

RESEARCH

Christian Schmidt

Interleaving Concepts for Digital-to-Analog Converters

Algorithms, Models,
Simulations and Experiments



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Experiments

With a foreword by Prof. Dr.-Ing. Friedel Gerfers

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Foreword

The internet data traffic increases between 30 and 60 % per year. This demand for high data rates is largely driven by new wireless applications, cloud storage, cloud computing, self-driving cars and smart-city technologies. Thus, the infrastructure for tomorrow's information and communication technology (ICT) requires not only higher bandwidth but also more energy-efficient data communications to accommodate the increasing amount of data traffic.

For this, optical data transmission is the key technology. Important components are the data converters performing the transformation between the digital and the analog domain and vice versa. Digital-to-analog converters (DACs) and analog-to-digital converters (ADCs) enable the utilization of real-time digital signal processing (DSP) for software-defined waveforms with both flexible modulation bandwidth and modulation formats, improved coding techniques as well as pre- and post-equalization possibilities.

As the energy-efficiency is becoming more and more important for optical networks, low-power complementary metal oxide semiconductor (CMOS) DACs are highly desired to generate the transmit signal. The CMOS DACs can be integrated with the CMOS DSP on a single chip. With an increased DAC bandwidth, less optical wavelengths are required to achieve the same capacity over the fiber. However, the analog bandwidth of CMOS DACs is fundamentally limited by technological constraints.

Mr. Schmidt addresses in his book two innovative DAC interleaving concepts to overcome the limitations of CMOS DACs enabling a new generation of high-speed DACs. A key question that this book tries to answer is how to enhance the performance of these state-of-the-art DACs. The DAC interleaving concepts enhancing both the bandwidth and the sampling rate are the analog multiplexer (AMUX)-DAC and the frequency interleaving (FI)-DAC concept. Both techniques are analyzed from a system-level perspective to overcome the bandwidth limitations of a single DAC.

The overall objective of this work is to analyze and examine the limitations and the potential of both the AMUX-DAC and the FI-DAC concept to gain a profound understanding of the approaches. The in-depth scientific analysis shows for the first time the possibilities and limitations of both techniques, which are primarily in the

analog components that are necessary for both methods. Methodically, analytical system models are created for both techniques, which are calibrated and verified by means of measurements. In this way, the effects of various disturbing effects are investigated. Furthermore, discrete-time system models are developed enabling the application of compensation techniques by a suitably adapted DSP.

This way, a reasonable decision can be made on the best approach to increase the network capacity by means of transmitter performance enhancement. Since, analog components are employed for the DAC interleaving, there is a trade-off between an enhanced analog bandwidth and a reduced effective number of bits (ENOB).

Overall, this book has an excellent combination of scientific improvements and practical advices in the area of interleaved DACs. Further, it provides valuable suggestions for additional investigations.

Prof. Dr.-Ing. Friedel Gerfers

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Christian Schmidt

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List of Symbols

Symbols

$a_{m,n}$	nonlinear polynomial coefficient for LO harmonic n
B	(signal) bandwidth
b	number of bits
$\underline{\mathbf{C}}(\boldsymbol{\mu})$	MIMO channel matrix
$\underline{\mathbf{C}}(f)$	MIMO channel matrix
C	Shannon capacity
CL	mixer conversion loss
$C_{ln}(\boldsymbol{\mu})$	MIMO channel frequency response
$C_n(f)$	Fourier transform of $c_n(t)$
$c_n(t)$	clock signal for AMUX input n
C_L	load capacitance
CL	mixer conversion loss
\mathbf{d}	digital signal vector
$\mathbf{D}(\boldsymbol{\mu}), \mathbf{D}_n(\boldsymbol{\mu})$	digital spectrum vector
$D(\boldsymbol{\mu}), D_n(\boldsymbol{\mu})$	digital spectrum
$d(k)$	digital signal
$d_n(k)$	digital signal of the n th transmitter/DAC
e_Q	quantization error
ENOB	effective number of bits
ENOB _{TI}	effective number of bits for TI-DAC
f	frequency
f_A, f_B	frequency interval
$f_{AMUX,m}$	AMUX clock frequency
f_c	(AMUX) switching frequency
$f_{co}, f_{co,n}$	filter cutoff frequency, frequency demultiplexer split frequency
f_{DAC}	DAC clock frequency
f_{LO}	local oscillator frequency

$f_{\text{LO,IQ},n}$	LO frequency for mixer n (I/Q mixer)
$f_{\text{LO,LSB},n}$	LO frequency for mixer n (RF mixer LSB)
$f_{\text{LO,USB},n}$	LO frequency for mixer n (RF mixer USB)
$f_{\text{LO},M_n,n}$	LO frequency for mixer n of type M_n
$f_{\text{LO},n}$	LO frequency for mixer n
$f_{\text{min},n}$	minimum frequency of sub-signal n
f_{max}	transistor maximum oscillation frequency
$f_{\text{max},n}$	maximum frequency of sub-signal n
$f_{\text{PN},1}, f_{\text{PN},2}$	lower and upper phase noise integration boundary
f_s	sampling rate
$f_{s,\text{analog}}$	sampling rate for analog simulation
$f_{s,n}$	sampling rate of DAC n
$f_{s,\text{tot}}$	total sampling rate
$f_{\text{sym,tot,target}}$	target signal symbol rate
$f_{\text{sym,tot}}$	symbol rate
f_T	transistor current gain cutoff frequency
$G(f)$	DAC frequency response
$g(t)$	DAC impulse response
$\hat{\mathbf{H}}_{\text{LS}}$	LS CE impulse response matrix
$H_{\text{B},n}^{\text{RC}}(f)$	raised cosine filter band-pass characteristic
$H_{\text{H},n}^{\text{RC}}(f)$	raised cosine filter high-pass characteristic
$H_{\text{L},n}^{\text{RC}}(f)$	raised cosine filter low-pass characteristic
$H_{\text{In}}(\mu)$	MIMO system frequency response
$H_{\text{SF},n}(f)$	digital demultiplexer frequency response
\mathbf{I}	identity matrix
I_{IN}	switch pair input current
I_{OUT}	DAC core output current
k	discrete time
\mathbf{K}_{D}	# of data samples vector
$\mathbf{K}_{\text{D,tot}}$	# of total data samples vector
$K_{\text{D,tot}}$	# of total data samples
$K_{\text{D},n}$	# of data samples for DAC n
K_{H}	length of channel impulse response
$K_{\text{O},n}$	# of additional samples for guard bands for DAC n

$K_{OH,n}$	# of additional samples for guard bands at high frequencies for DAC n
$K_{OL,n}$	# of additional samples for guard bands at low frequencies for DAC n
K_S	# of samples
$K_{S,n}$	# of samples for DAC n
$K_{S,tot}$	# of total samples
L	# of receivers for MIMO system
M	# of AMUX stages
$M_{AMUX-DAC}^{(a)}$	# of multiplications for AMUX-DAC DSP variant (a)
$M_{AMUX-DAC}^{(b)}$	# of multiplications for AMUX-DAC DSP variant (b)
$M_{CE,MIMO,f}$	# of multiplications for MIMO frequency domain CE
$M_{CE,MIMO,t}$	# of multiplications for MIMO time domain CE
$M_{Delay,f}$	# of multiplications for delay filter
$M_{FFT,c}^{sr}, M_{FFT,r}^{sr}$	# of multiplications for complex- and real-valued split-radix FFT
$M_{FFT}^{r2,m}$	# of multiplications for radix- 2^m FFT
M_{FI-DAC}	# of multiplications for FI-DAC DSP
$M_{FI,n}$	# of elements in the n th FI-DAC signal path
$M_{IFFT,r}^{sr}$	# of multiplications for real-valued split-radix IFFT
M_n	mixer type for signal path n
M_P	order of nonlinear polynomial
$M_{W,f}$	# of multiplications for frequency domain filter
$M_{W,MIMO,f,calc}$	# of multiplications for MIMO equalizer calculation
$M_{W,MIMO,f,diag}$	# of multiplications for MIMO equalizer (only diagonal elements)
$M_{W,MIMO,f}$	# of multiplications for MIMO equalizer
N	# of DACs, # of AMUX inputs, # of clock signals, # of sub-signals, # of transmitters
N_A	# of amplifiers in AMUX clock buffer
N_D	# of data samples for CE
N_{FFT}	FFT length
N_H	# of frequency bins for channel estimation
N_{IFFT}	IFFT length
N_{LO}	# of LO harmonics
N_P	degree of parallelization

N_W	# of frequency bins for frequency domain filter
NMSE	normalized mean squared error
$\hat{\mathbf{P}}$	inverse matrix of oversampling ratios
\mathbf{P}	matrix of oversampling ratios
$\tilde{\mathbf{P}}$	cofactor matrix of oversampling ratios
\tilde{p}_{1n}	first row of the cofactor matrix $\tilde{\mathbf{P}}$
P_{avg}	average constellation power
$P_{\text{AMP},n}$	power consumption of the n th amplifier
P_{DAC}	DAC's power consumption
$p_{H,n}$	oversampling ratio DAC n for high frequencies
$p_{H,n}^{\min}, p_{H,n}^{\max}$	lower and upper bound for oversampling ratio DAC n for high frequencies
P_{IN}	input power
$P_{\text{IN,ref}}$	input reference power
$P_{\text{LO},n}$	power consumption for the generation of the n th LO
$p_{L,n}$	oversampling ratio DAC n for low frequencies
$p_{L,n}^{\min}, p_{L,n}^{\max}$	lower and upper bound for oversampling ratio DAC n for low frequencies
$P_{m,n}$	spurious power
\hat{p}_{n1}	first column of the inverse matrix $\hat{\mathbf{P}}$
p_n	oversampling ratio DAC n
P_{TOT}	total power consumption
\mathbf{R}	received samples matrix for CE
R	resistance
$r(k)$	received discrete-time signal
R_C	collector resistance
R_E	emitter degeneration resistance
R_L	load resistance
\mathbf{S}	transmit data matrix for CE
\mathbf{S}_n	transmit data sub-matrix for CE of the n th transmitter
$s_{\text{CLK}}(t)$	clock signal
$s_{\text{IF}}(t)$	mixer IF input signal
s_{IN}	DAC input signal
$s_{\text{LO},n}(t)$	mixer LO signal's n th harmonic
$s_{\text{OUT}}(t)$	DAC behavioral model output signal, mixer behavioral model output signal

$s_{\text{OUT},n}(t)$	sub-signal n of behavioral mixer model
$s_{\text{RF}}(t)$	mixer RF output signal
SINAD	signal to interference, noise and distortion ratio
SNR	signal to noise ratio
$\text{SNR}_{\text{in},n}$	input SNR of the FI-DAC's analog processing system for the n th signal path
$\text{SNR}_{\text{out},n}$	output SNR of the FI-DAC's analog processing system for the n th signal path
$\text{SST}_{m,n,\text{ref}}$	spurious suppression reference table entry
$\text{SST}_{m,n}$	spurious suppression table entry
T	temperature
t	time
T_c	AMUX clock switching period
T_{IN}	time period of the input signal
T_s	sampling period
$\mathbf{V}(\boldsymbol{\mu})$	noise samples vector
$V(f), V_n(f)$	analog filter's frequency response
V_0	amplitude
V_{CC}	positive supply voltage
V_{CLK}	AMUX clock voltage
$V_{\text{clk},n}(f), V_{\text{clk}}(f)$	AMUX clock path frequency response
$v_{\text{clk},n}(t)$	AMUX clock path impulse response
V_{EE}	negative supply voltage
$v_{\text{FM},n}(t)$	frequency multiplexer impulse response
$V_{\text{IN1}}, V_{\text{IN2}}$	AMUX input voltages
$v_n(t)$	analog filter's impulse response
V_{OUT}	AMUX output voltage
$V_{\text{out,A}}(f), V_{\text{out,B}}(f)$	2:1 AMUX output filter frequency response sub-bands
$V_{\text{out}}(f)$	AMUX data output frequency response
$v_{\text{out}}(t)$	AMUX data output impulse response
V_{pp}	peak-to-peak amplitude
V_{T}	temperature voltage
$\underline{\mathbf{W}}(\boldsymbol{\mu})$	pre-equalizer frequency response matrix
$\underline{\mathbf{W}}_{\text{LMS},i}(\boldsymbol{\mu})$	LMS solution for pre-equalizer matrix
$\underline{\mathbf{W}}_{\text{MMSE}}(\boldsymbol{\mu}), \underline{\mathbf{W}}_{\text{ZF}}(\boldsymbol{\mu})$	MMSE and ZF solution for pre-equalizer matrix
$W(e^{j\Omega})$	pre-equalizer frequency response

w, w_R, w_I	complex number, its real part and its imaginary part
$W_{in}(\mu)$	MIMO equalizer coefficients
$\mathbf{X}(\mu)$	digital signal spectrum vector for multiple sub-bands
$\mathbf{X}(f)$	vector of DAC input spectra
$X(e^{j\Omega})$	DTFT of $x(k)$
$X(\mu), X_n(\mu)$	DFT of digital signal $x(k), x_n(k)$
$X(f)$	Fourier transform of $x(t)$
$x(k)$	digital (DAC input) signal
$X_A(f), X_B(f), X_Z(f)$	2:1 AMUX input sub-bands
$X_n(e^{j\Omega})$	DTFT of $x_n(k)$
$X_n(f)$	band-limited Fourier transform of $x_n(k)$
$x_n(k)$	digital input signal of the n th DAC
$\mathbf{Y}(\mu), Y(\mu)$	DFT of $y(k)$
$\mathbf{Y}(f)$	vector of AMUX output spectra
$Y(f)$	Fourier transform of $y(t)$
$y(k)$	sampled DAC output signal
$y(t)$	(combined) analog DAC output signal
$y_0(t)$	combined analog DAC output reference signal
$Y_A(f), Y_B(f), Y_Z(f)$	2:1 AMUX output sub-bands
$\mathbf{Y}_n(\mu), Y_n(\mu)$	part of sampled DAC output signal's spectrum
$\tilde{y}_n(t)$	upconverted analog signal
$Y_n(f)$	Fourier transform of $y_n(t)$
$y_n(t)$	analog output signal of DAC n
z, z_R, z_I	complex number, its real part and its imaginary part
$Z_{IN,LP}, Z_{IN,HP}$	input admittance of LPF and HPF
Z_{IN}	input admittance
Z_L	load admittance
α_{DAC}	DAC clock/sampling rate ratio
$\alpha_{LO,M_n,n}$	DAC clock/LO ratio
β_n	raised cosine roll-off factor
$\chi(e_Q)$	quantization error probability distribution
$\Delta V(t)$	time-variant amplitude deviations
Δ_Q	quantization interval amplitude
$\delta_\theta(t)$	rectangular function for Dirac distribution definition

$\Delta_{\text{SNR},m,n}$	SNR penalty of the m th element in the n th signal path
ε	2:1 AMUX clock line attenuation factor
Γ	set of mixer types for the FI-DAC
γ	tanh argument in AMUX clock buffer
κ	switch pair correction coefficient
Λ	set of DACs
λ	LMS update coefficient
Λ_n	set of DACs without the n th DAC
μ	discrete frequency of DFT
Ω	normalized angular frequency of the DTFT
ω_{IN}	angular input frequency
ϕ	phase shift
$\phi(t)$	time-variant phase
$\Phi_{\text{PN}}(f)$	one-sided phase noise spectrum
Ψ	constellation given as a set of symbols
ψ_i	symbols in the constellation
σ_{J}^2	average jitter power
σ_{N}^2	noise variance
σ_{Q}^2	average quantization noise power
σ_{S}^2	average signal power
τ	RMS jitter
$\tau_{\text{AMUX}}, \tau_{\text{AMUX},m}$	AMUX clock RMS jitter
τ_{DAC}	DAC clock RMS jitter
τ_{tot}	total AMUX-DAC RMS jitter
θ	height of rectangular function for Dirac definition
$\zeta_{\text{TC}}(f)$	thermal noise PSD for sampling switch ($k_{\text{B}}T/C$)
$\zeta_{\text{T}}(f)$	thermal noise PSD

Indices

i	index variable
l	index variable
m	index variable
n	index variable

Constants and imaginary unit

j	imaginary unit $\sqrt{-1}$
k_B	Boltzmann constant

Set symbols

\mathbb{N}_0	set of natural numbers including zero
$\mathbb{N}_{>0}$	set of natural numbers excluding zero
$\mathbb{R}_{>0}$	set of non-negative rational numbers
\mathbb{Z}	set of integers

Operators

$*$	convolution operator
$\langle \cdot \rangle$	expectation operator
$\lceil \cdot \rceil$	ceiling operator
\prod	product operator
\sum	summation operator
$\{\cdot\}^+$	operator selecting the positive frequencies of the spectrum
$\{\cdot\}^-$	operator selecting the negative frequencies of the spectrum
$\{\cdot\}^*$	complex conjugated operator
$\{\cdot\}^\dagger$	reversed frequency axis + complex conjugate operator
$\{\cdot\}^{-1}$	scalar/vector/matrix inverse operator
$\{\cdot\}^\ddagger$	vector/matrix conjugate transpose operator
$\{\cdot\}^T$	vector/matrix transpose operator
$\text{adj}(\cdot)$	adjugate matrix
$\text{cof}(\cdot)$	cofactor matrix
$\text{det}(\cdot)$	matrix determinant
$\mathfrak{F}\{\cdot\}$	Fourier transform
$\mathfrak{F}_{\text{DFT}}\{\cdot\}$	discrete Fourier transform

Functions

$\delta(t)$	Dirac delta distribution
$\text{rect}(t)$	rectangle function
$\text{sinc}(t)$	sinc function

Notation aspects

x, X	scalar in time and frequency domain
\mathbf{x}, \mathbf{X}	vector in time and frequency domain
$\underline{\mathbf{x}}, \underline{\mathbf{X}}$	matrix in time and frequency domain

List of Acronyms

ADC	analog-to-digital converter
AM	amplitude modulation
AMUX	analog multiplexer
AMUX-DAC	analog multiplexing DAC
ASIC	application-specific IC
AWG	arbitrary waveform generator
AWGN	additive white Gaussian noise
b2b	back-to-back
BER	bit error rate
BiCMOS	bipolar CMOS
BPF	band-pass filter
BPG	bit pattern generator
BPSK	binary phase shift keying
CE	channel estimation
CL	conversion loss
CMOS	complementary metal oxide semiconductor
CPU	central processing unit
CSI	channel state information
D/A	digital-to-analog
DAC	digital-to-analog converter
DBBS	De Bruijn binary sequence
DBI	digital bandwidth interleaving
DC	direct current
DEMUX	demultiplexer
DFT	discrete Fourier transform
DMT	discrete multi-tone
DMUX	digital multiplexer
DNL	differential nonlinearity
DSP	digital signal processing
DTFT	discrete-time Fourier transform
EDA	electronic design automation
ENOB	effective number of bits
EVM	error vector magnitude

FDE	frequency domain equalization
FEC	forward error connection
FET	field-effect transistor
FFT	fast Fourier transform
FI	frequency interleaving
FI-ADC	frequency interleaving ADC
FI-DAC	frequency interleaving DAC
FIR	finite impulse response
FoM	figure of merit
FPGA	field programmable gate array
GaAs	gallium arsenide
HBT	heterojunction bipolar transistor
HEMT	high electron mobility transistors
HHI	Fraunhofer Heinrich-Hertz-Institute
HPF	high-pass filter
IC	integrated circuit
ICT	information and communication technology
IDFT	inverse discrete Fourier transform
IF	intermediate frequency
IFFT	inverse fast Fourier transform
IIR	infinite impulse response
IMD	intermodulation distortion
IM/DD	intensity modulation and direct detection
IMP	intermodulation product
INL	integral nonlinearity
InP	indium phosphide
IP	internet protocol
I/Q	in-phase and quadrature
ISI	inter-symbol interference
LMS	least mean squares
LO	local oscillator
LPF	low-pass filter
LS	least squares
LSB	lower sideband
MIMO	multiple-input multiple-output
MINLP	mixed-integer nonlinear optimization program
MISO	multiple-input single-output
MMSE	minimum mean square error
MSE	mean square error
MZM	Mach-Zehnder modulator
NF	noise figure

NMSE	normalized mean squared error
NRZ	non-return-to-zero
OAWG	optical arbitrary waveform generation
OAWM	optical arbitrary waveform measurement
OFDM	orthogonal frequency division multiplexing
PA	power amplifier
PAM	pulse amplitude modulation
PAPR	peak-to-average power ratio
PDF	probability density function
PLL	phase-locked loop
PRBS	pseudo-random binary sequence
PSD	power spectral density
QAM	quadrature amplitude modulation
RF	radio frequency
RFD	relative frequency distribution
RLS	recursive least squares
RMS	root mean square
Rx	receiver
RZ	return-to-zero
SCIP	Solving Constraint Integer Programs
SE	single-ended
SFDR	spurious free dynamic range
Si	silicon
SiGe	silicon-germanium
SINAD	signal to noise and distortion ratio
SISO	single-input single-output
SNR	signal-to-noise ratio
SSB	single sideband
SSL	suspended stripline
SST	spurious suppression table
THD	total harmonic distortion
TI	time interleaving
TI-DAC	time interleaving DAC
TUB	Technische Universität Berlin
Tx	transmitter
USB	upper sideband
VNA	vector network analyzer
ZF	zero-forcing
ZIMPL	Zuse Institut Mathematical Programming Language
ZOH	zero-order-hold