

Fifth Edition

Singiresu S. Rao

Engineering Optimization

Theory and Practice

WILEY

Engineering Optimization

Engineering Optimization Theory and Practice

Fifth Edition

Singiresu S Rao

*University of Miami
Coral Gables, Florida*

WILEY

This edition first published 2020
© 2020 John Wiley & Sons, Inc.

Edition History

John Wiley & Sons Ltd (4e, 2009)

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, except as permitted by law. Advice on how to obtain permission to reuse material from this title is available at <http://www.wiley.com/go/permissions>.

The right of Singiresu S Rao to be identified as the author of this work has been asserted in accordance with law.

Registered Offices

John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, USA

Editorial Office

The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, UK

For details of our global editorial offices, customer services, and more information about Wiley products visit us at www.wiley.com.

Wiley also publishes its books in a variety of electronic formats and by print-on-demand. Some content that appears in standard print versions of this book may not be available in other formats.

Limit of Liability/Disclaimer of Warranty

While the publisher and authors have used their best efforts in preparing this work, they make no representations or warranties with respect to the accuracy or completeness of the contents of this work and specifically disclaim all warranties, including without limitation any implied warranties of merchantability or fitness for a particular purpose. No warranty may be created or extended by sales representatives, written sales materials or promotional statements for this work. The fact that an organization, website, or product is referred to in this work as a citation and/or potential source of further information does not mean that the publisher and authors endorse the information or services the organization, website, or product may provide or recommendations it may make. This work is sold with the understanding that the publisher is not engaged in rendering professional services. The advice and strategies contained herein may not be suitable for your situation. You should consult with a specialist where appropriate. Further, readers should be aware that websites listed in this work may have changed or disappeared between when this work was written and when it is read. Neither the publisher nor authors shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages.

Library of Congress Cataloging-in-Publication Data

Names: Rao, Singiresu S., 1944- author.

Title: Engineering optimization : theory and practice / Singiresu S Rao,
University of Miami, Coral Gables, FL, US.

Description: Fifth edition. | Hoboken, NJ, USA : John Wiley & Sons, Inc.,
2020. | Includes bibliographical references and index. |

Identifiers: LCCN 2019011088 (print) | LCCN 2019011277 (ebook) | ISBN
9781119454762 (Adobe PDF) | ISBN 9781119454793 (ePub) | ISBN 9781119454717
(hardback)

Subjects: LCSH: Engineering—Mathematical models. | Mathematical optimization.

Classification: LCC TA342 (ebook) | LCC TA342 .R36 2019 (print) | DDC
620.001/5196—dc23

LC record available at <https://lccn.loc.gov/2019011088>

Cover Design: Wiley

Cover Image: © Nayanba Jadeja/Getty Images

Set in 10.25/12pt TimesLTStd by SPi Global, Chennai, India

Printed in United States of America

10 9 8 7 6 5 4 3 2 1



Contents

Preface	xvii
Acknowledgment	xxi
About the Author	xxiii

1	Introduction to Optimization	1
1.1	Introduction	1
1.2	Historical Development	3
1.2.1	Modern Methods of Optimization	4
1.3	Engineering Applications of Optimization	5
1.4	Statement of An Optimization Problem	6
1.4.1	Design Vector	6
1.4.2	Design Constraints	7
1.4.3	Constraint Surface	7
1.4.4	Objective Function	8
1.4.5	Objective Function Surfaces	9
1.5	Classification of Optimization Problems	14
1.5.1	Classification Based on the Existence of Constraints	14
1.5.2	Classification Based on the Nature of the Design Variables	14
1.5.3	Classification Based on the Physical Structure of the Problem	15
1.5.4	Classification Based on the Nature of the Equations Involved	18
1.5.5	Classification Based on the Permissible Values of the Design Variables	27
1.5.6	Classification Based on the Deterministic Nature of the Variables	28
1.5.7	Classification Based on the Separability of the Functions	29
1.5.8	Classification Based on the Number of Objective Functions	31
1.6	Optimization Techniques	33
1.7	Engineering Optimization Literature	34
1.8	Solutions Using MATLAB	34
	References and Bibliography	34
	Review Questions	40
	Problems	41

2	Classical Optimization Techniques	57
2.1	Introduction	57
2.2	Single-Variable Optimization	57
2.3	Multivariable Optimization with no Constraints	62
2.3.1	Definition: r th Differential of f	62
2.3.2	Semidefinite Case	67
2.3.3	Saddle Point	67
2.4	Multivariable Optimization with Equality Constraints	69
2.4.1	Solution by Direct Substitution	69
2.4.2	Solution by the Method of Constrained Variation	71
2.4.3	Solution By the Method of Lagrange Multipliers	77

2.5	Multivariable Optimization with Inequality Constraints	85
2.5.1	Kuhn–Tucker Conditions	90
2.5.2	Constraint Qualification	90
2.6	Convex Programming Problem	96
	References and Bibliography	96
	Review Questions	97
	Problems	98

3 Linear Programming I: Simplex Method 109

3.1	Introduction	109
3.2	Applications of Linear Programming	110
3.3	Standard form of a Linear Programming Problem	112
3.3.1	Scalar Form	112
3.3.2	Matrix Form	112
3.4	Geometry of Linear Programming Problems	114
3.5	Definitions and Theorems	117
3.5.1	Definitions	117
3.5.2	Theorems	120
3.6	Solution of a System of Linear Simultaneous Equations	122
3.7	Pivotal Reduction of a General System of Equations	123
3.8	Motivation of the Simplex Method	127
3.9	Simplex Algorithm	128
3.9.1	Identifying an Optimal Point	128
3.9.2	Improving a Nonoptimal Basic Feasible Solution	129
3.10	Two Phases of the Simplex Method	137
3.11	Solutions Using MATLAB	143
	References and Bibliography	143
	Review Questions	143
	Problems	145

4 Linear Programming II: Additional Topics and Extensions 159

4.1	Introduction	159
4.2	Revised Simplex Method	159
4.3	Duality in Linear Programming	173
4.3.1	Symmetric Primal–Dual Relations	173
4.3.2	General Primal–Dual Relations	174
4.3.3	Primal–Dual Relations when the Primal Is in Standard Form	175
4.3.4	Duality Theorems	176
4.3.5	Dual Simplex Method	176
4.4	Decomposition Principle	180
4.5	Sensitivity or Postoptimality Analysis	187
4.5.1	Changes in the Right-Hand-Side Constants b_i	188
4.5.2	Changes in the Cost Coefficients c_j	192
4.5.3	Addition of New Variables	194
4.5.4	Changes in the Constraint Coefficients a_{ij}	195
4.5.5	Addition of Constraints	197
4.6	Transportation Problem	199

4.7	Karmarkar's Interior Method	202
4.7.1	Statement of the Problem	203
4.7.2	Conversion of an LP Problem into the Required Form	203
4.7.3	Algorithm	205
4.8	Quadratic Programming	208
4.9	Solutions Using Matlab	214
	References and Bibliography	214
	Review Questions	215
	Problems	216
5	Nonlinear Programming I: One-Dimensional Minimization Methods	225
5.1	Introduction	225
5.2	Unimodal Function	230
	ELIMINATION METHODS	231
5.3	Unrestricted Search	231
5.3.1	Search with Fixed Step Size	231
5.3.2	Search with Accelerated Step Size	232
5.4	Exhaustive Search	232
5.5	Dichotomous Search	234
5.6	Interval Halving Method	236
5.7	Fibonacci Method	238
5.8	Golden Section Method	243
5.9	Comparison of Elimination Methods	246
	INTERPOLATION METHODS	247
5.10	Quadratic Interpolation Method	248
5.11	Cubic Interpolation Method	253
5.12	Direct Root Methods	259
5.12.1	Newton Method	259
5.12.2	Quasi-Newton Method	261
5.12.3	Secant Method	263
5.13	Practical Considerations	265
5.13.1	How to Make the Methods Efficient and More Reliable	265
5.13.2	Implementation in Multivariable Optimization Problems	266
5.13.3	Comparison of Methods	266
5.14	Solutions Using MATLAB	267
	References and Bibliography	267
	Review Questions	267
	Problems	268
6	Nonlinear Programming II: Unconstrained Optimization Techniques	273
6.1	Introduction	273
6.1.1	Classification of Unconstrained Minimization Methods	276
6.1.2	General Approach	276
6.1.3	Rate of Convergence	276
6.1.4	Scaling of Design Variables	277

DIRECT SEARCH METHODS	280
6.2 Random Search Methods	280
6.2.1 Random Jumping Method	280
6.2.2 Random Walk Method	282
6.2.3 Random Walk Method with Direction Exploitation	283
6.2.4 Advantages of Random Search Methods	284
6.3 Grid Search Method	285
6.4 Univariate Method	285
6.5 Pattern Directions	288
6.6 Powell's Method	289
6.6.1 Conjugate Directions	289
6.6.2 Algorithm	293
6.7 Simplex Method	298
6.7.1 Reflection	298
6.7.2 Expansion	301
6.7.3 Contraction	301
INDIRECT SEARCH (DESCENT) METHODS	304
6.8 Gradient of a Function	304
6.8.1 Evaluation of the Gradient	306
6.8.2 Rate of Change of a Function Along a Direction	307
6.9 Steepest Descent (Cauchy) Method	308
6.10 Conjugate Gradient (Fletcher–Reeves) Method	310
6.10.1 Development of the Fletcher–Reeves Method	310
6.10.2 Fletcher–Reeves Method	311
6.11 Newton's Method	313
6.12 Marquardt Method	316
6.13 Quasi-Newton Methods	317
6.13.1 Computation of $[B_i]$	318
6.13.2 Rank 1 Updates	319
6.13.3 Rank 2 Updates	320
6.14 Davidon–Fletcher–Powell Method	321
6.15 Broyden–Fletcher–Goldfarb–Shanno Method	327
6.16 Test Functions	330
6.17 Solutions Using Matlab	332
References and Bibliography	333
Review Questions	334
Problems	336
7 Nonlinear Programming III: Constrained Optimization Techniques	347
7.1 Introduction	347
7.2 Characteristics of a Constrained Problem	347
DIRECT METHODS	350
7.3 Random Search Methods	350
7.4 Complex Method	351
7.5 Sequential Linear Programming	353

7.6	Basic Approach in the Methods of Feasible Directions	360
7.7	Zoutendijk's Method of Feasible Directions	360
7.7.1	Direction-Finding Problem	362
7.7.2	Determination of Step Length	364
7.7.3	Termination Criteria	367
7.8	Rosen's Gradient Projection Method	369
7.8.1	Determination of Step Length	372
7.9	Generalized Reduced Gradient Method	377
7.10	Sequential Quadratic Programming	386
7.10.1	Derivation	386
7.10.2	Solution Procedure	389
INDIRECT METHODS		392
7.11	Transformation Techniques	392
7.12	Basic Approach of the Penalty Function Method	394
7.13	Interior Penalty Function Method	396
7.14	Convex Programming Problem	405
7.15	Exterior Penalty Function Method	406
7.16	Extrapolation Techniques in the Interior Penalty Function Method	410
7.16.1	Extrapolation of the Design Vector X	410
7.16.2	Extrapolation of the Function f	412
7.17	Extended Interior Penalty Function Methods	414
7.17.1	Linear Extended Penalty Function Method	414
7.17.2	Quadratic Extended Penalty Function Method	415
7.18	Penalty Function Method for Problems with Mixed Equality and Inequality Constraints	416
7.18.1	Interior Penalty Function Method	416
7.18.2	Exterior Penalty Function Method	418
7.19	Penalty Function Method for Parametric Constraints	418
7.19.1	Parametric Constraint	418
7.19.2	Handling Parametric Constraints	420
7.20	Augmented Lagrange Multiplier Method	422
7.20.1	Equality-Constrained Problems	422
7.20.2	Inequality-Constrained Problems	423
7.20.3	Mixed Equality–Inequality-Constrained Problems	425
7.21	Checking the Convergence of Constrained Optimization Problems	426
7.21.1	Perturbing the Design Vector	427
7.21.2	Testing the Kuhn–Tucker Conditions	427
7.22	Test Problems	428
7.22.1	Design of a Three-Bar Truss	429
7.22.2	Design of a Twenty-Five-Bar Space Truss	430
7.22.3	Welded Beam Design	431
7.22.4	Speed Reducer (Gear Train) Design	433
7.22.5	Heat Exchanger Design [7.42]	435
7.23	Solutions Using MATLAB	435
References and Bibliography		435
Review Questions		437
Problems		439

8 Geometric Programming 449

8.1	Introduction	449
8.2	Posynomial	449
8.3	Unconstrained Minimization Problem	450
8.4	Solution of an Unconstrained Geometric Programming Program using Differential Calculus	450
8.4.1	Degree of Difficulty	453
8.4.2	Sufficiency Condition	453
8.4.3	Finding the Optimal Values of Design Variables	453
8.5	Solution of an Unconstrained Geometric Programming Problem Using Arithmetic–Geometric Inequality	457
8.6	Primal–dual Relationship and Sufficiency Conditions in the Unconstrained Case	458
8.6.1	Primal and Dual Problems	461
8.6.2	Computational Procedure	461
8.7	Constrained Minimization	464
8.8	Solution of a Constrained Geometric Programming Problem	465
8.8.1	Optimum Design Variables	466
8.9	Primal and Dual Programs in the Case of Less-than Inequalities	466
8.10	Geometric Programming with Mixed Inequality Constraints	473
8.11	Complementary Geometric Programming	475
8.11.1	Solution Procedure	477
8.11.2	Degree of Difficulty	478
8.12	Applications of Geometric Programming	480
	References and Bibliography	491
	Review Questions	493
	Problems	493

9 Dynamic Programming 497

9.1	Introduction	497
9.2	Multistage Decision Processes	498
9.2.1	Definition and Examples	498
9.2.2	Representation of a Multistage Decision Process	499
9.2.3	Conversion of a Nonserial System to a Serial System	500
9.2.4	Types of Multistage Decision Problems	501
9.3	Concept of Suboptimization and Principle of Optimality	501
9.4	Computational Procedure in Dynamic Programming	505
9.5	Example Illustrating the Calculus Method of Solution	507
9.6	Example Illustrating the Tabular Method of Solution	512
9.6.1	Suboptimization of Stage 1 (Component 1)	514
9.6.2	Suboptimization of Stages 2 and 1 (Components 2 and 1)	514
9.6.3	Suboptimization of Stages 3, 2, and 1 (Components 3, 2, and 1)	515
9.7	Conversion of a Final Value Problem into an Initial Value Problem	517
9.8	Linear Programming as a Case of Dynamic Programming	519
9.9	Continuous Dynamic Programming	523
9.10	Additional Applications	526
9.10.1	Design of Continuous Beams	526
9.10.2	Optimal Layout (Geometry) of a Truss	527

9.10.3 Optimal Design of a Gear Train	528
9.10.4 Design of a Minimum-Cost Drainage System	529
References and Bibliography	530
Review Questions	531
Problems	532

10 Integer Programming 537

10.1 Introduction	537
-------------------	-----

INTEGER LINEAR PROGRAMMING 538

10.2 Graphical Representation	538
10.3 Gomory's Cutting Plane Method	540
10.3.1 Concept of a Cutting Plane	540
10.3.2 Gomory's Method for All-Integer Programming Problems	541
10.3.3 Gomory's Method for Mixed-Integer Programming Problems	547
10.4 Balas' Algorithm for Zero-One Programming Problems	551

INTEGER NONLINEAR PROGRAMMING 553

10.5 Integer Polynomial Programming	553
10.5.1 Representation of an Integer Variable by an Equivalent System of Binary Variables	553
10.5.2 Conversion of a Zero-One Polynomial Programming Problem into a Zero-One LP Problem	555
10.6 Branch-and-Bound Method	556
10.7 Sequential Linear Discrete Programming	561
10.8 Generalized Penalty Function Method	564
10.9 Solutions Using MATLAB	569
References and Bibliography	569
Review Questions	570
Problems	571

11 Stochastic Programming 575

11.1 Introduction	575
11.2 Basic Concepts of Probability Theory	575
11.2.1 Definition of Probability	575
11.2.2 Random Variables and Probability Density Functions	576
11.2.3 Mean and Standard Deviation	578
11.2.4 Function of a Random Variable	580
11.2.5 Jointly Distributed Random Variables	581
11.2.6 Covariance and Correlation	583
11.2.7 Functions of Several Random Variables	583
11.2.8 Probability Distributions	585
11.2.9 Central Limit Theorem	589
11.3 Stochastic Linear Programming	589
11.4 Stochastic Nonlinear Programming	594
11.4.1 Objective Function	594
11.4.2 Constraints	595
11.5 Stochastic Geometric Programming	600

References and Bibliography	602
Review Questions	603
Problems	604

12 Optimal Control and Optimality Criteria Methods 609

12.1	Introduction	609
12.2	Calculus of Variations	609
12.2.1	Introduction	609
12.2.2	Problem of Calculus of Variations	610
12.2.3	Lagrange Multipliers and Constraints	615
12.2.4	Generalization	618
12.3	Optimal Control Theory	619
12.3.1	Necessary Conditions for Optimal Control	619
12.3.2	Necessary Conditions for a General Problem	621
12.4	Optimality Criteria Methods	622
12.4.1	Optimality Criteria with a Single Displacement Constraint	623
12.4.2	Optimality Criteria with Multiple Displacement Constraints	624
12.4.3	Reciprocal Approximations	625
	References and Bibliography	628
	Review Questions	628
	Problems	629

13 Modern Methods of Optimization 633

13.1	Introduction	633
13.2	Genetic Algorithms	633
13.2.1	Introduction	633
13.2.2	Representation of Design Variables	634
13.2.3	Representation of Objective Function and Constraints	635
13.2.4	Genetic Operators	636
13.2.5	Algorithm	640
13.2.6	Numerical Results	641
13.3	Simulated Annealing	641
13.3.1	Introduction	641
13.3.2	Procedure	642
13.3.3	Algorithm	643
13.3.4	Features of the Method	644
13.3.5	Numerical Results	644
13.4	Particle Swarm Optimization	647
13.4.1	Introduction	647
13.4.2	Computational Implementation of PSO	648
13.4.3	Improvement to the Particle Swarm Optimization Method	649
13.4.4	Solution of the Constrained Optimization Problem	649
13.5	Ant Colony Optimization	652
13.5.1	Basic Concept	652
13.5.2	Ant Searching Behavior	653
13.5.3	Path Retracing and Pheromone Updating	654
13.5.4	Pheromone Trail Evaporation	654
13.5.5	Algorithm	655

13.6	Optimization of Fuzzy Systems	660
13.6.1	Fuzzy Set Theory	660
13.6.2	Optimization of Fuzzy Systems	662
13.6.3	Computational Procedure	663
13.6.4	Numerical Results	664
13.7	Neural-Network-Based Optimization	665
	References and Bibliography	667
	Review Questions	669
	Problems	671
14	Metaheuristic Optimization Methods	673
14.1	Definitions	673
14.2	Metaphors Associated with Metaheuristic Optimization Methods	673
14.3	Details of Representative Metaheuristic Algorithms	680
14.3.1	Crow Search Algorithm	680
14.3.2	Firefly Optimization Algorithm (FA)	681
14.3.3	Harmony Search Algorithm	684
14.3.4	Teaching-Learning-Based Optimization (TLBO)	687
14.3.5	Honey Bee Swarm Optimization Algorithm	689
	References and Bibliography	692
	Review Questions	694
15	Practical Aspects of Optimization	697
15.1	Introduction	697
15.2	Reduction of Size of an Optimization Problem	697
15.2.1	Reduced Basis Technique	697
15.2.2	Design Variable Linking Technique	698
15.3	Fast Reanalysis Techniques	700
15.3.1	Incremental Response Approach	700
15.3.2	Basis Vector Approach	704
15.4	Derivatives of Static Displacements and Stresses	705
15.5	Derivatives of Eigenvalues and Eigenvectors	707
15.5.1	Derivatives of λ_i	707
15.5.2	Derivatives of Y_i	708
15.6	Derivatives of Transient Response	709
15.7	Sensitivity of Optimum Solution to Problem Parameters	712
15.7.1	Sensitivity Equations Using Kuhn–Tucker Conditions	712
15.7.2	Sensitivity Equations Using the Concept of Feasible Direction	714
	References and Bibliography	715
	Review Questions	716
	Problems	716
16	Multilevel and Multiobjective Optimization	721
16.1	Introduction	721
16.2	Multilevel Optimization	721
16.2.1	Basic Idea	721
16.2.2	Method	722
16.3	Parallel Processing	726

16.4	Multiobjective Optimization	729
16.4.1	Utility Function Method	730
16.4.2	Inverted Utility Function Method	730
16.4.3	Global Criterion Method	730
16.4.4	Bounded Objective Function Method	730
16.4.5	Lexicographic Method	731
16.4.6	Goal Programming Method	732
16.4.7	Goal Attainment Method	732
16.4.8	Game Theory Approach	733
16.5	Solutions Using MATLAB	735
	References and Bibliography	735
	Review Questions	736
	Problems	737

17 Solution of Optimization Problems Using MATLAB 739

17.1	Introduction	739
17.2	Solution of General Nonlinear Programming Problems	740
17.3	Solution of Linear Programming Problems	742
17.4	Solution of LP Problems Using Interior Point Method	743
17.5	Solution of Quadratic Programming Problems	745
17.6	Solution of One-Dimensional Minimization Problems	746
17.7	Solution of Unconstrained Optimization Problems	746
17.8	Solution of Constrained Optimization Problems	747
17.9	Solution of Binary Programming Problems	750
17.10	Solution of Multiobjective Problems	751
	References and Bibliography	755
	Problems	755

A Convex and Concave Functions 761

B Some Computational Aspects of Optimization 767

B.1	Choice of Method	767
B.2	Comparison of Unconstrained Methods	767
B.3	Comparison of Constrained Methods	768
B.4	Availability of Computer Programs	769
B.5	Scaling of Design Variables and Constraints	770
B.6	Computer Programs for Modern Methods of Optimization	771
	References and Bibliography	772

C Introduction to MATLAB® 773

C.1	Features and Special Characters	773
C.2	Defining Matrices in MATLAB	774
C.3	Creating m-Files	775
C.4	Optimization Toolbox	775

Answers to Selected Problems 777

Index 787

Preface

The ever-increasing demand on engineers to lower production costs to withstand global competition has prompted engineers to look for rigorous methods of decision making, such as optimization methods, to design and produce products and systems both economically and efficiently. Optimization techniques, having reached a degree of maturity by now, are being used in a wide spectrum of industries, including aerospace, automotive, chemical, electrical, construction, and manufacturing industries. With rapidly advancing computer technology, computers are becoming more powerful, and correspondingly, the size and the complexity of the problems that can be solved using optimization techniques are also increasing. Optimization methods, coupled with modern tools of computer-aided design, are also being used to enhance the creative process of conceptual and detailed design of engineering systems.

The purpose of this textbook is to present the techniques and applications of engineering optimization in a comprehensive manner. The style of prior editions has been retained, with the theory, computational aspects, and applications of engineering optimization presented with detailed explanations. As in previous editions, essential proofs and developments of the various techniques are given in a simple manner without sacrificing accuracy. New concepts are illustrated with the help of numerical examples. Although most engineering design problems can be solved using non-linear programming techniques, there are a variety of engineering applications for which other optimization methods, such as linear, geometric, dynamic, integer, and stochastic programming techniques, are most suitable. The theory and applications of all these techniques are also presented in the book. Some of the recently developed optimization methods, such as genetic algorithms, simulated annealing, particle swarm optimization, ant colony optimization, neural-network-based methods, and fuzzy optimization, do not belong to the traditional mathematical programming approaches. These methods are presented as modern methods of optimization. More recently, a class of optimization methods termed the metaheuristic optimization methods, have been evolving in the literature. The metaheuristic methods are also included in this edition. Favorable reactions and encouragement from professors, students, and other users of the book have provided me with the impetus to prepare this fifth edition of the book. The following changes have been made from the previous edition:

- Some less-important sections were condensed or deleted.
- Some sections were rewritten for better clarity.
- Some sections were expanded.
- Some of the recently-developed methods are reorganized in the form of a new chapter titled, *Modern methods of optimization*.
- A new chapter titled, *Metaheuristic Optimization Methods*, is added by including details of crow search, firefly, harmony search, teaching-learning, and honey bee swarm optimization algorithms.
- A new chapter titled, *Solution of optimization problems using MATLAB*, is added to illustrate the use of MATLAB for the solution of different types of optimization problems.

Features

Each topic in *Engineering Optimization: Theory and Practice* is self-contained, with all concepts explained fully and the derivations presented with complete details. The computational aspects are emphasized throughout with design examples and problems taken from several fields of engineering to make the subject appealing to all branches of engineering. A large number of solved examples, review questions, problems, project-type problems, figures, and references are included to enhance the presentation of the material.

Specific features of the book include:

- More than 155 illustrative examples accompanying most topics.
- More than 540 references to the literature of engineering optimization theory and applications.
- More than 485 review questions to help students in reviewing and testing their understanding of the text material.
- More than 600 problems, with solutions to most problems in the instructor's manual.
- More than 12 examples to illustrate the use of Matlab for the numerical solution of optimization problems.
- Answers to review questions at the web site of the book, <http://www.wiley.com/rao>.
- Answers to selected problems are given at the end of the book.

I used different parts of the book to teach optimum design and engineering optimization courses at the junior/senior level as well as first-year-graduate-level at Indian Institute of Technology, Kanpur, India; Purdue University, West Lafayette, Indiana; and University of Miami, Coral Gables, Florida. At University of Miami, I cover Chapter 1 and parts of Chapters 2, 3, 5, 6, 7, and 13 in a dual-level course entitled *Optimization in Design*. In this course, a design project is also assigned to each student in which the student identifies, formulates, and solves a practical engineering problem of his/her interest by applying or modifying an optimization technique. This design project gives the student a feeling for ways that optimization methods work in practice. In addition, I teach a graduate level course titled *Mechanical System Optimization* in which I cover Chapters 1–7, and parts of Chapters 9, 10, 11, 13, and 17. The book can also be used, with some supplementary material, for courses with different emphasis such as *Structural Optimization*, *System Optimization* and *Optimization Theory and Practice*. The relative simplicity with which the various topics are presented makes the book useful both to students and to practicing engineers for purposes of self-study. The book also serves as a reference source for different engineering optimization applications. Although the emphasis of the book is on engineering applications, it would also be useful to other areas, such as operations research and economics. A knowledge of matrix theory and differential calculus is assumed on the part of the reader.

Contents

The book consists of 17 chapters and 3 appendixes. Chapter 1 provides an introduction to engineering optimization and optimum design and an overview of optimization methods. The concepts of design space, constraint surfaces, and contours of objective function are introduced here. In addition, the formulation of various types of optimization problems is illustrated through a variety of examples taken from various fields of engineering. Chapter 2 reviews the essentials of differential calculus useful in finding

the maxima and minima of functions of several variables. The methods of constrained variation and Lagrange multipliers are presented for solving problems with equality constraints. The Kuhn–Tucker conditions for inequality-constrained problems are given along with a discussion of convex programming problems.

Chapters 3 and 4 deal with the solution of linear programming problems. The characteristics of a general linear programming problem and the development of the simplex method of solution are given in Chapter 3. Some advanced topics in linear programming, such as the revised simplex method, duality theory, the decomposition principle, and post-optimality analysis, are discussed in Chapter 4. The extension of linear programming to solve quadratic programming problems is also considered in Chapter 4.

Chapters 5–7 deal with the solution of nonlinear programming problems. In Chapter 5, numerical methods of finding the optimum solution of a function of a single variable are given. Chapter 6 deals with the methods of unconstrained optimization. The algorithms for various zeroth-, first-, and second-order techniques are discussed along with their computational aspects. Chapter 7 is concerned with the solution of nonlinear optimization problems in the presence of inequality and equality constraints. Both the direct and indirect methods of optimization are discussed. The methods presented in this chapter can be treated as the most general techniques for the solution of any optimization problem.

Chapter 8 presents the techniques of geometric programming. The solution techniques for problems of mixed inequality constraints and complementary geometric programming are also considered. In Chapter 9, computational procedures for solving discrete and continuous dynamic programming problems are presented. The problem of dimensionality is also discussed. Chapter 10 introduces integer programming and gives several algorithms for solving integer and discrete linear and nonlinear optimization problems. Chapter 11 reviews the basic probability theory and presents techniques of stochastic linear, nonlinear, and geometric programming. The theory and applications of calculus of variations, optimal control theory, and optimality criteria methods are discussed briefly in Chapter 12. Chapter 13 presents several modern methods of optimization including genetic algorithms, simulated annealing, particle swarm optimization, ant colony optimization, neural-network-based methods, and fuzzy system optimization. Chapter 14 deals with metaheuristic optimization algorithms and introduces nearly 20 algorithms with emphasis on Crow search, Firefly, Harmony search, Teaching-Learning and Honey bee swarm optimization algorithms. The practical aspects of optimization, including reduction of size of problem, fast reanalysis techniques and sensitivity of optimum solutions are discussed in Chapter 15. The multilevel and multiobjective optimization methods are covered in Chapter 16. Finally, Chapter 17 presents the solution of different types of optimization problems using the MATLAB software.

Appendix A presents the definitions and properties of convex and concave functions. A brief discussion of the computational aspects and some of the commercial optimization programs is given in Appendix B. Finally, Appendix C presents a brief introduction to Matlab, optimization toolbox, and use of MATLAB programs for the solution of optimization problems. Answers to selected problems are given after Appendix C.

Acknowledgment

I would like to acknowledge that the thousands of hours spent in writing/revising this book were actually the hours that would otherwise have been spent with my wife Kamala and other family members. My heartfelt gratitude is for Kamala's patience and sacrifices which could never be measured or quantified.

S.S. Rao
srao@miami.edu
April 2019

About the Author

Dr. S.S. Rao is a Professor in the Department of Mechanical and Aerospace Engineering at University of Miami, Coral Gables, Florida. He was the Chairman of the Mechanical and Aerospace Engineering Department during 1998–2011 at University of Miami. Prior to that, he was a Professor in the School of Mechanical Engineering at Purdue University, West Lafayette, Indiana; Professor of Mechanical Engineering at San Diego State University, San Diego, California; and Indian Institute of Technology, Kanpur, India. He was a visiting research scientist for two years at NASA Langley Research Center, Hampton, Virginia.

Professor Rao is the author of eight textbooks: *The Finite Element Method in Engineering*, *Engineering Optimization*, *Mechanical Vibrations*, *Reliability-Based Design*, *Vibration of Continuous Systems*, *Reliability Engineering*, *Applied Numerical Methods for Engineers and Scientists*, *Optimization Methods: Theory and Applications*. He coedited a three-volume *Encyclopedia of Vibration*. He edited four volumes of *Proceedings of the ASME Design Automation and Vibration Conferences*. He has published over 200 journal papers in the areas of multiobjective optimization, structural dynamics and vibration, structural control, uncertainty modeling, analysis, design and optimization using probability, fuzzy, interval, evidence and grey system theories. Under his supervision, 34 PhD students have received their degrees. In addition, 12 Post-Doctoral researchers and scholars have conducted research under the guidance of Dr. Rao.

Professor Rao has received numerous awards for academic and research achievements. He was awarded the *Vepa Krishnamurti Gold Medal for University First Rank* in all the five years of the BE (Bachelor of Engineering) program among students of all branches of engineering in all the Engineering Colleges of Andhra University. He was awarded the *Lazarus Prize for University First Rank* among students of Mechanical Engineering in all the Engineering Colleges of Andhra University. He received the First Prize in James F. Lincoln Design Contest open for all MS and PhD students in USA and Canada for a paper he wrote on *Automated Optimization of Aircraft Wing Structures* from his PhD dissertation. He received the *Eliahu I. Jury Award for Excellence in Research* from the College of Engineering, University of Miami in 2002; was awarded the *Distinguished Probabilistic Methods Educator Award* from the Society of Automotive Engineers (SAE) International for *Demonstrated Excellence in Research Contributions in the Application of Probabilistic Methods to Diversified Fields, Including Aircraft Structures, Building Structures, Machine Tools, Airconditioning and Refrigeration Systems, and Mechanisms* in 1999; received the American Society of Mechanical Engineers (ASME) *Design Automation Award for Pioneering Contributions to Design Automation, particularly in Multiobjective Optimization, and Uncertainty Modeling, Analysis and Design Using Probability, Fuzzy, Interval, and Evidence Theories* in 2012; and was awarded the *ASME Worcester Reed Warner Medal* in 2013 for *Outstanding Contributions to the Permanent Literature of Engi-*

neering, particularly for his Many Highly Popular Books and Numerous Trendsetting Research Papers. Dr. Rao received the *Albert Nelson Marquis Lifetime Achievement Award* for demonstrated unwavering excellence in the field of Mechanical Engineering in 2018. In 2019, the American Society of Mechanical Engineers presented him the J.P. Den Hartog Award for his Lifetime Achievements in research, teaching and practice of Vibration Engineering.

Introduction to Optimization

1.1 INTRODUCTION

Optimization is the act of obtaining the best result under given circumstances. In design, construction, and maintenance of any engineering system, engineers have to take many technological and managerial decisions at several stages. The ultimate goal of all such decisions is either to minimize the effort required or to maximize the desired benefit. Since the effort required or the benefit desired in any practical situation can be expressed as a function of certain decision variables, *optimization* can be defined as the process of finding the conditions that give the maximum or minimum value of a function. It can be seen from Figure 1.1 that if a point x^* corresponds to the minimum value of function $f(x)$, the same point also corresponds to the maximum value of the negative of the function, $-f(x)$. Thus, without loss of generality, optimization can be taken to mean minimization, since the maximum of a function can be found by seeking the minimum of the negative of the same function.

In addition, the following operations on the objective function will not change the optimum solution x^* (see Figure 1.2):

1. Multiplication (or division) of $f(x)$ by a positive constant c .
2. Addition (or subtraction) of a positive constant c to (or from) $f(x)$.

There is no single method available for solving all optimization problems efficiently. Hence a number of optimization methods have been developed for solving different types of optimization problems. The optimum seeking methods are also known as *mathematical programming techniques* and are generally studied as a part of operations research. *Operations research* is a branch of mathematics concerned with the application of scientific methods and techniques to decision-making problems and with establishing the best or optimal solutions. The beginnings of the subject of operations research can be traced to the early period of World War II. During the war, the British military faced the problem of allocating very scarce and limited resources (such as fighter airplanes, radar, and submarines) to several activities (deployment to numerous targets and destinations). Because there were no systematic methods available to solve resource allocation problems, the military called upon a team of mathematicians to develop methods for solving the problem in a scientific manner. The methods developed by the team were instrumental in the winning of the Air Battle by Britain. These methods, such as linear programming (LP), which were developed as a result of research on (military) operations, subsequently became known as the methods of operations research.

In recent years several new optimization methods that do not fall in the area of traditional mathematical programming have been and are being developed. Most of

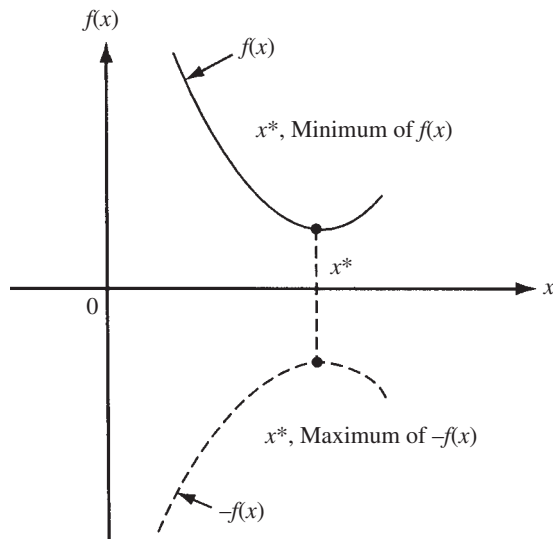


Figure 1.1 Minimum of $f(x)$ is same as maximum of $-f(x)$.

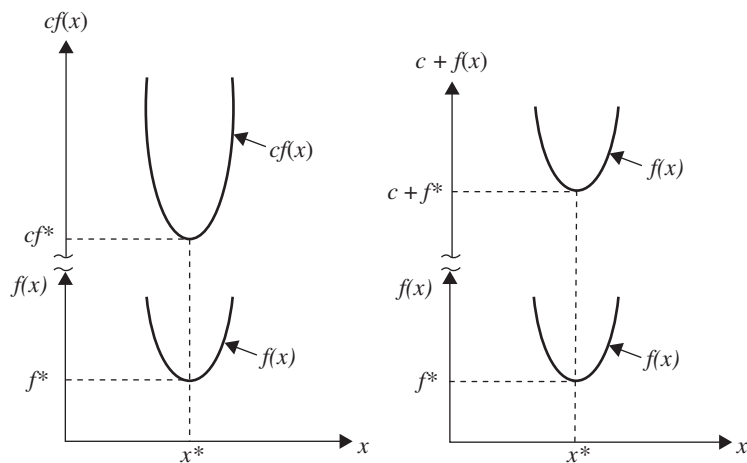


Figure 1.2 Optimum solution of $cf(x)$ or $c + f(x)$ same as that of $f(x)$.

these new methods can be labeled as metaheuristic optimization methods. All the metaheuristic optimization methods have the following features: (i) they use stochastic or probabilistic ideas in various steps; (ii) they are intuitive or trial and error based, or heuristic in nature; (iii) they all use strategies that imitate the behavior or characteristics of some species such as bees, bats, birds, cuckoos, and fireflies; (iv) they all tend to find the global optimum solution; and (v) they are most likely to find an optimum solution, but not necessarily all the time.

Table 1.1 lists various mathematical programming techniques together with other well-defined areas of operations research, including the new class of methods termed metaheuristic optimization methods. The classification given in Table 1.1 is not unique; it is given mainly for convenience.

Mathematical programming techniques are useful in finding the minimum of a function of several variables under a prescribed set of constraints. Stochastic process techniques can be used to analyze problems described by a set of random variables

Table 1.1 Methods of Operations Research.

Mathematical programming or optimization techniques	Stochastic process techniques	Statistical methods
Calculus methods	Statistical decision theory	Regression analysis
Calculus of variations	Markov processes	Cluster analysis, pattern recognition
Nonlinear programming	Queueing theory	Design of experiments
Geometric programming	Renewal theory	Discriminate analysis (factor analysis)
Quadratic programming	Simulation methods	
Linear programming	Reliability theory	
Dynamic programming		
Integer programming		
Stochastic programming		
Separable programming		
Multiobjective programming		
Network methods: Critical Path Method (CPM) and Program (Project) Management and Review Technique (PERT)		
Game theory		
<i>Modern or nontraditional optimization techniques (including Metaheuristic optimization methods)</i>		
Genetic algorithms	Bat algorithm	Salp swarm algorithm
Simulated annealing	Honey Bee algorithm	Cuckoo algorithm
Ant colony optimization	Crow search algorithm	Water evaporation algorithm
Particle swarm optimization	Firefly algorithm	Passing vehicle search algorithm
Tabu search method	Harmony search algorithm	Runner-root algorithm
	Teaching-learning algorithm	Artificial immune system algorithm
	Fruitfly algorithm	Neural network-based optimization
		Fuzzy optimization

having known probability distributions. Statistical methods enable one to analyze the experimental data and build empirical models to obtain the most accurate representation of the physical situation. This book deals with the theory and application of mathematical programming techniques suitable for the solution of engineering design problems. A separate chapter is devoted to the metaheuristic optimization methods.

1.2 HISTORICAL DEVELOPMENT

The existence of optimization methods can be traced to the days of Newton, Lagrange, and Cauchy. The development of differential calculus methods of optimization was possible because of the contributions of Newton and Leibnitz to calculus. The foundations of calculus of variations, which deals with the minimization of functionals, were laid by Bernoulli, Euler, Lagrange, and Weierstrass. The method of optimization for constrained problems, which involves the addition of unknown multipliers, became known by the name of its inventor, Lagrange. Cauchy made the first application of the steepest descent method to solve unconstrained minimization problems. Despite these early contributions, very little progress was made until the