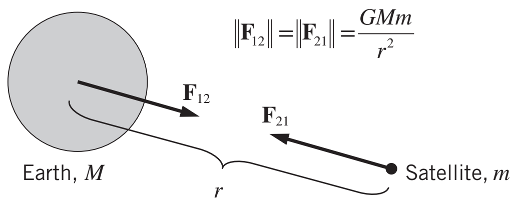

Figure 2.1 Newton's law of universal gravitation.

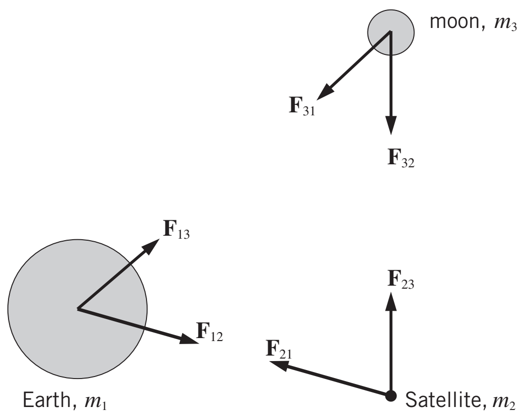

Figure 2.2 Gravitational forces for a three-body system.

$\mathbf{F}_{21}$ and the satellite attracts Earth with force $\mathbf{F}_{12}$. The reader should note that Eq. (2.1) presents the magnitude of the mutually attractive gravitational forces.

Figure 2.2 shows a schematic diagram of a three-body system (Earth, satellite, moon) with mutual gravitational forces among all three bodies. It should be clear from Figure 2.2 that $\mathbf{F}_{ij} = -\mathbf{F}_{ji}$. Equation (2.1) shows that the magnitudes are equal, or $\|\mathbf{F}_{ij}\| = \|\mathbf{F}_{ji}\|$. It is not difficult to imagine a diagram similar to Figure 2.2 with several (or N) gravitational bodies (however, an N-body diagram is very cluttered). The goal of this chapter (and the objective of this textbook) is to determine the motion of the satellite. Hence, a reasonable approach (similar to methods used in a basic dynamics course) would be to apply Newton's second law to a free-body diagram of the satellite. Applying Newton's second law to satellite mass m_2 for the three-body problem illustrated in Figure 2.2 yields

$$m_2\ddot{\mathbf{r}}_2 = \mathbf{F}_{21} + \mathbf{F}_{23} \tag{2.2}$$

where $\ddot{\mathbf{r}}_2$ is the satellite's acceleration vector relative to an *inertial* frame of reference or a frame that does not accelerate or rotate (we will use the over-dot notation to indicate a time derivative, e.g., $\dot{\mathbf{r}} = d\mathbf{r}/dt$ and $\ddot{\mathbf{r}} = d^2\mathbf{r}/dt^2$). We can extend Eq. (2.2) to an N-body system

$$m_2\ddot{\mathbf{r}}_2 = \sum_{\substack{j=1 \\ j \neq 2}}^{N} \mathbf{F}_{2j} \tag{2.3}$$

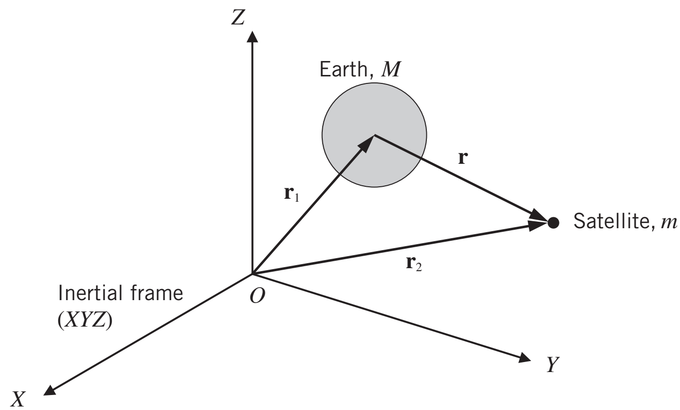

Figure 2.3 Two-body system.

Clearly, Eq. (2.3) is reduced to Eq. (2.2) when $N = 3$ as in Figure 2.2. Integrating Eq. (2.3) allows us to obtain the satellite's motion [velocity $\dot{\mathbf{r}}_2(t)$ and position $\mathbf{r}_2(t)$] in an N-body gravitational field. However, we cannot obtain analytical solutions of the general N-body problem [note that the inverse-square gravity (2.1) is a nonlinear function]. We must employ numerical integration schemes (such as Runge–Kutta methods) to obtain solutions to the N-body problem.

It is possible, however, to obtain analytical solutions for the satellite's motion if we only consider two bodies. These closed-form solutions will provide the basis for our analysis of space vehicle motion throughout this textbook. Figure 2.3 shows a two-body system comprising the Earth (mass M) and satellite (mass m). Coordinate system XYZ is an inertial Cartesian frame that does not rotate or accelerate. Vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ are the inertial (absolute) positions of the Earth and satellite relative to the XYZ frame. The position of the satellite *relative* to the Earth is easily determined from vector addition:

$$\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 \tag{2.4}$$

If the mutual gravitational forces are the only forces in the two-body system, then applying Newton's second law to each mass particle yields

$$\text{Earth: } M\ddot{\mathbf{r}}_1 = \frac{GMm}{r^2}\left(\frac{\mathbf{r}}{r}\right) \tag{2.5}$$

$$\text{Satellite: } m\ddot{\mathbf{r}}_2 = \frac{GMm}{r^2}\left(\frac{-\mathbf{r}}{r}\right) \tag{2.6}$$

Note that $\mathbf{r}/r$ is a unit vector pointing from the Earth's center to the satellite (hence $-\mathbf{r}/r$ is the direction of the Earth's attractive gravitational force on the satellite). Adding Eqs. (2.5) and (2.6) yields

$$M\ddot{\mathbf{r}}_1 + m\ddot{\mathbf{r}}_2 = \mathbf{0} \tag{2.7}$$

Integrating Eq. (2.7), we obtain

$$M\dot{\mathbf{r}}_1 + m\dot{\mathbf{r}}_2 = \mathbf{c}_1 \tag{2.8}$$

where $\mathbf{c}_1$ is a vector of integration constants. Equation (2.8) is related to the velocity of the center of mass of the two-body system. To show this, let us express the inertial position of the two-body system's center of mass:

$$\mathbf{r}_{cm} = \frac{M\mathbf{r}_1 + m\mathbf{r}_2}{M + m} \tag{2.9}$$

Taking the time derivative of Eq. (2.9), we see that Eq. (2.8) is equal to the product of the total mass ($M + m$) and the velocity of the center of mass. Therefore, we can conclude that the center of mass $\mathbf{r}_{cm}$ is not accelerating.

Our goal is to develop a governing equation for the satellite's motion relative to a single gravitational body M. Let us take the second time derivative of the relative position vector, Eq. (2.4):

$$\ddot{\mathbf{r}} = \ddot{\mathbf{r}}_2 - \ddot{\mathbf{r}}_1 \tag{2.10}$$

Next, we use Eqs. (2.5) and (2.6) to substitute for the absolute acceleration vectors of the Earth and satellite:

$$\ddot{\mathbf{r}} = \frac{GM}{r^2}\left(\frac{-\mathbf{r}}{r}\right) - \frac{Gm}{r^2}\left(\frac{\mathbf{r}}{r}\right)$$

or

$$\ddot{\mathbf{r}} = -\frac{G(M+m)}{r^3}\mathbf{r} \tag{2.11}$$

Note that although the denominator is r^3, Eq. (2.11) is still an inverse-square law because $\mathbf{r}/r$ is a unit vector. Equation (2.11) is a vector acceleration equation of the relative motion for the two-body problem.

Let us complete the two-body equation of motion by making use of the previous results and the assumption that the satellite's mass m is negligible compared with the mass of the gravitational body M. This assumption is very reasonable; for example, the mass ratio of a 1,000 kg satellite and the Earth is less than $2(10^{-22})$. Hence, we may assume that the two-body system center of mass and the center of the Earth are coincident. Furthermore, because the center of mass is not accelerating we can place an inertial frame at the center of the gravitational mass M. Figure 2.4 shows this scenario where the origin O of the

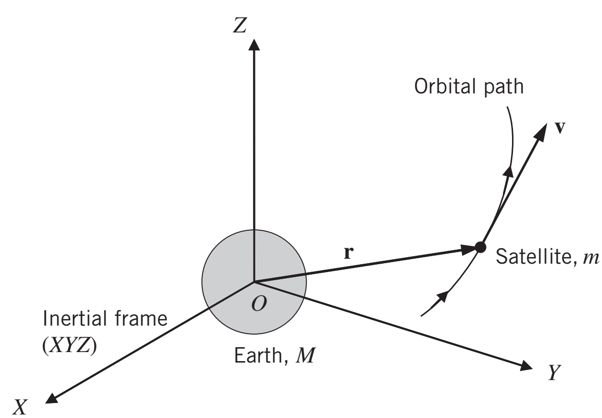

Figure 2.4 Two-body system with a body-centered inertial frame *XYZ*.