RESEARCH

Alexander Reiter

Optimal Path and Trajectory Planning for Serial Robots

Inverse Kinematics for Redundant Robots and Fast Solution of Parametric Problems

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Alexander Reiter

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List of Symbols and Abbreviations

Abbreviation	Description
DMS	direct multiple shooting
DOF	degree(s) of freedom
DSS	direct single shooting
EE	end-effector
EoM	equations of motion
GNA	generalized nullspace augmentation
iff	if and only if
IK	inverse kinematics
JSD	joint space decomposition
KKT	Karush-Kuhn-Tucker
LICQ	linear equality constraint qualification
NLP	nonlinear programming
OCP	optimal control problem
PG	projected gradient
QP	quadratic programming
RG	reduced gradient
w.l.o.g.	without loss of generality
✓	correct, successful
✗	incorrect, failed

Abstract

This dissertation covers the topic of optimal path and trajectory planning for serial robots in general, and rigorously treats the challenging application of path tracking for kinematically redundant manipulators therein in particular. Furthermore, methods for fast computation of approximate optimal solutions to planning problems are presented.

As the theoretical foundation relevant to this thesis, basics of numerical optimization, B-spline curves and their use for the solution of optimal control problems using direct methods are discussed. Furthermore, the kinematics of serial robots is reviewed in depth. In particular, the inverse kinematics problem regarding kinematically redundant manipulators is considered. Therein, numerical optimization is applied not to find any solution to the path tracking problem, but rather to solve it in an optimal manner. As one contribution of this thesis, a method is presented that is particularly useful for this task. Therein, differential inverse kinematics is augmented by a weighted linear combination of the Jacobian nullspace basis, which allows to directly exploit a system's internal motion capabilities in accordance with the path tracking optimization goal. It is capable of resolving both, the path tracking task and the optimal inverse kinematics problem, simultaneously. The efficacy of this approach is demonstrated using numerical examples and experiments.

Optimal control problems require a certain level of complexity of the mathematical-physical models used therein to accurately reflect the corresponding real-world problem. As a result, the numerical optimization problem used to represent such a task, is typically time-consuming to solve. Moreover, certain parts of the problem setup may be subject to change and are therefore not known beforehand. A sudden parameter change in the time between computing the optimal solution and the task execution using this solution may result in a suboptimal, or even inappropriate system behavior. The goal is of course to obtain a solution to the modified problem. Computing an exact optimal solution is typically not possible due to large computation times, classical methods for finding approximate optimal solutions produce – if at all – inadmissible results for larger perturbations. As another contribution, this thesis presents a method that exhibits fast convergence to an admissible, nearly optimal solution in the case of larger parameter perturbations. Therein, a first-order approximation of an optimal solution for a perturbed optimization problem is obtained by means of parametric sensitivities. The possibility of an active set of constraints different from that of the nominal solution is addressed which is the main advantage over existing methods. The potential of this approach is shown by means of a comparison with established methods as well as numerical examples and corresponding experiments.

Kurzfassung

Diese Doktorarbeit behandelt optimale Bahn- und Trajektorienplanung für serielle Roboter im Allgemeinen und Pfadfolgestrategien für kinematisch redundante Roboter im Speziellen. Außerdem werden Näherungsmethoden für optimale Lösungen solcher Planungsaufgaben entwickelt, die im Vergleich zur exakten Lösungen eine bedeutend geringere Rechenzeit benötigen.

In dieser Arbeit werden – soweit für das Verständnis relevant – zunächst die Grundlagen zur numerischen Optimierung, zu B-Spline-Kurven, sowie die Vereinigung dieser Gebiete in Optimalsteuerungsproblemen in direkter Formulierung untersucht. Zusätzlich wird eine genaue Untersuchung der Kinematik von seriellen Robotern durchgeführt, wie sie in Pfadfolgeproblemen auftritt. Besonderes Augenmerk wird auf die Inverskinematik für kinematisch redundante Roboter gelegt. Dabei wird numerische Optimierung angewandt, um nicht nur irgendeine, sondern die optimale Lösung für dieses Problem zu finden. Als einer der Beiträge dieser Arbeit wird eine Methode vorgestellt, die für diese Aufgabestellung besonders gut geeignet ist, da darin optimales Pfadfolgen und das darin enthaltene Inverskinematikproblem simultan optimal gelöst werden können. Dabei handelt es sich um einen differentiellen Inverskinematikansatz, der mittels gewichteter Basisvektoren des Nullraums der Jacobi-Matrix erweitert wird. Diese Vorgangsweise erlaubt es, die Möglichkeit für interne Bewegungen eines Systems hinsichtlich des Optimierungsziels des übergeordneten Pfadfolgeproblems direkt auszunutzen. Die Effektivität dieses Ansatzes wird mittels einiger Rechenbeispiele und zugehöriger Experimente gezeigt.

Optimalsteuerungprobleme benötigen einen gewissen Detailgrad der darin verwendeten mathematisch-physikalischen Modelle, um die zugehörige Aufgabenstellung ausreichend genau abbilden zu können. Mit wachsender Genauigkeit steigt allerdings häufig auch die zur Lösung des Problems notwendige Rechenzeit an. Zusätzlich kann sich die Aufgabenstellung durch

sich ändernde Parameter beeinflusst werden, womit die Aufgabe zu einem parameterabhängigen Optimalsteuerungsproblem wird. Wenn Parameterschwankungen im Zeitraum zwischen der Bestimmung der optimalen Lösung und ihrer Ausführung liegen, so wird die neue Aufgabe im Allgemeinen durch diese Lösung nur suboptimal oder garnicht gelöst. Das Problem für die tatsächlich gültigen Parameter zu lösen ist oftmals aufgrund von Rechenzeiteinschränkungen nicht möglich. Einen Ausweg stellt eine Methode dar, die in kurzer Rechenzeit eine zulässige Näherung der optimalen Lösung des parameterabhängigen Optimalsteuerungsproblems liefert. Klassische Methoden sind hier hinsichtlich der erlaubten Größe der zulässigen Parameterstörung für diese Aufgabenstellung nur eingeschränkt nutzbar. Einen weiteren Beitrag dieser Arbeit stellt ein Ansatz dar, mit dessen Hilfe mit vergleichsweise geringem Rechenaufwand eine zulässige Näherung erster Ordnung der optimalen Lösung bestimmt werden kann. Diese wird mittels parametrischer Sensitivitäten aus dem Optimierungsproblem für nominale Parameterwerte berechnet. Der Ansatz zeichnet sich durch die Eigenschaft aus, dass auch eine Veränderung der Menge der aktiven Nebenbedingungen des Optimierungsproblems berücksichtigt werden kann, was einen Vorteil gegenüber bestehenden Methoden darstellt. Die Wirksamkeit dieser Methode wird durch Vergleichsrechnungen mit bewährten Methoden, sowie durch weitere numerische Beispiele und deren experimentelle Umsetzung demonstriert.

Chapter 1

Introduction

Due to the rapid progress in developing solutions for industrial applications of robotic manipulators, today it is not sufficient to simply fulfill a given task. Now it is required to solve a given problem in an *optimal* manner. To this end, not only foundational knowledge about kinematics and dynamics of multi-body systems and robotics sub-disciplines emerging therefrom are required, it also necessitates expertise in numerical optimization.

The present thesis provides contributions to sub-fields of kinematics and also of numerical optimization. In the former field, the inverse kinematics for a specific class of robotics is investigated. In the latter, numerical optimization, particular approximate solutions of parametric optimization problems are discussed. In order to facilitate understanding of these topics, basics regarding all required fields of knowledge are presented in brevity and references to state-of-the-art literature are provided.

An important portion of the present thesis is devoted to *kinematic redundancy*. In short, it is a property of the kinematic structure that robots of certain types exhibit. It allows a robot to perform additional, internal motion while executing a prescribed motion of its end-effector. That is, for a given motion of a robot's end-effector, the joint motion is ambiguous, thus enabling additional criteria to be pursued in addition to end-effector path tracking. The mention of kinematic redundancy in this thesis is twofold. On the one hand, it is discussed in the kinematics chapter in Section 3.5 where its basic properties and consequences are reviewed. In particular,

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inverse kinematics methods for such kinematic structures are examined. On the other hand, kinematic redundancy is exploited in course of optimal path tracking in Chapter 5 to increase the task execution performance.

Outline

In Chapter 2, numerical basics relevant for the rest of the work are presented. Therein, elementary knowledge regarding numerical optimization is repeated in Section 2.1. In particular, parametric optimization problems are discussed wherein an iterative scheme to compute approximate solutions to such problem is proposed as one contribution of this work. Furthermore, B-spline curves are reviewed in Section 2.2 which are then used for general optimal control problems in Section 2.3.

Chapter 3 treats kinematic properties of serial robots, i.e. the problem of forward kinematics as well as the more challenging inverse kinematics. Therein, kinematically non-redundant and redundant manipulators are considered and solutions on position-, velocity- and higher derivative level are presented. A further contribution of this thesis is a particular approach to compute a solution to the inverse kinematics problem for kinematically redundant manipulators on derivative level.

Optimal path and trajectory planning, cf. Chapter 4, are robotic applications of the above. Therein, the evolution of a robot's joint positions (and by virtue of the kinematic chain) the end-effector, is computed to fulfill a specified task, e.g. point-to-point motion. This is accomplished using different strategies, a single-step strategy wherein the geometric and temporal aspects are treated simultaneously, and a multi-stage approach where these subtasks are solved subsequently. Furthermore, in order to allow for rapid computation of the above, the iterative approximation scheme for optimal solutions using parametric sensitivities is applied.

Chapter 5 discusses optimal path tracking. In contrast to path and trajectory planning, therein, a path is prescribed and needs to be tracked by the robot's end-effector in an optimal way. With the previously discussed inverse kinematics for kinematically redundant manipulators and optimal control theory, optimal solutions not only regarding the inverse kinematics resolution, but also for the path tracking problem are obtained. For kinematically non-redundant robots, neither the inverse kinematics, nor the path tracking problem are in the scope of this thesis.

Chapter 6 provides final remarks on this thesis and concludes with a brief summary of the work presented herein.

In Appendix A, an alternative parametrization of B-spline curves in terms of the arc length is discussed. A software package that allows for evaluation and construction of such functions is also proposed therein. Appendix B presents additional information about the dynamics of multi-body systems, their representation as a system of coupled differential-algebraic equations and their numerical solution. Furthermore, a software library that facilitates code generation regarding the numerical solution of such problems is introduced.

Examples

In order to ease understanding, theoretical parts are frequently complemented by numerical examples of typical applications. The robots featured therein are of various types, planar and spatial, kinematically non-redundant and redundant, as well as industrial manipulators and lab prototypes. The following brief overview provides an introduction to the robots mentioned throughout this work.

Example 1.1: SCARA4 (description)

The SCARA4 manipulator is a kinematically redundant lab prototype that is shown in Figures 1.1 and 1.2. It comprises of four links, serially connected by four actuated revolute joints as well as a prismatic joint with a gripper attachment. In this thesis, only the planar structure with four revolute joints is considered, cf. the reduced presentation in Figure 1.2. The vertical prismatic joint with the gripper attachment are not part of the model.

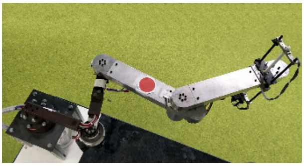

Figure 1.1: SCARA4 Lab Prototype.

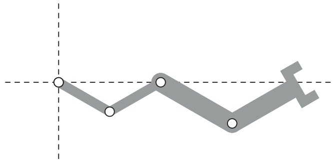

Figure 1.2: SCARA4 Planar Model.

Example 1.2: Stäubli TX90L (description)

The Stäubli TX90L is a spatial industrial robot, cf. Figure 1.3 that consists of six revolute joints in a serial structure and features a spherical wrist. In this setup it is placed on top of a linear axis, i.e. a prismatic joint, establishing inherent kinematic redundancy (which can be disabled by simply locking the prismatic joint).

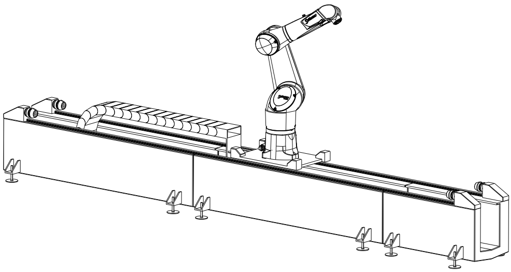

Figure 1.3: Stäubli TX90L Industrial Manipulator On Linear Axis (Contour Rendering).

Example 1.3: Comau Racer R3 (description)

The Comau Racer R3 is a kinematically inherently non-redundant, spatial, industrial robot, cf. Figure 1.4 that consists of six revolute joints in a serial structure and features a spherical wrist.

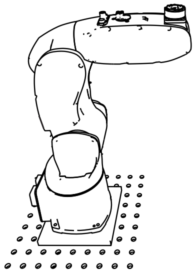

Figure 1.4: COMAU Racer R3 Industrial Manipulator (Contour Rendering).

In this thesis a number of numerical examples are discussed including their respective computation times. These computations were implemented in *MATLAB R2015b* on *Windows* running on an *Intel Xeon E3-1245V3* CPU.

Aim & Contributions

The two main sets of contributions in this thesis are the following:

- Fast solution approximation using parametric sensitivities. In certain applications, the computation time available to find a solution to a complex optimization problem is limited, e.g. for a robotics path trajectory planning task by a real-time system with a fixed cycle time. To that end, in many cases, solutions to such problems cannot be obtained by traditional optimization schemes which gives rise to computationally less demanding methods that compute approximations to the solution of an optimization problem. Such an approach requires an existing solution to a nominal problem as an initial guess to which the actual situation presents a deviation or perturbation. A frequent limitation to such approaches is the size of the allowed deviation. This work proposes an iterative scheme using parametric sensitivities that allows for large perturbations. Of course, it cannot only be applied to above robotic tasks, cf. Chapter 4, but is applicable to more general problems, cf. the (successful) comparison to more established approaches in the academic examples from Section 2.1.3. Publications of the author in this field are the conference papers [112, 116].