Lecture Notes in Electrical Engineering 596

Vančo Litovski

Electronic Filters

Theory, Numerical Recipes, and Design Practice based on the $\mathcal{RM}$ Software

Springer

Lecture Notes in Electrical Engineering

Volume 596

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ISSN 1876-1100 ISSN 1876-1119 (electronic)
Lecture Notes in Electrical Engineering
ISBN 978-981-32-9851-4 ISBN 978-981-32-9852-1 (eBook)
https://doi.org/10.1007/978-981-32-9852-1

This Springer imprint is published by the registered company Springer Nature Singapore Pte Ltd.
The registered company address is: 152 Beach Road, #21-01/04 Gateway East, Singapore 189721, Singapore

To the loving memory of my son
Leonid Litovski

Preface

Filter design as a research subject dates almost a century now. Of course, it was coming as part to the development of the radio and telecommunication industry which became ubiquitous now. Substantial number of solutions was offered in transfer function synthesis as well as in the physical realization which gradually shifted from analogue to a digital one with the ability to be even implemented as software. In that, the interest got shifted mainly towards new technologies while gradually narrowing the scope and the versatility of the approximation solutions. As a consequence, modern filter design is based on reduced set of transfer functions being catalogued and available broadly.

It was our filling that in the historical moment, we are living catalogues and nomograms are not the best way to produce optimal solutions. That is valid especially having in mind the fact that the computers became affordable to anyone and that practically all electrical engineers are fluent in programming languages. We think that the theory and numerical receipts given, anyone may produce any practically feasible transfer functions starting with the proper set of requirements. Similar stands for the circuit or system synthesis.

Having that in mind, this book is to be looked upon as a supportive tool for software development for filter design in both transfer function synthesis (approximation process) and system synthesis (physical design).

It is divided into three parts. The context is created within the first part by introducing the reader into the basic concepts of signals and filters including the $\mathcal{RM}$ software as a supportive tool used for creation of the examples within the book. Then, the transfer function synthesis is addressed which encompasses a wide variety of solutions. It was our intention here not only to describe the solution as such but to give full information for the reader to start software development for the proper approximant by his own. That is supported within every chapter by a paragraph named “Developer’s corner” containing advice and transferring experience related to the practical implementation of the concepts just described. In addition, examples are given of relatively high complexity allowing the reader to verify his own solution. In the third part, the system synthesis of analogue and digital systems is visited. Here again, a wide variety of technologies is encompassed allowing the designer to cover most of the frequency spectrum from very low audio frequencies up to tenths of GHz. There is something specific, however, within this part. Namely, in most cases parallel solutions are advised and synthesis concepts are described.

The author is using the opportunity to thank the research staff of the Laboratory for Electronic Design Automation (LEDA) of the Faculty of Electronic Engineering at the University of Niš, Serbia, for their lifelong and everlasting collaboration and support.

Niš, Serbia Vančo Litovski

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About the Author

Prof. Vančo Litovski was born in 1947 in Rakita, South Macedonia, Greece. He graduated from the Faculty of Electronic Engineering in Niš in 1970, and obtained his M.Sc. in 1974, and his Ph.D. in 1977. He was appointed as a teaching assistant at the Faculty of Electronic Engineering in 1970 and became a Full Professor at the same faculty in 1987. He was elected as Visiting Professor (honoris causa) at the University of Southampton in 1999. From 1987 until 1990, he was a consultant to the CEO of Ei, and was Head of the Chair of Electronics at the Faculty of Electronic Engineering in Niš For 12 years. From 2015 to 2017, he was a researcher at the University of Bath. He has taught courses related to analogue electronics, electronic circuit design, and artificial intelligence at the electro-technical faculties in Priština, Skopje, Sarajevo, Banja Luka, and Novi Sad. He received several awards including from the Faculty of Electronic Engineering (Charter in 1980, Charter in 1985, and a Special Recognition in 1995) and the University of Niš (Plaque 1985). Prof. Litovski has published 6 monographs, over 400 articles in international and national journals and at conferences, 25 textbooks, and more than 40 professional reports and studies.

His research interests include electronic and electrical design and design for sustainability, and he led the design of the first custom commercial digital and research-oriented analogue CMOS circuit in Serbia. He has also headed 8 strategic projects financed by the Serbian and Yugoslav governments and the JNA, and has participated in several European projects funded by the governments of Germany, Austria, UK, and Spain, and the EC as well as the Black See Organization of Economic Cooperation (BSEC).

Part I
Basics

Chapter 1
Introduction

> Give a man a fish, and you feed him for a day. Teach a man to fish, and you feed him for a lifetime.

The story of filter design is almost hundred years old now. It follows two paths, one related to transfer function synthesis and the other related to physical implementation synthesis.

While there had to be physical realization for whatever function was developed, due to technological limitations, the branch related to transfer function synthesis (which relied on mathematics) was leading, often facing the "physical realization" obstacle. It grew fast since the first Butterworth's [1] introduction of the maximally flat amplitude characteristic. Solutions were found for equi-ripple amplitude approximation both in the passband and in the stopband of the filter. The problem of linear phase was addressed and solved allowing for subsequent solutions satisfying mixed amplitude and phase requirements. Finally, direct time domain solutions were reported [2].

It was really very difficult to manipulate complicated complex functions in the pre computer era. The introduction of computers in the sixties (and later on) of the last century revolutionized the transfer function synthesis and most of the main problem were now solved with acceptable complexity. Let just mention the problem of frequency transformations [3] proposed by Orchard and Temes and the introduction of iterative design [4].

Later on, with further advent of the computers and digital integrated circuit technology, theory of digital signal processing and digital filter's transfer function synthesis was introduced to allow for an explosion of the telecommunication systems [5].

Nevertheless, after all these years and almost unlimited number of contributions to the subject of transfer function synthesis, even the most advanced contemporary sources are based on a very limited set of solutions (such as Butterworth, Chebyshev, Elliptic, Inverse Chebyshev, and Thomson) and use formulas, nomograms or pre-prepared tables [6–8] for the design. In [9] the authors explicitly state that the transfer function synthesis will be based on existing nomograms. In fact, the real transfer function synthesis became a privilege of a very limited number of

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https://doi.org/10.1007/978-981-32-9852-1_1

scholars who have all the mathematical knowledge, the technological experience, and programming capabilities.

That became an important issue in the progress of the transfer function synthesis research and development frequently leading to non-optimal solutions. Namely, having the wrong feeling that one is fully informed on the subject, research results were published (by world renown publishers) in which explicit ignorance is demonstrated. We will illustrate this claim here with several examples with the intention to draw the attention of the filter design community and to suggest more responsible reviewing. Of course, to avoid negative advertising we will not cite the exact sources. As the first example we will mention the inadequate computation of the phase characteristics. After definition of the transfer function in the form

$$H(\mathrm{j}\omega) = \frac{m_1(\mathrm{j}\omega) + j \cdot n_1(\mathrm{j}\omega)}{m_2(\mathrm{j}\omega) + j \cdot n_2(\mathrm{j}\omega)} \tag{1.1}$$

the authors derive the expression for the phase characteristic as

$$\varphi(\mathrm{j}\omega) = \arctan\left[\frac{n_1 \cdot m_2 - n_1 \cdot m_1}{m_1 \cdot m_2 + n_1 \cdot n_2}\right]. \tag{1.2}$$

Even if we skip the question of a phase being a function of a complex variable and attribute that to negligence, there is a fundamental problem with the method of calculating the phase. Namely, assuming the ms and ns are real function of a real variable (which must be), the maximum (minimum) asymptotic value of the phase expressed by (1.2) may be $+\pi/2$ (or $-\pi/2$, depending on the sign of the expression in the brackets). That is to be opposed to the right value of

$$\varphi(\omega)_{|\omega\to\infty} = -(k + l) \cdot \pi/2, \tag{1.3}$$

for a filter with k zeros (complex in the right half plane or imaginary on the ω-axis or mix of these two) and l complex poles in the left half plane. So, a tenth order all-pass filter will have asymptotic value of the phase equal to -10π. Any tenth order polynomial filter will, of course, have $-10\pi/2$.

The next example is related to basic knowledge in which no distinction is made between a circuit schematic and the corresponding transfer function. The authors claim the circuit of Fig. 1.1 is the schematic of an elliptic filter.

In fact it is a sixth order ladder filter (using maximum number of inductances and minimum number of capacitances?!) with two transmission zeros at the real part of the ω-axis which, depending on the element values, may perform as so many different transfer functions, among others, as the Inverse Chebyshev, for example. Since it is of even order and has equal terminations, however, it is hard to believe that it is realizable as an elliptic filter. This is not the only example which may be encountered in literature where the topology of a network is associated to a single type of approximant since, probably, that approximant is the only one known to the author.

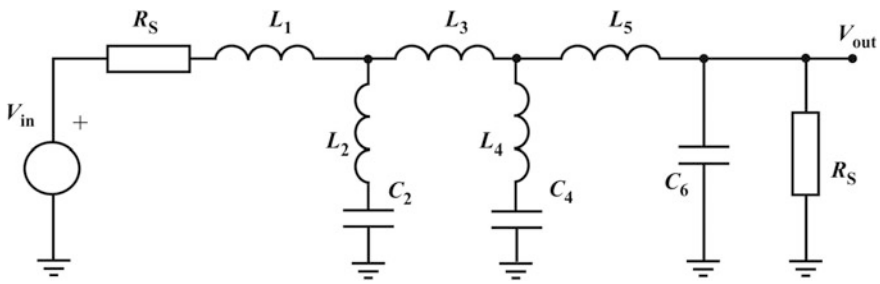

Fig. 1.1 A sixth order LC ladder filter

Finally, one may read about "ultra selective" linear-phase low-pass FIR filters what sounds as a thunder. In fact the authors are reporting a transfer function exhibiting stopband attenuation of 100 dB while the transition region of the filter is 20 times wider than the pass-band width. That means that even the 40th harmonic of a spectral component belonging to the middle of the passband is not filtered. Leaving alone the need for 100 dB of attenuation, a question arises as to: is that a filter at all?

On the physical realization branch, probably with some delay, equally explosive developments could be witnessed. Starting with RLC elements [10], it went first through active RC synthesis [11] which was boosted by the introduction of the integrated operational amplifier [12].

The CMOS technology [13], like the Tesla's alternating currents in the 19th century, became the propellant of the modern society in any respect of the word. It enabled explosion of the information-telecommunication (IT) technologies and did democratize the computing and communication to an unconceivable extent. As for the filter design, first, by introducing the microprocessor the digital filtering era was indicated [14]. Then, by development of powerful programmable devices the digital filtering became affordable. Finally, the custom design of digital integrated circuits allowed for any skilled laboratory to routinely design systems with embedded digital filtering as a subsystem.

Analog CMOS was rising in parallel [15]. In addition to the possibility of embedding the analog filter into a mixed-mode integrated system, fully new ideas were promoted and realized such as switched capacitor (SC) circuits and transconductance-C (Gm-C) circuits which allowed analog signal processing by integrated CMOS circuits at very low and very high frequencies, respectively.

The modern state of the things in the IT may be characterized by immense rise of the awareness of the necessity for every citizen to be capable of writing programming code. One may say that now every electrical and electronic engineer is fluent in one or more programming languages. That, however, does not map into the philosophy of modern filter design. Still, nomograms and tables, generated in the last half of the 20th century, are ruling the trade.

Having that in mind, this book has several main goals.

The first one is to bring forward most of the knowledge and algorithms necessary for successful software development for filter design. In that sense our intention is to give enough information and algorithmic capacity to the reader for he/she, being fluent in modern programming languages and contemporary electronics, to became capable to create his own software for every problem encountered in his/hers everyday professional life.

By acquiring the know-how explicated in this book the reader will be capable to produce approximants (transfer functions of filters) with unique properties accommodated to his/hers problem to a maximal extent. In that sense we want to stress

- The critical monotonic passband amplitude characteristics (CMACs).
- The linear phase filters approximating constant group delay in an equi-ripple manner
- The all-pass and band-pass approximants exhibiting group delay approximating a horizontal and inclined line.
- The amplitude correction algorithms enabling improved both linear phase and selective polynomial prototypes.
- The modified elliptic.
- The low-pass and band-pass group delay correctors.
- The simultaneous group delay and amplitude approximation of band-pass filters.

On the other side our intention was to make the reader capable to create in an existing design environment. To that end we will exemplify all the algorithms and procedures, belonging to transfer function synthesis or to physical system synthesis, on the case of use of the $\mathcal{RM}$ software [16, 17].

In that, one should have in mind that this is not a book on electronics. It is about filter design which means only limited information about the circuits will be given. Namely, one may usually find in filter design books complete information about semiconductor technology, transistor and passive devices design and modelling, and synthesis of low-complexity circuit (maximum to second order cells) leaving the issues of complex synthesis to the reader. Instead, we will try to skip the common knowledge and minimize the theory on synthesis of basic electronic circuits and offer to the potential software developer and filter designer what we think are the best building blocks for proper applications so trying to concentrate on complex real-life solutions and architectures.

The book is divided in three parts.

In the first part we first deal with basics of signals and filtering. Definitions will be given and the basic terminology explained. Attempts will be made not to leave any ambiguities related to the meaning of the terms. As a specialty we introduce for the first time quantification of the selectivity while insisting on design being as selective as possible (the ones saving bands of frequencies are named ecological designs). Next comes the definition of the transfer function and calculations of frequency and time domain responses. Here analog transfer functions are considered only. A specialty in this paragraph is the calculation of the phase response which is frequently misunderstood as exemplified above. We insist that for all phases of design: function approximation, function analysis and physical synthesis, factored form of the transfer function is the only way to represent it. That allows for an integrated approach. Issues related to the transfer function definitions are completed by the description of the frequency transformations in the analog domain. The first part ends with a brief description of the potentials, structure, and functionality of the $\mathcal{RM}$ software for filter design which will be used to exemplify the algorithms and the physical synthesis methods.

The second part is devoted to transfer function synthesis. It encompasses a very wide spectrum of approximants fulfilling requirements in the frequency domain obeying amplitude, group delay or both amplitude and group delay requirements. It is in this book for the first time a complete description and information for synthesis is given for the low-pass selective polynomial filters with critical monotonic amplitude characteristic (CMAC) in the passband. The non-monotonic amplitude characteristic in the passband are studied via the Chebyshev filter while a modification is described which allows for modified amplitude characteristic of the even order Chebyshev filters. These proceedings are followed by comparisons in order to complete the knowledge about polynomial selective filters.

It is a fact that extending the transfer function of a polynomial filter by a finite transmission zero at the ω-axis brings more benefits to the amplitude characteristic than increasing the order of the polynomial filter. That, however, is not advertized in the literature and no books may be found describing the procedure of introducing such zeros. Here we do that in two variants: with distinct and with multiple new transmission zeros. Of course, special cases of these functions are the so called Inverse Chebyshev and the Elliptic filters. Here we introduce the term modified elliptic filters since the procedure we are proposing allows for independent values of the maximum passband and minimum stopband attenuations with number of attenuation zeros equal or smaller than maximum.

Algorithms and examples of synthesis of linear phase low-pass filters exhibiting maximally flat and equi-ripple approximation of constant group delay are discussed next. Synthesis of all-pass low-pass filter is described, too. Algorithms and procedures are given for extending these transfer function with transmission zeros at the ω-axis to improve selectivity while preserving the phase linearity. The rest of this part of the book is related to solutions obeying group delay requirements as the primary ones. First, synthesis of group delay correctors (both low-pass and band-pass) is visited. Then, band-pass all-pass filters approximating a line in maximally flat sense are described. The line here may be horizontal and inclined (rising or falling). Finally, direct synthesis of selective band-pass filters with a group delay approximating a line in maximally flat sense is introduced.

For every approximation type described in this part of book complete information and advice is given enabling the reader to develop his own transfer function synthesis program. Note, for every algorithm and procedure, being inherently iterative, initial solution creation is advised. Detailed examples are accompanying every chapter so allowing the reader to check his own results in every phase of the software action. Readers having no experience in optimization or having difficulties to apprehend the iterative processes recommended in this book, are advised to first read [16].

As for the physical synthesis, to which the third part is devoted, the following technologies are encompassed. Passive LC cascade circuit synthesis; active RC cascade circuit synthesis; parallel active RC circuit synthesis; Gm-C filter synthesis based on LC prototypes; parallel active SC circuit synthesis; and parallel synthesis of IIR digital filters based on bilinear transformation of analog prototypes. For every each of them detailed information is given allowing the reader having a transfer function represented in a factored form to develop his/hers own synthesis program. As in the case of transfer function synthesis, examples are given in every chapter allowing the software developer to check his/hers results in every phase of the transformation of the input data on the transfer function into element values of the schematic.

A special feature of this topic is the parallel synthesis. Namely, the cascade synthesis is not avoided in two basic cases as the passive LC (including the Gm-C filters emanating from them) and the active RC but, since for realization of cascade synthesis laborious procedures are needed which still involve uncertainties, the parallel alternative is offered. It is simple to implement and in some cases advantageous.

The book is supported by 34 video presentations/animations which are on public display [17, 18] (the second was introduced since some of the countries have no access to YouTube). There details on the capacities and ways of implementation of the $\mathcal{RM}$ software are given. We expect that this and the "Developers corner" introduced in every chapter of this book will rise the confidence of the readers for both to take the adventure and develop his/hers own software or to effectively use of existing one.

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Chapter 2
Basics of Signals and Filtering

Types of signals are introduced first: continuous time and discrete time. Transposition into the frequency domain via the Fourier transform is used to allow for introduction to the signal processing. By manipulation of the spectrum of an example signal the idea of filtering and filters is introduced. Rationale for the relocation of the spectrum by modulation is given. The term and a measure for selectivity are defined and "green design" is introduced. Categorization of the filters is given based on the way how is the signal processed (continuous time and discrete time); how the signal is represented: analog and digital; where the position of the passband is (low-pass, band-pass, high-pass, all-pass, and band-stop); and what is the technology used for physical implementation (discrete passive LC, discrete and integrated active RC, integrated active SC, integrated active Gm-C, and software realized and integrated digital). Practical issues such as frequency, time, and element values normalization as well as calculator-less decibel (and inverse) calculations are included.

2.1 Introduction

The topic of filtering is rather multidisciplinary in the sense that encompasses serious mathematical apparatus and equally serious technological knowledge. Of course, knowledge of circuit theory and signal processing should not be undermined. That is why preparing a book on filter design is an action full of responsibilities. When starting this book, we decided to try to simplify the explanations as much as possible in order to allow the reader to acquire only the knowledge which is indispensible for the subject. That means that we will try to use examples being as general as possible and in the same time as simple as possible. On the other side an attempt will be made to encompass as much information as possible in order not to bring the designer's horizon too near and so discourage him before starting.

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https://doi.org/10.1007/978-981-32-9852-1_2

Following these consideration this chapter will be an introduction to signal processing and filter design. It will serve as a place for definition the basic terms and concepts.

2.2 Signals

A signal is a function of one or more variables that conveys information about some (usually physical) phenomenon [1]. For the purpose of this book, a signal will be function of a single variable (one-dimensional), the time, t. According to the variation of their time variable and their amplitude, signals can be either continuous-time or discrete-time, analog or discrete amplitude, or digital. This classification relates to the way signals are either processed, stored, or both. According to whether the signals exhibit repetitive behavior or not they are classified as periodic or aperiodic signals [2].

Figure 2.1a depicts a continuous time or an analog signal. Both the time and the voltage variables are allowed to take any value. Analog signals are natural signals and, strictly speaking, all physical signals are analog.

When it come to representation of the signals, e.g. in a computer, problems arise first due to the finite number of significant figures available for representation. This property is referred to as the resolution. No matter how large is the computer word (say 64 bit) only discrete values of both time and the signal may be represented. In between two neighboring values is a difference which is represented by the value of the least significant bit. In addition, storing signal values which are so near to each other (in time and value) may represent a serious request for memory. So, in order to save memory, one discretizes the time, the discretization interval being Δt, and allows for the complete signal to be stored in a limited amount of memory space. Such a signal, depicted in Fig. 2.1b, is still analog since the voltage is allowed to take any value. The process of conversion of a continuous time signal into a discrete time one is referred to as sampling. The final signal is a sampled one. The quantity

$$fs = 1/\Delta t \tag{2.1}$$

is referred to as the sampling frequency and it is expressed in Hz. The reciprocal is usually stated as sampling period $T_s = \Delta t$. For example, $f_s = 25$ kHz means 25,000 samples per second and the sampling period is $T_s = 1/25{,}000$ s $= 40$ µs.

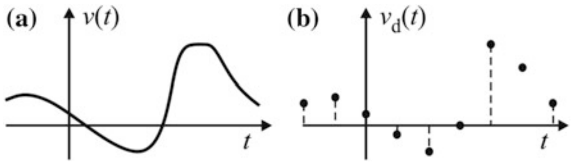

Fig. 2.1 Signals. **a** Continuous and **b** discrete time

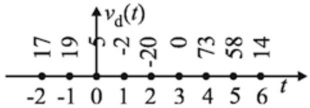

Fig. 2.2 Discretized in time and digitized in value signal

If the instantaneous value of the voltage taken in discrete time intervals is discretized (the term quantized is in use, too), we in fact get a digital signal. The discrete values of the voltage are represented by finite number of figures and, usually, in binary form which allows for storing in a computer. For example, the discretized signal depicted in Fig. 2.1b is converted into a digital one and depicted in Fig. 2.2. Here, to simplify, decimal system is used for representation of the voltage digital values which means that the quantization interval was $\Delta v = 1$ V.

In Fig. 2.2 the discretized time lost its identity. In fact the samples are enumerated and since the sampling time interval Δt is known it may be restored easily. It is obvious that representation of the signal depicted in Fig. 2.2 in a computer is an easy task. Not only that, it will take negligible amount of memory. Both, simplifying the representation and reducing the memory requirements, are what the computer needs. The price for that, however, is lost information in both coordinates. To protect the accuracy one should try to use as small Δv and Δt as possible.

In the next we will very shortly describe the activities frequently referred to as signal processing in all three domains mentioned above.

2.3 The Spectrum of a Signal

Looking to the signal of Fig. 2.1a we see that there is not much to be done with it except to extend its voltage value i.e. to amplify it or to perform sampling and enter the discrete time domain.

In fact, there is much more which is not visible at first sight. The time domain representation we are considering is hiding many properties of the signal which may be modified, corrected, improved etc. To come to these properties we need to change the domain and from the (natural) time domain to switch to (artificial) frequency domain. For that purpose we use the so called Fourier transform which allows for a full insight on the signal and enables implementation of means for processing it.

In the next we will introduce this transform and proceed by use of the means it offers to create context for the need for filtering and ways of doing it.

Given an arbitrary *periodic* continuous time signal $x(t)$ with a period T, according to the Fourier transform it may be represented in a form of an infinite series so as

$$v(t) = c_0 + \sum_{k=1}^{\infty} 2 \cdot \mathrm{Re}\left\{c_k e^{\mathrm{j}k\omega_0 t}\right\}. \tag{2.2}$$

In the above equation

$$\omega_0 = 2 \cdot \pi / T, \tag{2.3}$$

$\mathrm{j} = \sqrt{-1}$, and c_k, k = 0, 1, …, ∞ is a set of constants needed to be determined in order for (2.2) to be valid.

If complex quantities are properly represented by real and imaginary parts a new form of the Fourier series may be created as

$$v(t) = c_0 + 2 \cdot \sum_{k=1}^{\infty} |c_k| \cdot \cos(k\omega_0 t + \theta_k) \tag{2.4}$$

or

$$v(t) = c_0 + \sum_{k=1}^{\infty} [a_k \cdot \cos(k\omega_0 t) + b_k \cdot \sin(k\omega_0 t)] \tag{2.5}$$

where $c_k = |c_k| \cdot e^{\mathrm{j}\theta_k}$, $a_k = \mathrm{Re}\{2 \cdot c_k\}$ and $b_k = -\mathrm{Im}\{2 \cdot c_k\}$. Equation (2.5) is referred to as the *trigonometric form* of the Fourier transform.

Here are the formulas determining c_k:

$$c_k = \frac{1}{T} \int_{t_0}^{t_0 + T} v(t) \cdot e^{\mathrm{j}k\omega_0 t} \cdot \mathrm{d}t \tag{2.6}$$

with

$$c_0 = \frac{1}{T} \int_{t_0}^{t_0 + T} v(t) \cdot \mathrm{d}t \tag{2.7}$$

representing the average value of the signal over the period T. t_0 in (2.6) and (2.7) is arbitrary instant for starting the integration of the periodic signal $v(t)$.

The coefficient c_0 being the average, c_1 is referred to as the first or the main harmonic of the series while c_k for $k > 1$ is referred to as the kth harmonic.

From (2.4) and (2.5) we may see that each complex sinusoid is associated with a particular frequency (which is some integer multiple (k) of the fundamental frequency (ω_0)). So, these coefficients indicate at which frequencies the signal is concentrated. In this way, the Fourier series representation provides a means for

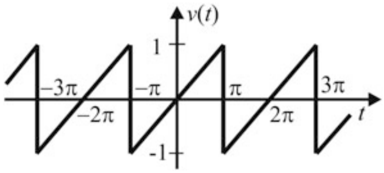

Fig. 2.3 The example saw-tooth signal

measuring the frequency content of a signal. The distribution of the signal over different frequencies is referred to as the *frequency spectrum* of the signal.

Various ways exist to illustrate the frequency spectrum of a signal. Typically, we plot the Fourier series coefficients as a function of frequency. Since, in general, the Fourier series coefficients are complex-valued; we usually display this information using two plots. One plot shows the amplitude of the coefficients as a function of frequency. This is called the amplitude spectrum. The other plot shows the arguments of the coefficients as a function of frequency. In this context, the argument is referred to as the phase, and the plot is called the phase spectrum of the signal.

We will now elaborate an example which will introduce us into the subject of analog signal processing.

Consider the saw-tooth signal, depicted in Fig. 2.3, which is represented by the following set of expressions

$$v(t) = \frac{t}{\pi} \quad \text{for } -\pi < t \le \pi, \tag{2.8}$$

and

$$v(t + 2\pi \cdot l) = v(t) \quad \text{for } l = 0, 1, \ldots, \infty. \tag{2.9}$$

Since $T = 2\pi$, $\omega_0 = 1$ rad/s. The coefficients of the trigonometric form of the Fourier series related to this signal are

$$a_k = 0 \quad \text{for } k \ge 0 \tag{2.10}$$

and

$$b_k = 2 \cdot \frac{(-1)^{k+1}}{\pi \cdot k} \quad \text{for } k \ge 1. \tag{2.11}$$

After substitution in (2.5) we get

$$v(t) = \sum_{k=1}^{\infty} \left[2 \cdot \frac{(-1)^{k+1}}{\pi \cdot k} \cdot \sin(kt) \right]. \tag{2.12}$$

We will first pay attention to the values of the coefficients b_k. Figure 2.4a depicts the first 20 coefficients and shows that they have alternative signs and decrease

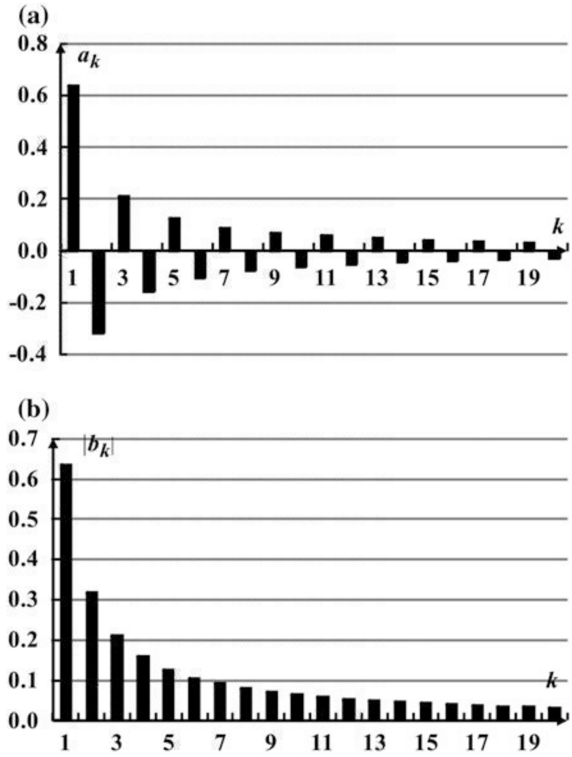

Fig. 2.4 Frequency domain representation of the saw-tooth signal. **a** The b_k coefficients up to the twentieth and **b** modulus of the b_k coefficients—the amplitude spectrum

hyperbolically with increase of k. That means that the higher order coefficients have smaller influence to the overall sum.

Figure 2.4b depicts the amplitude spectrum of the saw-tooth signal with components up to the 20th. Looking to this diagram we come to ideas as to how an analog signal may be processed. Simply, by manipulating the components of the spectrum! Since manipulation may, in some case, mean elimination of a component from the spectrum the term filtering comes forward. We say that that component was eliminated by a filter. Starting with that any manipulation of the spectrum is referred to as filtering and the means for performing filtering is referred to as a filter.

Let us now study the influence of some filters to the time domain presentation of the saw-tooth signal. Consider a filter function defined as

$$f(k) = \begin{cases} 1 & for \quad k \le m \\ 0 & for \quad k > m \end{cases} \tag{2.13}$$

which is depicted in Fig. 2.5a. We will refer to this type of filtering function as *ideal amplitude low-pas filter*. The value m represents in fact a multiple of ω_0 (i.e. $m \cdot \omega_0$) and is referred to as the cut-off frequency of the low-pass filter.

After multiplying the original signal with the filtering function one gets