Philosophy's Loss of Logic to Mathematics
An Inadequately Understood Take-Over
Studies in Applied Philosophy, Epistemology and Rational Ethics

Volume 43

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Philosophy’s Loss of Logic to Mathematics
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Essays originally published in Korean have been translated into English by me. I have made small changes, mainly stylistic, to most of the articles published internationally. In updating the book’s chapter, I have striven for an overall consistency and coherence, I hope with satisfactory success. Supplementary readings are proposed in the Epilogue.


Daejeon, Korea (Republic of) 

Woosuk Park
Acknowledgements

While I am indebted to all my teachers in logic and philosophy, I am especially grateful to Jorge J. E. Garcia and John Corcoran for their unfailing support. Nino Cocchiarella’s detailed and much appreciated comments led to invaluable improvements of several of the book’s chapters. As always, I have benefited from the criticism and encouragement of John Woods and Lorenzo Magnani.
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Chapter 1
Introduction

This is a collection of my papers on the history and philosophy of logic and mathematics published for the last thirty years. Virtually all the chapters tackle some particular logical, methodological, epistemological, and ontological issues that are not entirely clear in official history of modern logic. In retrospect, there were some very good reasons for me to be fascinated by the particular issues and the philosophers at the earlier stages of my research. Topically speaking, these chapters can be grouped under four parts. Part 1, which deals with Gottlob Frege, was motivated to understand what aspects of his logic were truly innovative in its revolution against the Aristotelian logic. Part 2 treats Hilbert and his associates and followers in the hope to understand the revolutionary change in the axiomatic method. Against that background, Part 3 discusses how to understand Tarski and Gödel as the towering figures whose problems are still with us. Finally, part 4 invokes some of the most influential positions in contemporary philosophy of mathematics. Both Maddy and Shapiro can be understood in terms of their reactions to Benacerraf’s challenge to platonism in mathematics. Even though this part ends with a chapter on a renaissance philosopher, the main question is raised from our current situation in science and mathematics. Largely due to the foundational approaches in the first half of the twentieth century philosophy of mathematics, we do not fully understand the problems of application of mathematics. By introducing Biancani and Aristotelian philosophy of mathematics to the forefront, I want to hint at the urgent need to reconsider the Aristotelian position in logic and mathematics, which disappeared almost completely from the scene without good reasons in the early twentieth century.

There are already many textbooks in philosophy of mathematics and philosophy of logic, not to mention logic. The uniqueness of my book lies in its attempt to understand the philosophical problems raised in logic and mathematics in historical context. Rather than plunging into the unending disputes between different parties, I try to focus on the main issues that motivated the giants who established modern logic and philosophy of mathematics. By avoiding unnecessary technicalities of
logic and mathematics, it will be accessible to any reader who wants to understand modern philosophy of logic and mathematics.

Part 1, which deals with Gottlob Frege, was motivated to understand what aspects of his logic were truly innovative in its revolution against the Aristotelian logic. There are several surprising facts around this theme. First of all, Frege was a professional mathematician throughout his career. How was it possible for a mathematician to become the father of modern logic, analytic philosophy, and cognitive science? Secondly, we have reason to be surprised to witness the all too late revolution in logic. How was it possible for one man to revolutionize a field of scientific research dominated by the Aristotelian paradigm for more than two millennia? Finally, if Frege’s reform is truly revolutionary, why has he not been well known to the intellectuals, if not to the general public? In other words, why the influence of modern logic has not been felt widely and unmistakably?

Frege frequently complains that others are ignorant of the distinction between “falling under” and “subordination”. This criticism is not only directed against the philosophers who are under the influence of Aristotelian logic but also against the mathematicians of his time. In Chap. 2, I shall show that this distinction must be the vantage point for understanding Frege in both historical and philosophical contexts. Strangely, this distinction is not studied extensively nowadays. There are some good reasons for this. First, ironically, it is so well established as to become a triviality. Secondly, some people think that Frege’s criticism of the aggregate view of sets is outdated. Consequently, we cannot understand why this distinction was so important to Frege. In what problem situation did Frege formulate this distinction? Were there any rival theories of predication? Was this distinction an ad hoc device for Frege in order to establish other important theses? What would happen if we lack this distinction? This chapter aims at a partial answer to these questions.

In Chap. 3, I shall compare the ontologies of Scotus, Frege, and Bergmann, thereby indicating the inseparable connection between logic and ontology. If N. Cocchiarella’s recent discussion of Frege’s function-correlate is correct, we have reason to assimilate Frege’s ontology to the Avicennian-Scotistic tripartite ontology of individuals, universals, and common natures in themselves. Further, to the extent that Scotus’ ontology is similar to Frege’s ontology, we may have indirect evidence concerning how Bergmann would think about such an interpretation of Scotus’ haecceitas ontology. I want to show that current treatments of individuation can be seriously challenged by the possible return of the common nature. My strategy will be as follows. In section A, I shall discuss Cocchiarella’s thesis regarding Frege’s function-correlates. In section B, by using Cocchiarella’s thesis, I shall try to compare Frege’s ontology with the Avicennian-Scotistic tripartite ontology. Some of the similarities and differences between these two ontologies will become clearer in the process. In section C, I shall examine Bergmann’s interpretation of Frege’s ontology. After having drawn attention to how Bergmann criticizes Frege’s introduction of concept-correlates and value-ranges, we may understand, in section D, how Bergmann would view Frege’s and Scotus’ tripartite ontologies. Hopefully the peculiarity of Bergmann’s theory of
universals and his bare particular theory of individuation will stand out clearly against the background of Frege’s and Scotus’ ontologies.

Chapter 4 continues to get further insights from Cocchiarella’s history and philosophy of logic in understanding the contrast of Aristotelian and Fregean logic. Recently Cocchiarella proposed a conceptual theory of the referential and predicatable concepts used in basic speech and mental acts (Cocchiarella 1998). This theory is interesting in itself in that singular and general, complex and simple, and pronominal and nonpronominal, referential concepts are claimed to be given a uniform account. Further, as a fundamental goal of this theory is to generate logical forms that represent the cognitive structure of our speech and mental acts, as well as logical forms that represent only the truth conditions of those acts, it is an indispensible part of Cocchiarella’s conceptual realism as a formal ontology for general framework of knowledge representation. In view of the recent surge of interest in his formal ontology by cognitive scientists and AI people, at least, Cocchiarella’s theory of reference deserves careful examination. Above all, however, the utmost value of Cocchiarella’s theory of reference must be found in its challenge against what he calls “the paradigm of reducing general reference to singular reference of logically proper names” that pervades the 20th century (Cocchiarella 1998, 170).

The aim of the present chapter is to provide an impressionistic sketch of Cocchiarella’s challenge.

Part 2 treats Hilbert and his associates and followers in the hope to understand the revolutionary change in the axiomatic method. Even though it is almost common-sensical to cite the big three, i.e., logicism, formalism, and intuitionism, in the foundations of mathematics, through Russell and Wittgenstein, philosophers have paid much more attention to logicism compared to other schools in the foundations of mathematics. This is unfortunate, for one might give up the fine opportunity to secure a broader perspective on history of mathematics and the real practice of working mathematicians. There must be lots of lessons we can get from the detailed history of the interaction between mathematics and philosophy.

In this vein, Chap. 5 intends to examine the widespread assumption, which has been uncritically accepted, that Zermelo simply adopted Hilbert’s axiomatic method in his axiomatization of set theory. What is essential in that shared axiomatic method? And, exactly when was it established? By philosophical reflection on these questions, we are to uncover how Zermelo’s thought and Hilbert’s thought on the axiomatic method were developed interacting each other. As a consequence, we will note the possibility that Zermelo, in his early as well as late thought, had views about the axiomatic method entirely different from that of Hilbert. Such a result must have far-reaching implications to the history of set theory and the axiomatic method, thereby to the philosophy of mathematics in general.

Encouraged by the recent surge of interest in logical positivism, I shall discuss in Chap. 6 the problem of the relationship between Hilbert and the logical positivism. To what extent and in what respects were logical positivists indebted to Hilbert? In particular, what exactly did they learn from the notion of implicit definition as the core of Hilbert’s axiomatic method? In order to answer these questions, I shall first try to fathom Michael Friedman’s mind about them. Secondly, I shall attempt to
determine whether Carnap’s approach to theoretical terms in science is nothing but an application of Hilbert’s axiomatic method, as Bernays claims.

Since Bernays has not drawn scholarly attention that he deserves, in Chap. 7, it would be worth a while to continue to probe the questions raised in Chaps. 5 and 6. Only quite recently, the reevaluation of his philosophy, including the projects of editing, translating, and reissuing his writings, has just started. As a part of this renaissance of Bernays studies, this chapter tries to distinguish carefully between Hilbert’s and Bernays’ views regarding the axiomatic method. We shall highlight the fact that Hilbert was so proud of his own axiomatic method on textual evidence. Bernays’ estimation of the place of Hilbert’s achievements in the history of the axiomatic method will be scrutinized. Encouraged by the fact that there are big differences between the early middle Bernays and the later Bernays in this matter, we shall contrast them vividly. The most salient difference between Hilbert and Bernays will shown to be found in the problem of the uniformity of the axiomatic method. In the same vein, we will discuss the later Bernays’ criticism of Carnap, for Carnap’s project of philosophy of science in the late 1950s seems to be a continuation and an extension of Hilbert’s faith in the uniformity of the axiomatic method.

Part 3 discusses how to understand Tarski and Gödel as the towering figures whose problems are still with us. They are not only the great logicians of all time but also the wonderful source of information to the history of logic in the twentieth century. I will hint at what kind of unconventional approaches might shed light on the history of logic in this regard.

Chapter 8 is basically an extensive review of Patterson’s monumental book on Tarski’s Philosophy of logic and language (Patterson 2012). We still do not know against what historical/philosophical background and motivation Tarski’s definition of logical consequence was introduced, even if it has had such a strong influence. In view of the centrality of the notion of logical consequence in logic and philosophy of logic, it is rather shocking. There must be various intertwined reasons to blame for this uncomfortable situation. There has been remarkable progress achieved recently on the history of analytic philosophy and modern logic. In view of the recent developments of the controversies involved, however, we will have to wait years to resolve all these uneasiness. In this gloomy situation, Douglas Patterson’s recent study of Tarski’s philosophy of language and logic seems to have the potential to turn out to be a ground breaking achievement. This chapter aims at uncovering the state-of-the-art and fathoming the future directions of the research in this problem area by examining critically some unclear components of Patterson’s study.

In recent years there has been a surge of interest in Gödel’s ontological proof of the existence of God. In spite of all this extensive concern, it is not certain whether there is any improvement in understanding the motivations of Gödel’s ontological proof. Why was Gödel so preoccupied with completing his own ontological proof? To the best of my knowledge, no one has dealt with this basic question seriously enough to answer it. In Chap. 9, I propose to examine Gödel’s ideas against a somewhat larger background in order to understand his motivation for establishing the ontological proof. I shall point out that the value of Gödel’s proof is to be found in the possible role of his proof of the existence of God in his philosophy as a whole.
as well as in its relative merit as an ontological proof. Hopefully, my guiding question as to Gödel’s motivation will turn out to be extremely fruitful by enabling us to fathom his mind regarding God and mathematics. After all, we are interested in Gödel’s ontological proof because it a proof presented by the great mathematician. At the same time, we may fathom how Gödel understood the axiomatic method indirectly from his ontological proof.

Part 4 invokes some of the most influential positions in contemporary philosophy of mathematics. Why am I counting Penelope Maddy and Stewart Shapiro as representing the most influential positions? Well, in view of the fact that it is simply impossible to do justice to all philosophies of mathematics in contemporary philosophy, selection is unavoidable. So, my choice must have been made by what I believe to be the most promising direction for the future philosophy of mathematics, i.e., Aristotelian philosophy of mathematics. Maddy and Shapiro have more than enough aspects to be discussed under the general rubric of Aristotelian mathematics.

Chapter 10 is an attempt to probe the question as to why Maddy gave up mathematical realism and moved to her own version of mathematical naturalism. According to one widespread hypothesis, Maddy’s change of mind was brought up by her criticism of Quine-Putnam indispensability argument. Though quite convincing, it is not good enough to explain why one has to give up mathematical realism. The analogy of science and mathematics will instead be shown to be the better perspective to fathom Maddy’s changing beliefs. For this purpose, we have to understand to what extent Maddy’s thought in her realist years, which was strongly influenced by Quine and Gödel, was governed by the analogy between science and mathematics. Also, we have to understand why Maddy gave up the analogy, and thereby gave up mathematical realism. Finally, some criticisms against Maddy’s abandonment of the analogy will be examined so as to hint at the reasons why I believe Maddy’s intellectual journey in mathematical ontology is rather regress than progress.

In some sense, both ontological and epistemological problems related to individuation have been the focal issues in the philosophy of mathematics ever since Frege. However, such an interest becomes manifest in the rise of structuralism as one of the most promising positions in recent philosophy of mathematics. The most recent controversy between Keränen and Shapiro seems to be the culmination of this phenomenon. Rather than taking sides, in Chap. 11, I propose to critically examine some common assumptions shared by both parties. In particular, I shall focus on their assumptions on (1) haecceity as an individual essence, (2) haecceity as a property, (3) the classification of properties, and thereby (4) the search for the principle of individuation in terms of properties. I shall argue that all these assumptions are mistaken and ungrounded from Scotus’ point of view. Further, I will fathom what consequences would follow, if we reject each of these assumptions.

We can witness the recent surge of interest in the controversy over the scientific status of mathematics among Jesuit Aristotelians around 1600. Following the lead of Wallace, Dear, and Mancosu, I propose to look into this controversy in more detail. For this purpose, I shall focus on Biancani’s discussion of scientiae mediae in his dissertation on the nature of mathematics. From Dear’s and Wallace’s
discussions, we can gather a relatively nice overview of the debate between those who championed the scientific status of mathematics and those who denied it. But it is one thing to fathom the general motivation of the disputation, quite another to appreciate the subtleties of dialectical strategies and tactics involved in it. It is exactly at this stage when we have to face some difficulties in understanding the point of Biancani’s views on scientiae mediae. Though silent on the problem of scientiae mediae, Mancosu’s discussions of the Jesuit Aristotelians’ views on potissima demonstrations, mathematical explanations, and the problem of cause are of utmost importance in this regard, both historically and philosophically. In Chap. 12, I will carefully examine and criticize some of Mancosu’s interpretations of Piccolomini’s and Biancani’s views in order to approach more closely what was really at stake in the controversy.

Largely due to the foundational approaches in the first half of the twentieth century philosophy of mathematics, we do not fully understand the problems of application of mathematics. By introducing Biancani and Aristotelian philosophy of mathematics to forefront, I want to hint at the urgent need to reconsider the Aristotelian position in logic and mathematics, which disappeared almost completely from the scene without good reasons in the early twentieth century. In the Epilogue, I will hint at how to do more interesting and exciting researches for such a project.

References

Part I The Fregean Legacy
Chapter 2
Frege’s Distinction Between “Falling Under” and “Subordination”

Abstract Frege frequently complains that others are ignorant of the distinction between “falling under” and “subordination”. This criticism is not only directed against the philosophers who are under the influence of Aristotelian logic but also against the mathematicians of his time. I shall show that this distinction must be the vantage point for understanding Frege in both historical and philosophical contexts. Strangely, this distinction is not studied extensively nowadays. There are some good reasons for this. First, ironically, it is so well established as to become a triviality. Secondly, some people think that Frege’s criticism of the aggregate view of sets is outdated. Consequently, we cannot understand why this distinction was so important to Frege. In what problem situation did Frege formulate this distinction? Were there any rival theories of predication? Was this distinction an ad hoc device for Frege in order to establish other important theses? What would happen if we lack this distinction? This chapter aims at a partial answer to these questions.

Keywords Aristotle · “Falling under” · Frege · Predication · Subordination

1 Introduction

Frege frequently complains that others are ignorant of the distinction between “falling under” and “subordination” (Angelelli 1967, p. 107, 125n). This criticism is not only directed against the philosophers who are under the influence of Aristotelian logic but also against the mathematicians of his time (Jourdain 1912, 204; Angelelli 1967, 107–108; Frege 1893 in Geach and Black 1952, 149). Isn’t
this the hallmark of Frege? As long as we honor Frege as the founder of modern symbolic logic, and as long as we mention the reunion of logic and mathematics in order to prove the superiority of new logic, this distinction must be the vantage point for understanding Frege in both contexts, i.e., in the history of philosophy and the foundations of logic and mathematics.

Strangely, this distinction is not studied extensively nowadays.\(^1\) There are some good reasons for this. First, ironically, it is so well established as to become a triviality. This distinction is rather given to many, and they tend to think that there must be predecessors who anticipated it. Secondly, some people think that Frege’s criticism of the aggregate view of sets is outdated (Resnik 1980, p. 206). According to them, we have a new aggregate view of sets which is immune to Frege’s criticism, i.e., the iterative concept of set (Boolos 1971, 486–487). In other words, Frege’s criticism of 19th century mathematicians’ failure to draw the distinction is no longer interesting.

Consequently, we cannot understand why this distinction was so important to Frege. In what problem situation did Frege formulate this distinction? Were there any rival theories of predication? Was this distinction an ad hoc device for Frege in order to establish other important theses? What would happen if we lack this distinction? This chapter aims at a partial answer to these questions. In Sect. 2, the distinction between “falling under” and “subordination” is briefly introduced. In Sect. 3.1, I shall examine the hypothesis that in Aristotelian logic predication was interpreted as Fregean “subordination” together with some possible objections to it. In Sect. 3.2, I shall discuss how Frege criticized his contemporary mathematicians for their failure to draw the distinction. In Sect. 4, I shall fathom why the distinction was so important to Frege.

2 The Distinction

Perhaps the most elaborate presentation of the distinction can be found in Frege (1892) (Geach and Black 1952, p. 51).\(^2\) Here he began from distinguishing an object’s falling under a first-level concept from a concept’s falling within a

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\(^1\)Ignacio Angelelli and Nino B. Cocchiarella seem to be rare exceptional cases. Angelelli devotes a chapter on this distinction, and scrutinizes the question why it was lacking in traditional philosophy. As a consequence, this chapter is full of interesting information. However, in general, he tends to presuppose the correctness of the distinction rather than contesting its merit critically. He never presents clearly what the distinction itself is. Nor does he examine Frege’s criticism of 19th century mathematicians in detail. For example, he never quotes from Dedekind’s own works (Angelelli 1967). Cocchiarella’s analysis is full of logical and philosophical insights. Since he deals with all important issues related to Frege, however, it may not be a kind introduction to the distinction between “falling under” and “subordination” for beginners or general public.

\(^2\)The distinction is of early origin in the development of Frege’s thought. In several places we can find it out. For example, at the beginning of §53, he drew the distinction between properties of a concept and the mark of the concept. And, immediately he added the following explanation:
second-level concept. From this he concludes that the distinction of concept and object holds with all its sharpness. Then follows the distinction between ‘property’ and ‘mark’. In order to contrast them, he resorted to the distinction between “falling under” and “subordination”. I quote an entire paragraph, for this is our key text:

I call the concepts under which an object falls its properties; thus
‘to be Φ is a property of Γ’
Is just another way of saying;
‘Γ falls under the concept of aΦ’.
If the object Γ has the properties Φ, X, and Ψ, I may combine them into Ω; so that it is the same thing if I say that Γ has the property Ω, or, that Γ has the properties Φ, X, and Ψ.
I then call Φ, X, and Ψ marks of the concept Ω, and at the same time, properties of Γ. It is clear that the relations of Φ to Γ and to Ω are quite different, and that consequently different terms are required. Γ falls under the concept Φ; but Ω, which is itself a concept, cannot fall under the first-level concept Φ; only to a second-level concept could it stand in a similar relation. Ω is, on the other hand, subordinate to Φ. (Frege 1892, p. 51. Emphases are mine)

In the same place, he used the following example. We may say that 2 is a positive whole number less than 10.

Here

to be a positive number (Φ),
to be a whole number (X),
to be less than 10 (Ψ),
appear as properties of the object 2, and also as marks of the concept positive whole number less than 10 (Ω).
This is neither positive, nor a whole number, nor less than 10. It is indeed subordinate to the concept whole number, but does not fall under it.

By using standard notation of modern symbolic logic, we can understand it as follows:

2 falls under being a whole number. X₂
Being a positive whole number less than 10 is subordinate to being a whole number. ∀x (∀Ωx→Xx)

In other words, the distinction between “falling under” and “subordination” is closely related to the distinction between singular propositions and universal

“These latter (the characteristics of which make up the concept) are properties of things which fall under the concept, not of the concept” (Frege 1884, 64e7). Moreover, at the end of the section, he made it clear that we should not confuse the relation of one concept’s falling under (within) another higher concept with the subordination of species to genus (Frege 1884, 65e7).
propositions. What a trivial distinction! Surprisingly, however, this trivial distinction had been lacking until Frege established it. From the history of logic, we hear that traditional logicians (in particular, in the post-Renaissance period) interpreted singular propositions as A-type propositions, i.e., universal affirmative propositions (Copi 1982, p. 239; Prior 1963, 160). Also, we can see how the pioneers of modern symbolic logic were so proud of this innovation.

Russell reported that the enlightenment he derived from Peano had come from two purely technical advances. He did not forget to indicate the fact that both of these advances had been made by Frege. And, these two alleged advances were the two sides of the very distinction between “falling under” and “subordination”:

The first advance consisted in separating propositions of the form ‘Socrates is mortal’ from propositions of the form ‘All Greeks are mortal’…. The second important advance that I learnt from Peano was that a class consisting of one member is not identical with that one member. (Russell 1959, p. 52)

Then, first, we should confirm whether there had been no such distinction in philosophy as well as in mathematics.

3 The Non-existence of the Distinction

3.1 Traditional Logic

3.1.1 Ambiguity of ‘be’

It is a commonsense that there is ambiguity of the verb ‘be’ in Indo-European languages. We usually distinguish ‘is of predication’ and ‘is of identity’. Also, we might indicate the usage of ‘is of existence’. Of course, Frege understood the distinction clearly. He distinguished between ‘‘is’ as a mere copula, as in the proposition ‘The sky is blue’” and ‘‘is’ which has the sense of ‘is identical with’ or ‘is the same as’ (Frege 1884, 69e). Some people even emphasize Frege’s distinction between the meaning of the verb ‘be’ as “one of the most widely accepted inventions of his and an

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3As John Corcoran indicates, the idea that singulars are logically equivalent to universals is not the same as the idea that “falling under” is the same as “subordination”. Following Quine, we can treat Ps as logically equivalent to ∃x(x = s ⊃ Px). Even among the schoolmen we can find a thinker who interpreted “Socrates is running” as “Everything that is Socrates is running” (Prior 1963, p. 160). In this respect, it is by no means clear why traditional logicians (in particular in the post-Renaissance period) treated singular propositions as universal propositions. If it is the result of their theory which considers “falling under” as “subordination” (i.e., as will be examined in the next section), then it is still more dubious why it was treated that way more clearly in post-Renaissance period.

4As Cocchiarella points out, here I am leaving unexplained how traditional logicians took predication to function in E, I, O type propositions (Cocchiarella 2015b).
integral part of most current treatments of semantics” (Hintikka 1979, p. 72).\(^5\) However, such an insight cannot be said a breakthrough. Some people say that even Plato clearly distinguished the various meanings of the verb ‘be’ (Ackrill 1957).\(^6\)

Then, there seems to be no hope for tracing the reason why traditional logicians failed to draw the distinction between ‘falling under’ and ‘subordination’ from the distinction between ‘is of predication’ and ‘is of identity’. Were they simply ignorant of the subdivision of predication into ‘falling under’ and ‘subordination’? There is no doubt that the ambiguity of the verb ‘be’ plays a significant role in making matters complicated in predication theory, old and new. But it is not easy to pin down exactly what is wrong.

### 3.1.2 Aristotle: “Falling Under” as “Subordination”

As is well-known, in his Categories 5.2, Aristotle treated genera and species as secondary substances. According to Kneale and Kneale, this blurs “the all-important distinction between singular and general propositions” (Kneale and Kneale 1962, p. 31). In Aristotelian framework, the proposition “Man is an animal” is the paradigm case of predication. In Fregean language, this amounts to saying “the class of man is subordinate to the class of animals”. Thus far, there is no problem. But, while Frege would interpret “Socrates is a man” and “Socrates is an animal” as exemplifying “falling under”, Aristotle would treat them as cases of subordination: “This individual man is a man”, and “This individual man is an animal”, respectively. In the former, an individual man is subordinate to a lowest species. In the latter, only derivatively can we say “An individual man is an animal”, because the class of man is subordinate to the class of animals:

Whenever one thing is predicated of another as of a subject, all things said of the subject also. For example, man is predicated of the individual man, and animal of man; so animal

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\(^5\)The following quote from Hintikka (1981) seems extremely informative: “The sharpest specific difference between Frege’s logical notation and the ideas of his predecessors lies in his treatment of verbs for being. Such verbs are, according to Frege and his followers, ambiguous in that they have to be translated into the logical notation in at least four different ways: (i) by the identity sign ‘=’ (the ‘is’ of identity); (ii) by the existential quantifier (the ‘is’ of existence); (iii) by predicative juxtaposition (the ‘is’ of predication or the copula), and (iv) by a general implication (the ‘is’ of class inclusion). In 1914 Bertrand Russell (1914) called this fourfold distinction ‘the first serious advance in real logic since the time of the Greeks’, and it has been incorporated in all the usual systems of first-order logic (lower predicate calculus, quantification theory). Hence everybody who has been using the notation of first-order logic for the purpose of semantical representation is committed to the ambiguity of ‘is’. This applies to linguists and philosophers otherwise as unlike each other as George Lakoff, Noam Chomsky, W. V. Quine, Donald Davidson, and Ludwig Wittgenstein. Frege’s distinction has thus become one of the most widely accepted inventions of his and an integral part of most current treatments of semantics (Hintikka 1981, p. 72).

\(^6\)Corcoran informed me of the fact that Russell cites De Morgan (1847, 49–53) instead of Frege (Russell 1903, 64n).
will be predicated of the individual man also- - -for the individual man is both a man and an
animal. (Aristotle, *Categories*, 3, 1b 10, Ackrill 1963)

From these facts we might formulate a working hypothesis that Aristotle treated “falling under” as “subordination”, even though we cannot be sure whether Aristotle was simply ignorant of the distinction between “falling under” and “subordination”, or schemingly assimilated “falling under” to “subordination”. In the following, I shall test this hypothesis.

### 3.1.3 Predication in *Quid* and in *Quale*

Against the hypothesis that Aristotelian logic treats any predication as subordination in Fregean sense, a doubt naturally arises about whether the distinction between predication *in quid* and *in quale* is a counter-example. Can’t we pair “falling under” with *in quale* on the one hand, and “subordination” with *in quid* on the other hand? Let us check whether this works.

We can find a crystal-clear formulation of the distinction between *in quid* and *in quale* in Porphyry (1975, p. 8, Porphyry 1966). Of course, we can go back to Aristotle.\(^7\) Be that as it may, according to these old logicians, predication *in quid* is in answer to the question “What is it?”, while predication *in quale* is in answer to the question “Of what sort is it?”. So, they distinguished (1) “Socrates is a man” from (2) “Socrates is white”, and distinguished (3) “Man is an animal” from (4) “Man is mortal”.

At first sight, the example sentence of predication *in quale* seems to be also a good example of “falling under”: (2) “Socrates is white”. If it were the case that only singular propositions are treated as *in quale*, then our attempt to assimilate *in quale* to “falling under” would be successful. Unfortunately, however, the sentence (3) “Man is an animal” is also an example of *in quale*, and in this example a lower species is subordinate to a higher genus. Now the old logicians find out a lower thing’s subordination to a higher species in singular propositions such as (2) “Socrates is white”:

Differentia is predicated of species and individuals …. Porphyry is predicated of the species of which it is the property and of the individuals under the species. And accident is predicated of species and individuals. (Porphyry 1975, p. 21, Porphyry 1966)

We may further demonstrate the point that, in *in quale* as well as in *in quid*, lower things (or species) is subordinate to a higher species (or genus) and thus that in both cases we can see Fregean “subordination” by paraphrasing the examples given above into old logicians’ awkward language of addition as follows:

(1’) This snubnosed white *man* called Socrates is a *man*.

(2’) This snubnosed *white* man called Socrates is *white*.

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\(^7\) This distinction must originate from Aristotle. As a matter of fact, in *Topics* 102a 32, we can find Greek equivalent of “in eo quod quid est”. Interestingly enough, I am unable to find out Aristotle’s equivalent of “in eo quod quale est”.