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Contemporary Computational Mathematics – A Celebration of the 80th Birthday of Ian Sloan

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Dedicated to Ian H. Sloan on the occasion of his 80th birthday.

Preface

On June 17, 2018, Professor Ian Hugh Sloan will celebrate his 80th birthday. We are delighted in wishing him well on this happy occasion. Ian has been a teacher, a mentor, a research collaborator, and a very dear friend to many of us.

We decided to give Ian a special birthday present in the form of this book as a tribute to his many important contributions in various areas of computational mathematics. At this point, we wish to thank the colleagues who contributed to this book as authors and/or referees. In fact, the response of sending papers to celebrate the 80th birthday of Ian was so great and from so many colleagues that it was indeed a difficult job for us to limit the number of pages of the book. We are very grateful to Springer Verlag that they agreed from the very beginning that the number of pages of the book will not be an issue.

The book consists of nearly 60 articles written by international leaders in a diverse range of areas in contemporary computational mathematics. These papers highlight impact and many achievements of Ian in his distinguished academic career. The papers also present the current state of knowledge in such areas as quasi-Monte Carlo and Monte Carlo methods for multivariate integration, multi-level methods, finite element methods, uncertainty quantification, spherical designs and integration on the sphere, approximation and interpolation of multivariate functions, and oscillatory integrals and in general in information-based complexity and tractability, as well as in a range of other topics.

This book tells an important part of the life story of the renowned mathematician, family man, colleague, and friend who has been an inspiration to so many of us.

We believe that the best way to begin this book is by presenting a few words about Ian. We are also sure that the reader will enjoy reading the family perspectives on Ian by his wife Jan, his children Jenni and Tony, and his grandchildren Sam, Gus, Mack, Corrie, and Kiara. (Granddaughter Eliza missed the opportunity to contribute due to travelling.)

Ian Hugh Sloan was born on June 17, 1938, in Melbourne, Australia. He did his schooling at Scotch College, Melbourne, and Ballarat College. The father of Ian was a senior mathematics master at Scotch College and later Principal at Ballarat and apparently took good care of the background on mathematical education of his son.

Then Ian was educated at the University of Melbourne, where he obtained BSc in 1958 in physics and BA (hons) in 1960 in pure and applied mathematics. Ian met his future wife Jan at the University of Melbourne, and they were married in 1961. He obtained MSc at the University of Adelaide in 1961 in mathematical physics for a thesis entitled "*Ionization in Nebulae*". Ian was supervised in Adelaide by Professor Herbert Green who was one of Australia's first professors of mathematical physics. It is worth mentioning that Ian completed his master's degree in record time of 7 months. Ian received his PhD in 1964 at the University of London in theoretical physics based on the thesis "*Electron Collisions by Neutral and Ionized Helium*" and was supervised by renowned mathematical physicist Professor Sir Harrie Massey who worked on the Manhattan Project and at the Australian Woomera Rocket Range. Ian completed his PhD again in record time of just 30 months. Part of his PhD work involved computations on an early mainframe machine in Manchester.

Ian Sloan started his professional career as a research scientist for the Colonial Sugar Refining Company, during 1964–1965, in Melbourne. Since 1965 Ian Sloan has been at the University of New South Wales as a member of the School of Mathematics. He was appointed Lecturer in 1965 and became involved in research in theoretical nuclear physics. Ian had a very good start at UNSW and published 10 single authored papers in the first 5 years. He was promoted to Senior Lecturer in 1968. His research focus was shifting from theoretical physics to applied mathematics, especially towards numerical analysis, first for integral equations relevant to scattering theory and then to computational mathematics mostly of multivariate integration and approximation. He was promoted to Associate Professor in 1973 and was appointed to a personal Chair in Mathematics in 1983 and then to Scientia Professor in 1999. He served as Head of the School of Mathematics of UNSW from 1986 to 1990 and from 1992 to 1993.

Ian had many visiting positions during his career. He was associated, in particular, with (in alphabetical order) Cornell University, ESI in Vienna, Hong Kong Polytechnic University, IBM Paris, ICERM in Providence in the USA, King Fahd University of Petroleum and Minerals in Saudi Arabia, Mittag-Leffler Institute in Stockholm, Newton Institute at Cambridge, Politecnico di Torino, Technical University of Vienna, University of Bath, University of Maryland (two sabbaticals 1971–1972 and 1979–1980), University of Stuttgart, and Weierstrass Institute in Berlin.

Ian Sloan received many honours and awards during his academic career. In 1993 he was elected a Fellow of the Australian Academy of Science; in 1997 he was awarded the ANZIAM Medal of the Australian Mathematical Society; during 1998–2000 he was the President of the Australian Mathematical Society; in 2001 he received the Australian Academy of Science's Thomas Ranken Lyle Medal; in 2001 he was awarded the Centenary Medal; in 2002 he shared the inaugural George Szekeres Medal of the Australian Mathematical Society with Alf van der Poorten of Macquarie University; during 2003–2007 he was the President of the International Council for Industrial and Applied Mathematics (ICIAM); in 2005 he received the Information-Based Complexity (IBC) Prize; in the June 2008 Queen's Birthday Honours, he was appointed an Officer of the Order of Australia (AO); in 2009 he became a Fellow of the Society for Industrial and Applied Mathematics (SIAM); in 2012 he became a Fellow of the American Mathematical Society; and in 2014 he was elected a Fellow of the Royal Society of New South Wales (FRSN).

Ian Sloan has been serving on editorial boards of many international computational mathematical journals. These include *Journal of Integral Equations and Applications* (1987–2012), *SIAM Journal on Numerical Analysis* (1991–1997 and 2003–2012), *Journal of Complexity* (1999–2009 as Associate Editor and since 2009 as Senior Editor), *Numerische Mathematik* (2004–2014), *Advances in Computational Mathematics* (2000–2015), *Computational Methods in Applied Mathematics* (2000–2015), *Chinese Journal of Engineering Mathematics* (2007–), *International Journal of Geomathematics* (2011–), *International Journal for Mathematics in Industry* (2013–), and *Foundations of Computational Mathematics* (2015–).

Ian Sloan loves to work with other people. The list of his collaborators is very impressive, and many of them contributed papers to this book. He was a PhD advisor of Reginald Cahill (1971), John Aarons (1972), Ivan Graham (1980), Stephen Joe (1985), Sunil Kumar (1987), Yi Yan (1989), Thanh Tran (1994), Timothy Langtry (1995), Thang Cao (1995), Yi Zeng (1998, co-supervisor), Josef Dick (2004), Kassem Mustapha (2004, co-supervisor), Benjamin Waterhouse (2007), Paul Leopardi (2007), Jan Baldeaux (2010), Cong Pei An (2011, co-supervisor), James Nichols (2013), Andrew Chernih (2013), Yu Guang Wang (2015), Alexander Gilbert (current, co-supervisor), and Yoshihito Kazashi (current).

Ian Sloan has so far published more than 280 peer-reviewed papers in leading journals of theoretical physics and computational mathematics, book chapters, and refereed conference proceedings, as well as one book with Stephen Joe entitled *Lattice Methods for Multiple Integration* published by Oxford University Press in 1994. His papers cover various areas such as the numerical solution of integral equations, boundary integral equations, numerical integration, interpolation and approximation of multivariate functions, partial differential equations with random coefficients, and information-based complexity and tractability. The list of Ian's publications is included later in this book. He is one of a select few on the 2001 Thompson ISI list of highly cited authors.

Professor Ian Sloan has made outstanding contributions to mathematical research over the last 50 years. Ian's impact is felt widely today; the Bencze-Redish-Sloan equation and the Sloan iteration for integral equations (see the article by Thanh Tran in this monograph) have been named after him. Further key contributions were the introduction of weighted spaces and the study of tractability and inventing the component-by-component construction of lattice rules.

As many of us know, Ian loves to travel. He almost always travels with his wife Jan, and it is sometimes easier to meet Ian and Jan abroad than in Sydney. He has travelled to all parts of the world giving invited talks on his work really almost everywhere. He is making many friends during his trips and has many interesting, not always mathematical, stories to tell about his travels. More importantly, Ian does not slow down. He is as energetic and active today as he was years ago. Ian Sloan is a role model and inspiration to his friends and colleagues.

In addition to most authors of this book who also served as referees, the following people also served as referees for the book: Michael Feischl, Alexander Gilbert, Juan Gonzalez Criado del Rey, Michael Griebel, Thomas Hou, James Hyman, Stephen Joe, Pierre LÉcuyer, Klaus Ritter, Robert Schaback, Frank Stenger, Kosuke Suzuki, Mario Ullrich, Clayton Webster, Takehito Yoshiki, and Penchuan Zhang. We sincerely thank all authors and referees for their contributions.

We are also grateful to Martin Peters of Springer Verlag for his strong support of this book from the very beginning and for making it possible that every contributor receives a free copy of the book.

Josef Dick
Frances Y. Kuo
Henryk Woźniakowski

Family Perspectives

Jan Sloan (Wife)

The Ian Sloan I know is always in a hurry and never has enough time. There is always a paper to finish, a deadline to meet, someone to see, or something to do. I understand that this started early: he began school a few days after turning four because he grew impatient watching other children through the school fence. (His father was a mathematics teacher and at that time was a house master of a boarding house which happened to be next to the primary school.) A few years later, we spent our honeymoon in Adelaide so that he could put in an appearance at morning tea in the Math Physics Department, to allow him to complete a master's degree in two terms—morning tea satisfied the "minimum three-term" requirement! Then he completed a PhD in theoretical physics at the University College London in two and a half years, with his final oral exam taken in a taxi. The oral in a taxi was because his supervisor, Professor Sir Harrie Massey, was in even more of a hurry: he was an important person in the space programme at Cape Canaveral (now Cape Kennedy), so was rarely in London—indeed Ian saw him only six times during his PhD, or seven if you count the taxi. But there was another reason for urgency, namely, that Ian wanted to finish his PhD in superfast time.

I believe his PhD experience made him exceptionally self-reliant and independent. He never needs others to tell him what to do.

After a memorable two and a half years in London, we returned to Australia for Ian to take up a research position in an industrial lab (to which he owed some loyalty since they had paid him to do the PhD in London). Ian says this period of industrial research was the most miserable period of his working life, since in reality there was nothing for him to do. (His industry boss allowed him to go gracefully, saying "We have to accept that some people are not cut out for research.") After a gloomy 10 months in industrial research, he was rescued by an advertisement for a casual teaching position at the University of New South Wales. And after more than 50 years, he is still there!

The person I know is courageous and of strong character. If something needs to be said, he will say it. A choice example came early in his career at UNSW when he told his Head of Department that he should either do his job properly or resign. I suggested, but to no avail, that this was not the right way to advance his career. He is still stirring trouble, recently organising his colleagues in a collective letter of complaint to the Vice Chancellor. I can tell him that this is unwise, but it makes no difference.

He is also uncompromisingly honest. Our children will remember that they were never allowed to use his university-supplied paper and pencils.

He is always checking facts, correcting spelling and pronunciation, and finding errors—no doubt a desirable characteristic professionally, but one not always appreciated at home. Another challenging habit is that he is always reading the fine print, advertisements, and plaques on the wall, anything at all. It is a bad idea to take him food shopping, as he reads all the labels.

Surprisingly, he has a terrible memory. He often tells me of introducing himself in friendly fashion, only to get the response “I know who you are, we met yesterday”.

Many collaborators over the years, first in physics, then later in mathematics, have commented that he has great energy. That is a side that I rarely see, but it is true that he will often be at his desk sending emails in the early hours of the morning. And during our many overseas travels, I am often woken by the clicking of the computer keys when all sensible people are asleep.

Travel has always been an important part of our lives. This is especially true now that children are grown up, and I am not working and Ian is not teaching, but even in the early days, the two year-long sabbaticals we spent at the University of Maryland were important to us—and to our two children who spent formative years in American schools. Through our many travels, we have made the most beautiful and long-lasting friendships.

He is an enthusiastic reader, with a wide range of interests, and he often has five books on the go at one time. (One grandson inherited the interest, and when small always walked with a book in hand.) Both of us are keen on music. For Ian, this used to take the form, when the children were small, of playing Chopin etudes and Beethoven sonatas on the piano in the evening. We attend many concerts, and both love theatre. For us this started all those years ago in London, where for two and a half years we attended one stellar performance or another almost every week.

Ian often talks to me about his various projects, in spite of my complete lack of mathematical training and aptitude. Such talk is not about mathematical details, but rather about strategy, and about the human and intellectual struggles. And I like to know who among our friends is involved in each project.

He seems to me to change fields more often than most people. Privately, he tells me that he is something of a butterfly: he sips the nectar then moves on.

Over the years I have watched from the sidelines the evolution of many projects and papers, first in physics then later in mathematics. Often there is drama, through the struggle to get ideas worked out and papers written and accepted, and sometimes (though rarely) the pain of rejection. But I have also shared in the pleasure of the many awards and signs of recognition he has received over the years.

And I have watched him prepare many talks. It is clear to me that he takes great effort to communicate his ideas and results clearly, and I believe that his colleagues agree.

In spite of his passion for research, he has always been willing to take on administrative burdens, often against my advice. He has been Head of this and Chair of that, always with apparent success. An extreme example early in his mathematical life was taking on the Editorship of the journal that later became the *ANZIAM Journal*, at a time when the journal was in disarray, with the previous Editor having resigned, recommending that the journal be closed down. The journal still exists and is apparently doing well.

Along the way we have found time to have two children and by now six grandchildren, all of whom are just fine, and a source of great pride to both of us. Ian claims to be a good family man, in spite of never having enough time, and says we should judge by the results. I am not so sure, but will ask the family to pass their own judgement.

Jenni Johnson (Daughter)

. . . and judge we do. . . somewhat mercilessly at times!

Ian (or dad as I like to call him, despite his best attempts to level the playing field by getting us to call him Ian) is a conundrum. The man at home is a constant source of amusement, often leading the way by laughing at himself, a very endearing characteristic which I have decided to emulate, as it helps get one through otherwise difficult situations. I recall him telling us about his first attempt to use a lapel microphone for one of his lectures. As the lecture progressed and he became more and more animated, he found himself wrapped up in the cord, struggling to stay upright and keep control of the students, who were vastly amused.

He can burst with energy if there is something stimulating to argue about or discuss; arms will fly, the eyes become beady, the veins on his rather proud dome pulse with excitement, and the voice volume rises. Just as easily, he will be off to sleep in a flash if there is too much “small talk”—so it is a constant challenge for us all to keep things interesting. Some might say he has a short attention span, but I am not so sure. I think it’s an efficiency measure to keep his brain agile for things that really matter.

While his brain is agile, he has very poor body awareness, and this sometimes gets him into trouble both at home and in far-flung places. He recently injured his knee playing tennis, but was only really aware of the problem the next day when he described “struggling to get out of the bed sheets”. Most lesser mortals would have described pain or swelling as the first symptom and usually at the time of the accident. He has been known to walk into walls leaving a permanent nose imprint (to the delight of children and grandchildren) or walk around with socks filled with blood oblivious to a recent insult.

He manages to move around the world mixing with the best. He is adventurous and curious, keeping us well entertained with his stories of international largesse. Yes of course there is always a talk or a workshop to prepare, a paper to review, a student to meet, a keynote address, an important conference to prepare for, and the endless awards and accolades he receives for his work. However the best stories come from his adventures overseas. You may find him trapped in the mines of Chile, losing his navel in a hospital in Poland, on the shores of Lake Baikal in Russia drinking vodka out of a glass rifle, or racing uncontrollably across the sand dunes of Saudi Arabia in a four wheel drive. There is always a funny story to be told and some more friends to be collected along the way.

He adapts to his environment like a chameleon, feeling just as at home in China as in Italy, Korea, or the USA. He learns the essentials of the language and picks up a few colleagues along the way to assist with the immersion process. He is on and off planes at the drop of a hat, yet seems to bounce back to be back at work on the same day. He manages a black tie affair at the Academy of Science, a family “muck in”, or is equally comfortable in his “gardening gear and matching glasses”. I think this is because he is humble, extremely tolerant, and as I have heard from others “a true gentleman”.

One of his most treasured abilities is multitasking. He will often be reading five books at one time, arranging travel for his next adventure, writing several papers, arranging the next season’s tickets to the opera, and watching the football all at the same time. In order to get through all this, the 24 h of the day he has available must be maximally utilised; he is well known to be stomping around at 3:00 am writing down the skeletal thoughts of his next paper. He will always have paper and a pencil available readily just in case a thought needs to be captured.

I believe this extreme behaviour comes from a desire to keep learning and continues to be relevant. He strives to make a difference. I think this characteristic has been handed down to the next generations, something for which we are very grateful. The long hours and high productivity are really not a chore for him. Work and life seem to be like sweet treats in a cake shop—all to be sampled and enjoyed or passed off as an “interesting experience”. (For him “interesting” is a deceptive adjective: it can well mean he didn’t like it!)

So maths, science, and family are his passions, but he also enjoys music, theatre, food, wine, and sport even though this is not his personal forte. He enjoys all the children’s sports and is a proud grandparent. He has recently taken up tennis again and has joined a whole series of clubs, really quite strange, as he seemed to have an intolerance of this kind of activity as a younger man. There is always a surprise in store for us!

I find it difficult to reconcile the man we know, somewhat distracted, warm hearted, generous, and humble, with the man and his stellar academic career. We are very proud to have been part of it all!

Tony Sloan (Son)

My father has an eclectic sense of humour. Rather than saying "time flies", he'll rush around like a mad professor screaming "tempus fugit". In my field of accountancy, we say that small amounts of money are "immaterial". Ian's mathematical translation of this term is "epsilon". So I learned about "epsilon" (very small) and "infinity" (more than very large) early in my life. When Ian received his Lyle Medal a decade or so ago, he roared with laughter, almost uncontrollably, when someone said that something was getting "ever closer to infinity". When George Bush was talking about weapons of mass destruction, Ian was proudly holding his own weapon of math instruction—his calculator.

He gave me a tee shirt, channelling Maxwell's equations, which he thought was hilarious. It read: "And God Said

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad \nabla \cdot \mathbf{E} = \mathbf{0},$$
$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \qquad \nabla \cdot \mathbf{B} = \mathbf{0}.$$

And Then There Was Light". Once he borrowed it when he was teaching Maxwell's equations, and at the appropriate moment, in the style of Superman, he stripped off his outer shirt to reveal the message.

While most people have graduated and have proper jobs in the real world, he sticks with maths because he says it's the only job which really counts. But I took a different point of view, and often used to ask him: "When are you going to get a real job?"

Age shall not weary him, technology shall not defeat him, and retirement is just moving ever closer to infinity for Ian. As he progresses around the bell curve of life, and slowly transforms from a vertical bar into a more cuddly version, higher honours no doubt await him, Professor to the math gods perhaps or maybe "Sir Cumference", with the Queen's blessing.

Sam Johnson (Grandson)

For much of my early childhood, I was confused as to what my grandfather (or, as he prefers to sign his emails to the family, Ian/Dad/Papa) actually did for his job. I had been told that he was a "very smart man who likes maths", but much of my experiences with him seemed to involve English. For example, Papa always had a strong compulsion to drill into the heads of his grandchildren the correct spelling of "raspberry". He would always make sure to identify the silent "p", and would clap in delight the first time one of us spelled it correctly. (Jan adds: the word "raspberry" is special to Ian because, as I discovered early and to my great delight, he himself didn't know it contained a silent "p", and only believed me after checking the dictionary.)

Another strong image of mine as a child is going to Ian's office. As I took after my grandfather in my bookishness, he seemed to take great amusement in taking out various papers that he had been working on and seeing if I could read and pronounce the titles. While at the time (and to this day) the words I was saying appeared to be nonsense, nonetheless he made sure that by the end of the session I had come away being able to pronounce a new word.

Later in life, I came to the belief that rather than being a professor, my grandfather was actually a professional traveller, as he seemed to spend the majority of his time in a different country. You can imagine my shock when I eventually found out that he was journeying to all these exotic locales to sit with colleagues and discuss mathematical principles. I did find, however, that the trips were not quite as boring as I had imagined, as I discovered when I was invited to take a holiday with both my grandparents to South America in early 2013: there we travelled in zodiacs, gazed at a glacier calving, and saw amazing mountains.

Ian has always been an enigma to me and the other grandchildren, somehow managing to be an incredibly interesting person, despite his work making sense to nobody at all except for those he works with. One always finds out new things when having a discussion with him, as he seems to have a vast array of knowledge on a colossal number of topics.

Upon reflection, the initial description of my grandfather seems to be very apt: he is indeed a "very smart man who likes maths". However, Papa always tempers a brilliant mind and a sharp wit with a caring and loving heart, who never fails to make time for his family. I have always seen my grandfather as one who has never stopped writing, reading, loving, or pondering, and I hope he never will.

Gus Sloan (Grandson)

It was not until early 2016 that I realised how revered Ian is among his colleagues, when one of them, a publican at my part-time workplace, read the surname on my security licence. "Sloan is a very famous name at UNSW", he said to me—"but I am sure you've no interest". Don't be so quick to judge sir, as I explained I was his grandson. From then on he frequently reminded me of how intelligent Ian was, with an ability to think critically and inform others clearly.

That reminds me of when I was 13 and struggled with maths at school, and Ian would spend hours teaching me different tricks to do calculations faster, despite it being far too simple he would still give me his time. I owe much of my success in school to these early lessons, without such I probably wouldn't have succeeded in the Higher School Certificate. I still use these skills today.

He is always keen to hear of what is happening in our lives despite having a more interesting life than all of us combined. In the past year I have made a conscious effort to educate myself on difficult mathematical concepts so that I would be able to ask him questions at family meetings, such as the Banach-Tarski paradox to which I was able to provide a measly amount of information and Ian was able to clarify and make it interesting for me.

Because my grandparents are constantly travelling around the world, we only see each other a few times a year at family gatherings. But Ian never changes, always with a smile on his dial and a glass of red in his hand and those hauntingly beady eyes. He always wants to chat about his latest adventure or some political issue, except when he falls asleep in the middle of a party. His days of watching me play rugby have now ended since my playing days are behind me, but his interest in watching the Australian rugby Wallabies is still there in spades.

I look forward to the future as we both grow older and wiser, perhaps there will come a day where I am the one answering questions from him!

Mack Sloan (Grandson)

Not even the human calculator himself could have predicted how his life would turn out all these years later. Although, from a young age, he had an undoubted mathematical ability and passion, it would have been impossible for him to think that he would be travelling the world, teaching and researching what he loves, with whom he loves, only to arrive home again to be with the family he loves.

It is interesting to ponder how the small, "epsilon" actions in the past can have big implications for the present. One of these moments was when Jan and Ian started dating. Jan wanted to party on her birthday, but Ian had a very important exam the next day: he needed to be fresh, alert, and prepared. And so, he was faced with the choice: does he rest so he can perform well on the looming exam, or does he instead have a night out with Jan? He bravely declined the party invitation, survived the relationship crisis, and unsurprisingly performed the best in the class for the exam. But what would have happened if he had gone to the party? Would he still be a mathematician? Would he have met someone else other than Jan at the party? Would he have had the privilege to travel the world? Would he come to live a happy life with a beautiful wife, two children, and six grandchildren?

Much like Ian, the butterfly effect is complex, yet very interesting. Not until I stepped back and looked into Ian's accomplishments did I begin to understand how much Ian has impacted my life, and our families. The decisions he chose to make, and the ones he chose not to make, have turned out to be good ones and to have had a positive impact on each of our lives.

I don't know where Ian is travelling half the time, or what area of maths he is in, but it seems to me that he is living a life that many could only dream of. Through his commitment and dedication to both academia and Jan, he is truly a gentleman. As I grow older and wiser, and I reflect on my work and study habits, I realise that my ability to focus and my modus operandi are directly due to Ian. While I may never have his brain power, I can see more clearly every day the enormous impact Ian, my

Papa, has had on my life in terms of problem solving and deep thinking. We are all very proud to be his grandchildren and to enjoy his tales and good humour.

Corrie Sloan (Grandson, Aged 12)

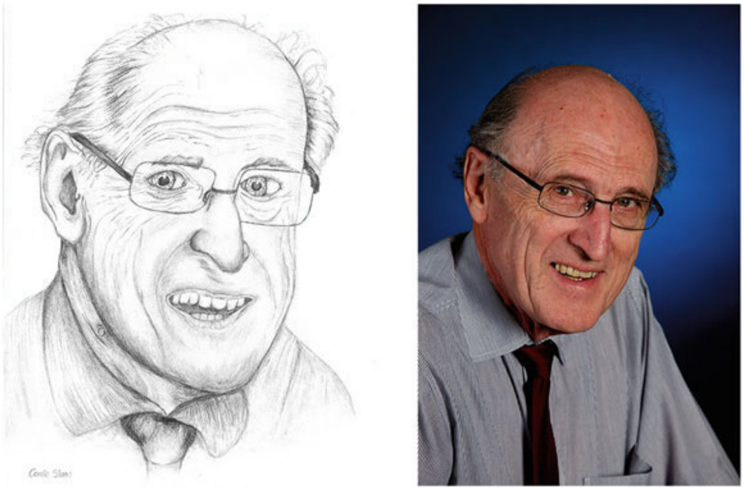
When I was young, I wondered what job my grandfather, Ian, did. I always thought it was strange that my Dad graduated before him, but I was continually reminded that Ian works at the university, so doesn't need to leave. I remember that everyone told me that my grandfather (Papa) was so smart and that he was a scientist, so I pulled out my iPad and googled Ian Sloan. I saw hundreds of pictures of him, but the strangest one of all was of him in his well-cut garden drinking his famous red wine. I still have no idea how he got that picture onto the Internet. Whenever I go to Papa's for a sleepover, the first thing I see is him on his hands and knees working in the garden, picking up some sticks or digging up a plant. But whenever I see him, he's always got a jolly smile and rosy cheeks ready to greet me. Papa makes the best home-made creamy and delicious porridge I have ever tasted. Whenever I am sleeping over, his porridge is the one thing to get me up in the morning.

Whenever Papa has the chance to come and watch me play sport or come to grandparents' day at school, he is there and never in a hurry. I remember the time when Papa and Janny came to one of my representative rugby games, and I scored two tries in the corner where they were standing. I looked up at Papa and Janny and they both clapped and smiled right at me.

When my sister, Kiara, and I go over to Papa's place, he always takes us on an adventure into the bushland. My Dad tells me that when he was little he used to always go into the bush where we go with Papa. He said one time he went to his favourite rock to rest on, slept for about 10 min, and then woke up but realised he was looking right into the eyes of a brown snake. My Dad stayed still for a few minutes and the snake went away. I always ask Papa if he's seen a brown snake but he says no. When we are in the bush, we always see lots of animals rustling in the bushes and birds chirping in the trees. I always enjoy walking through the bush with my grandfather, Papa.

When we have family gatherings at Papa's house, it is always a day to remember. With Papa's great preaching and Janny's great cooking, it always turns out fun. All the grandchildren go outside and play touch footy or kick tennis balls, while the adults stay inside and Papa talks about some political problem. I don't think any of us has seen Papa without a glass of red wine in his grasp or a smile on his face. He always makes us laugh.

Papa is a loving grandfather, a wordsmith, a preacher, a funny clown, and a math God. I look forward to the time when I learn all of my grandfather's tricks and math skills so I too can take part in the mathematical and political discussions that happen inside Papa's house.



Corrie's sketch of Papa and a photo taken in 2011

Kiara (Button) Sloan (Granddaughter, Aged 9)

I'm always asking Papa math questions, and he always answers them correctly, I think! I'm not sure because he says things in completely different ways—since he's a University Professor, he says them in a university way instead of a Year 4 context. So I ask, what does that mean? what does that mean? Over and over.

Since Papa is such a genius, he always needs to go to meetings about math stuff, and those meetings are ALWAYS in another state or country. But it makes it all the merrier when Papa and Janny come back and tell us all about their trip.

Papa always takes us on awesome bush walks like the last time Corrie and I were at Papa's house, Papa took us on a bush walk around a lake. He said if we kept walking we could get to Newcastle, hundreds of kilometres away. I'm so pleased we didn't end up walking to Newcastle!

Papa comes to nearly all my dance and singing concerts and performances. The only time he doesn't come is when he is overseas on a work trip. But I love it when Papa and Janny get back and can come to my performances and see how much I have improved.

Papa is the best grandfather any girl could ask for. He is kind and funny, always has a huge smile, and is jolly good at making porridge. I think if he opened up a porridge shop he would be famous. He's the best Papa anyone could be, and I wouldn't want to change one single thing about him.

Family

We want to thank sincerely all of those who have contributed to this memorable celebration of Ian's 80th. We especially thank Henryk, Frances, and Josef and all of those who have contributed to this volume.

Jan Sloan and Family

A Fortunate Scientific Life

In the course of a long career, my scientific directions have seen many changes. I am often asked: why so many changes? To me there is often no great change: there is always a connecting thread. But what is also true is that I have grasped every opportunity to learn about something new and interesting. And over the years, I have been helped to move in new directions by many wonderful friends and collaborators. In large part this essay is a homage to those friends and collaborators.

But in truth my early research years were much more solitary. My PhD research at the University College London was concerned with the theory and computation of the scattering of electrons by atoms. There were experts about in the Department, but there was little tradition of collaboration or communication. Indeed I recall being less than amused to find out, well into the PhD research, that another student had been given a project overlapping very significantly with my own. That gave extra incentive, if any were needed, to finish my PhD quickly.

My first significant publication, from those PhD years, was [157], a paper appearing in the Proceedings of the Royal Society (something for a young researcher to be proud of) concerned with an improved method for computing scattering cross sections for electrons hitting upon simple atoms.

One very fortunate aspect of my early career was its timing, in that the first general purpose electronic computer available to university researchers in the UK (a big beast of a machine at the University of Manchester) had just become available. (This was fortunate because until that time a PhD would typically include 6 months of laborious hand calculations to solve numerically one simple integro-differential equation.) In that new era, students were able to write Fortran programmes on paper tape, to be transferred to Manchester overnight. We learned to be experts at patching paper tape.

After returning to Australia, and an unhappy year in an industrial research laboratory, I joined the Applied Mathematics Department at the University of New South Wales. At that time it was in everything but name a theoretical physics department, under the leadership of John M. Blatt, who after a very distinguished career in nuclear physics was beginning to dabble in other areas, including control theory and economics. Probably it was under his influence that I started to work on scattering problems in nuclear physics, especially quantum mechanical problems involving a small number of protons or neutrons. At first the going was tough, because when I began I knew nobody working in the field, either in Australia or anywhere else in the world.

Nevertheless, I managed to make some progress and attract some attention in low-energy scattering problems in nuclear physics. An important milestone for me came through an invitation in the early 1970s to spend a sabbatical year at the University of Maryland. I remain grateful to my host, Gerry Stephenson (later at Los Alamos National Laboratory). In the nuclear theory group there, I learned for the first time about the stimulating effect of a strong group environment, a lesson I have since taken to heart. My friend E.F. (Joe) Redish was an enthusiastic member of that group.

For me that year at Maryland was highly productive. In those years the Faddeev equations (devised by the great Russian physicist L.D. Faddeev) were attracting great interest. They are a beautiful set of integral equations which allow the nuclear three-body problem (e.g., the problem of two neutrons and one proton, or equivalently of a deuteron, i.e., heavy hydrogen, nucleus and an incident neutron) to be solved on a computer essentially exactly. One significant fruit of my time at Maryland was the paper [170], in which I developed analogous equations for four particles, rather than the three particles of the Faddeev equations. While an increase from three to four might not seem much, the problem does become harder, not only mathematically but also computationally—it is still hard to obtain computationally exact solutions for more than three particles.

Later the four-particle equations were generalised to any number of particles, independently by both D. Bencze and Joe Redish, to make what are still recognised as the Bencze-Redish-Sloan equations. But by then my own interests had moved elsewhere.

In those physics years, the principal mathematical tools for those of us working on scattering problems were integral equations. There were many good idea floating around, but I often felt that those good ideas were accompanied by a cavalier attitude to the question of proof. I liked the fact that in physics one can take space to explain the ideas behind an approximation scheme, but sometimes this was at the cost of a simple proof. Still, I enjoyed my time in this area.

For me a turning point came in the mid-1970s when, for reasons I no longer remember, I decided to develop and write up some of the ideas for solving integral equations for publication in a numerical analysis journal. It was something of a shock to me (knowing as I did the more relaxed publication standards in physics) to find my paper in trouble with a referee. (The referee later turned out to be L.M. (Mike) Delves, who before my time had been a staff member at the University of New South Wales, and indeed whose old golf clubs were left behind in my first office at UNSW.) The referee thought there were some good ideas, but noted (very appropriately, as I now think) that there were no proofs and, more importantly from my point of view, wanted the paper rewritten in a way that would (in my view) have buried the intuition behind the method. That rewriting was something I was not willing to do, so in the end the paper appeared as [265] in the *Journal of Computational Physics*.

That experience made me determined to prove the merit of my ideas, but to do that I had first to master the theory of the numerical solution of integral equations, which I did through the books of my later friends Philip Anselone and Ken Atkinson.

In the end I was able to prove the merit of what is now often called the "Sloan iteration" for integral equations of the second kind. In brief, the essence of a second-kind integral equation is that the unknown solution appears on both the left and right sides of the defining equation. (On the left the solution is on its own; on the right it appears under an integral sign.) It is therefore natural, if one already has some approximation to the solution, to substitute that approximation into the right-hand side of the equation, and so generate another approximation, hopefully a better one. I was eventually able to prove, under more or less natural conditions, that if one starts with a so-called Galerkin approximation (which I will not explain!), then the iterated Galerkin approximation generated in that way always converges faster than the original Galerkin approximation as the dimension of the approximating space increases. A dramatic description is that the new iterated approximation is "superconvergent".

The original superconvergence work appeared principally in the references [174, 176]. To me there was special pleasure in the fact that the second of those papers appeared in the journal *Mathematics of Computation*, the very journal that had (deservedly!) given my first venture such a hard time.

By the time that particular interest had been worked through, I discovered that I had somehow drifted out of physics, essentially because I was too busy elsewhere. But there were challenges for me in moving into numerical analysis, because I was no longer in the first flush of youth, yet at the time I made my unsuccessful submission as above I knew not a single person in the world in the field of numerical analysis. Admittedly there were Australian experts, some even in the area of integral equations (I am thinking of David Elliott, Bob Anderssen, Frank de Hoog, and Mike Osborne, all of them later good friends), but at that time I am sorry to say that I had never heard of any of them. This was in essence because of the remarkable cultural separation that exists between physics and mathematics: in the main they publish in different journals, attend different conferences, and almost never meet each other. For me, I recall that until 1975 I had never attended a conference with mathematics in the title and since my undergraduate days had never been in a mathematics department other than my own.

But all of that was to change quickly. In (I think it was) 1976 I was an invited speaker at the Australian Applied Mathematics Conference. Within a year or two of that, I was Editor of the Australian journal of the applied mathematicians (now the *ANZIAM Journal*) and in subsequent years have been heavily involved, to my pleasure, in all aspects of Australian and indeed international mathematics.

In particular, I find it pleasing to be able to report that this not-so-young new boy was accepted remarkably quickly into the professional community of numerical analysts. Many people helped in this—Philip Anselone and Kendall Atkinson certainly, also the great Ben Noble who happened to be at my lecture on "New

Methods for Integral Equations" at the Australian Applied Mathematics Conference, Zuhair Nashed who invited me to a meeting of the American Mathematical Society, Günther Hämmerlin, who invited me to more than one Oberwolfach conference, and many others, to all of whom I am forever grateful.

In the early 1980s, I first met Vidar Thomée, a renowned authority on the numerical solution of parabolic problems (e.g., the time-dependent problem of heat diffusion in some region). In the course of time, we published together some six papers, with him patiently teaching me the modern theory of partial differential equations and me contributing, I hope, some insight into integral equations, numerical integration, and superconvergence. Our first joint paper [241] was concerned with superconvergence for integral equations, but with the analysis informed by the analysis of finite element methods for partial differential equations. In addition to collaborating on papers, Thomée also introduced me to the whole new world of finite element methods, a world that included subsequent friends Lars Walhlbin and Al Schatz at Cornell, who took me further into the world of PDE and superconvergence [152].

In the early 1980s, I became interested in high-dimensional numerical integration, something that has become a major theme for me in recent years. It came about this way. While visiting old physics friends at Flinders University in Adelaide, the late Ian McCarthy introduced me to the amazing number-theoretic methods of Korobov and others, which they were trying out experimentally in their atomic physics codes. I had never heard of these things and was immediately captivated by them. But early on I had the idea that they could be extended from the classical constructions (nowadays called lattice rules of rank one) to more general constructions, so I applied, successfully, for an Australian Research Council grant to work on such a generalisation, which led eventually to the publication [224]. The classical constructions of point sets were to my eyes like crystal lattices, in that they are unchanged under special overall shifts (or translations), if you think of the point set as extended indefinitely. That paper, with my postdoc Philip Kachoyan, for the first time allowed the points to be any set which is invariant under translations. In developing a theory for such general lattice rules, I could take advantage of earlier experience in teaching the theory of group representations in quantum mechanics, and was also helped by looking back at my old books on solid-state physics, where the "dual lattice" plays an important role (e.g., in the scattering of X-rays from crystals). Incidentally, that is another paper which experienced great troubles in the refereeing process, but no doubt in the end the paper was all the better for it.

In 1984 I had an opportunity to present ideas on lattice methods for numerical integration at an international congress in Leuven. There I met James Lyness, who suggested that we could work together on this topic for 10 years and expect to publish one paper per year. James (regrettably now deceased) was a colourful personality, and a renowned expert on numerical integration, with whom as it happens I had shared an office at UNSW in my first year or two. While we never reached our 10 papers, we did some interesting work together, and in particular in [225] managed to classify all lattice rules according to "rank", a new concept at the time.

In 1987 I spent an important sabbatical at the University of Stuttgart with Wolfgang Wendland. Under his influence I became more interested in what are called boundary integral equations. (In brief, these might be described as ways of solving certain differential equation for a bounded region by an integral equation that lives only on the boundary.) For a number of years, this was a major preoccupation. One aspect was the development of "qualocation" methods (a quadrature-based extension of collocation methods), notably in a paper with Wendland, [248].

I have always been interested in the methods of approximation, which is the foundation subject for all work on the approximate solution of differential equations, integral equations, and numerical integration. In the early 1990s, having often used polynomial approximations for integral equations and numerical integration, I became interested in questions of polynomial approximation on spheres and other manifolds. I was intrigued by this conundrum: that whereas approximation of a periodic function on an interval (or what is equivalent, for a function on a circle), the approximation known as interpolation (which just means fitting a (trigonometric) polynomial through the function values at equally spaced points), has properties as good as those of the more famous orthogonal projection, yet for spheres of dimension more than one this is not the case: indeed interpolation on spheres remains to this day very problematic; see [286]. Yet a discrete approximation with the right properties (but admittedly using more points than interpolation) is available. This approximation, now called "hyperinterpolation", appeared in the paper [203]. I must say I was rather proud of this paper (I remember presenting it in an animated way to the significant mathematician Werner Reinboldt on the only occasion I met him. He urged me to find a name different from interpolation: hence the name "hyperinterpolation"). That paper had more trouble getting past the referees than almost any other paper I have written. Nevertheless it continues to attract some interest.

I began working with Robert Womersley in the late 1990s, often on polynomial approximation and point distribution problems on spheres. This has been an extremely fruitful partnership, in which we each bring different attributes: from me analysis and from him very high level skills in optimization and high performance computing. One early fruit was the paper [253], in which we proved that the hyperinterpolation approximation can be optimal in a space of considerable practical importance (that of continuous functions).

My interest in approximation has been invigorated from time to time by stumbling across fascinating ideas well known to others but not to me, such as "radial basis functions" and more recently "needlets". On the first of these, I had the opportunity to work with Holger Wendland, an authority on RBFs and the inventor of a special class of localised RBFs that carry his name. With him we were able to develop a successful theory for Wendland RBFs of progressively smaller scale: the idea was to use thinner and thinner RBFs (but correspondingly more and more of them) to get successive corrections to an initial approximation. Of the several papers we wrote on this topic, I especially like our recent "Zooming-in" paper [130]. (It is not as recent as it seems, having been lost for 4 years in the refereeing process.) In this project, as in many other projects over the years, my former student Q. Thong Le Gia was a valued participant. I said former “student”, but actually Thong only did an undergraduate (honours) degree at UNSW before heading to the USA; nevertheless he did his undergraduate work well enough to result in a joint paper [122], in which hyperinterpolation was extended to spheres in an arbitrary number of dimensions. Needlets, invented by Joe Ward and Fran Narcowich (now friends), are something similar, but are localised (and “spiky”) polynomials. A recent paper [284] developed a fully constructive theory of needlet approximation, one that needs only function values at discrete points on the sphere. Other participants in that project were Robert Womersley and my recent student and continuing collaborator Yu Guang Wang.

Another recent influence in the broad area of approximation theory has been Edward Saff, an expert on potential theory and on energies and point distributions on the sphere. By combining different areas of expertise, we were able (together with Robert Womersley and Johann Brauchart) to come up with a new concept (of so-called QMC designs) of point distributions on the sphere; see [29]. (The essence is that instead of characterising point distributions by geometrical properties, or e.g., by minimal energy, now sequences of point sets are characterised by asymptotic convergence properties.) Time will tell how useful this concept is.

I remained interested in high-dimensional integration problems and, around 1980, was invited by Oxford University Press to write book on the subject. At the time I had an excellent colleague (and former student) Stephen Joe. We delayed writing a book on lattice methods until we thought we had the subject wrapped up (though it turned out we were quite wrong—the topic was far from finished). The book eventually appeared as [1] and remains a useful reference to the classical theory.

In 1994 I had the good fortune of meeting Henryk Woźniakowski, a world leader in the field of information-based complexity. We soon started to work seriously on problems of high-dimensional integration. At that time I was familiar with the work in the 1950s and 1960s of the number theorists, on lattice and other methods for integration in many variables. They typically worked with an arbitrary number of variables, but (like the numerical analysts of the time) paid little or no attention to the way the accuracy reduces, or the cost increases, as the number of variables increases: the sole interest was instead in what happens as the number of function-evaluation points increases. Henryk, in contrast, always asked: what happens as the number of variables increases? It turned out that this was a very good question. In our first paper [259], we were able to show something surprising, that in one of the most popular theoretical settings of the number theorists, while there was no flaw in their predictions of the rate of convergence for a large enough number of function values, in the worst case there would be no improvement until the number of points was impossibly large. (Technically, the required number of points was approximately 2 raised to the power of the number of dimensions. Try it out for 100 dimensions!)

In the second paper with Woźniakowski, we took seriously an idea that for problems with large numbers of variables, those variables might not be equally important. We thought that if the variables are ordered in order of importance, we should be able to quantify that decreasing importance by assigning to each variable a parameter (a "weight"), with the weights becoming progressively smaller. We were able to carry through that programme, to the point of being able, for an important setting, to characterise completely the condition on the weights needed to get a result independent of dimension, that is, of the number of variables. (Technically, we were able to find a necessary and sufficient condition for the worst case error to be independent of the dimension.) That work, appearing in [260], gave us much pleasure and has been the foundation for a large amount of subsequent work by ourselves and others.

I have been privileged for the past two decades to have two outstanding young colleagues in high-dimensional computation, Frances Kuo (a student of Stephen Joe, and hence my doctoral granddaughter) and Josef Dick (who earlier came from Salzburg to be my PhD student). Together we wrote a major review [52] of certain methods for high-dimensional integration. I have had many good students, but Josef was exceptional, in that shortly before he was due to submit his thesis he abandoned the work already done and wrote a new thesis on novel joint work with Friedrich Pillichshammer. He became the teacher and I the student.

Our interest in high-dimensional problems led us a few years ago into a joint project with an Australian merchant bank. (Many problems in mathematical finance are high-dimensional because they involve many, even infinitely many, random variables.) I think the truth is that we made no contribution to the bank's bottom line, but the experience had a lasting influence on our subsequent research, because none of our high-dimensional theories can explain the apparent success of some of our methods for so-called option pricing. (The problem with options is that their value is considered to be zero if the final price drops below an agreed "strike price". For that reason the functions that need to be integrated have a kink, which places them outside almost all existing theory.) With Frances Kuo and Michael Griebel, we made some progress in finding theoretical answers to this conundrum in [69], and at the moment we are proposing a practical cure in joint work with Hernan Leövey and Andreas Griewank.

Many years earlier, my first PhD student after my physics days was Ivan Graham, who came from Northern Ireland. In 2007, as a Professor at the University of Bath, he directed my attention to the field of partial differential equations (PDEs) with random coefficients, as a burgeoning source of very challenging high-dimensional problems. Christoph Schwab from ETH Zurich was another who directed my interest in that direction. Such PDE application has become a major interest for all of us in subsequent years. I especially like the experimental paper [65] and the theoretical paper [116], both with Frances Kuo and with Ivan Graham and colleagues Dirk Nuyens and Robert Scheichl in the first paper and Christoph Schwab in the second. This work continues.

Is there a consistent theme? Perhaps there is, to the extent that many problems I have worked on are governed by the question of what can be done, and proved, when the underlying problems of physics and mathematics are intrinsically infinite-dimensional, yet our computations can use only a finite amount of information (e.g., of function values at points) and a limited amount of computer resources. But the truth is that I have always found that one interesting problem leads to another, and I have just done whatever I have found interesting and the things my colleagues have helped me to do.

In the limited space of this essay, I cannot do justice to my more than 100 lifetime collaborators and my many students and postdoctoral fellows. Among the influential collaborators not mentioned already are Mark Ainsworth, Xiaojun Chen, Ronald Cools, Mahadevan Ganesh, Michael Giles, Rolf Grigorieff, Rainer Kress, Kerstin Hesse, Fred Hickernell, Hrushikesh Mhaskar, Harald Niederreiter, Philip Rabinowitz, Alastair Spence, Sergei Pereverzyev, Siegfried Prössdorf, Ernst Stephan, Xiaoqun Wang, and Grzegorz Wasilkowski. I am grateful to all of them, and more, for the lessons they have taught me and for the wonderful ideas to which they have introduced me.

Ian H. Sloan

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