Fifth Edition

Singiresu S. Rao

Engineering Optimization

Theory and Practice

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Preface

The ever-increasing demand on engineers to lower production costs to withstand global competition has prompted engineers to look for rigorous methods of decision making, such as optimization methods, to design and produce products and systems both economically and efficiently. Optimization techniques, having reached a degree of maturity by now, are being used in a wide spectrum of industries, including aerospace, automotive, chemical, electrical, construction, and manufacturing industries. With rapidly advancing computer technology, computers are becoming more powerful, and correspondingly, the size and the complexity of the problems that can be solved using optimization techniques are also increasing. Optimization methods, coupled with modern tools of computer-aided design, are also being used to enhance the creative process of conceptual and detailed design of engineering systems.

The purpose of this textbook is to present the techniques and applications of engineering optimization in a comprehensive manner. The style of prior editions has been retained, with the theory, computational aspects, and applications of engineering optimization presented with detailed explanations. As in previous editions, essential proofs and developments of the various techniques are given in a simple manner without sacrificing accuracy. New concepts are illustrated with the help of numerical examples. Although most engineering design problems can be solved using nonlinear programming techniques, there are a variety of engineering applications for which other optimization methods, such as linear, geometric, dynamic, integer, and stochastic programming techniques, are most suitable. The theory and applications of all these techniques are also presented in the book. Some of the recently developed optimization methods, such as genetic algorithms, simulated annealing, particle swarm optimization, ant colony optimization, neural-network-based methods, and fuzzy optimization, do not belong to the traditional mathematical programming approaches. These methods are presented as modern methods of optimization. More recently, a class of optimization methods termed the metaheuristic optimization methods, have been evolving in the literature. The metaheuristic methods are also included in this edition. Favorable reactions and encouragement from professors, students, and other users of the book have provided me with the impetus to prepare this fifth edition of the book. The following changes have been made from the previous edition:

- Some less-important sections were condensed or deleted.
- Some sections were rewritten for better clarity.
- Some sections were expanded.
- Some of the recently-developed methods are reorganized in the form of a new chapter titled, *Modern methods of optimization*.
- A new chapter titled, *Metaheuristic Optimization Methods*, is added by including details of crow search, firefly, harmony search, teaching-learning, and honey bee swarm optimization algorithms.
- A new chapter titled, *Solution of optimization problems using MATLAB*, is added to illustrate the use of MATLAB for the solution of different types of optimization problems.

Features

Each topic in *Engineering Optimization: Theory and Practice* is self-contained, with all concepts explained fully and the derivations presented with complete details. The computational aspects are emphasized throughout with design examples and problems taken from several fields of engineering to make the subject appealing to all branches of engineering. A large number of solved examples, review questions, problems, project-type problems, figures, and references are included to enhance the presentation of the material.

Specific features of the book include:

- More than 155 illustrative examples accompanying most topics.
- More than 540 references to the literature of engineering optimization theory and applications.
- More than 485 review questions to help students in reviewing and testing their understanding of the text material.
- More than 600 problems, with solutions to most problems in the instructor's manual.
- More than 12 examples to illustrate the use of Matlab for the numerical solution of optimization problems.
- Answers to review questions at the web site of the book, http://www.wiley.com/rao.
- Answers to selected problems are given at the end of the book.

I used different parts of the book to teach optimum design and engineering optimization courses at the junior/senior level as well as first-year-graduate-level at Indian Institute of Technology, Kanpur, India; Purdue University, West Lafayette, Indiana; and University of Miami, Coral Gables, Florida. At University of Miami, I cover Chapter 1 and parts of Chapters 2, 3, 5, 6, 7, and 13 in a dual-level course entitled *Optimization in Design*. In this course, a design project is also assigned to each student in which the student identifies, formulates, and solves a practical engineering problem of his/her interest by applying or modifying an optimization technique. This design project gives the student a feeling for ways that optimization methods work in practice. In addition, I teach a graduate level course titled *Mechanical System Optimization* in which I cover Chapters 1–7, and parts of Chapters 9, 10, 11, 13, and 17. The book can also be used, with some supplementary material, for courses with different emphasis such as *Structural Optimization*, *System Optimization* and *Optimization Theory and Practice*. The relative simplicity with which the various topics are presented makes the book useful both to students and to practicing engineers for purposes of self-study. The book also serves as a reference source for different engineering optimization applications. Although the emphasis of the book is on engineering applications, it would also be useful to other areas, such as operations research and economics. A knowledge of matrix theory and differential calculus is assumed on the part of the reader.

Contents

The book consists of 17 chapters and 3 appendixes. Chapter 1 provides an introduction to engineering optimization and optimum design and an overview of optimization methods. The concepts of design space, constraint surfaces, and contours of objective function are introduced here. In addition, the formulation of various types of optimization problems is illustrated through a variety of examples taken from various fields of engineering. Chapter 2 reviews the essentials of differential calculus useful in finding the maxima and minima of functions of several variables. The methods of constrained variation and Lagrange multipliers are presented for solving problems with equality constraints. The Kuhn–Tucker conditions for inequality-constrained problems are given along with a discussion of convex programming problems.

Chapters 3 and 4 deal with the solution of linear programming problems. The characteristics of a general linear programming problem and the development of the simplex method of solution are given in Chapter 3. Some advanced topics in linear programming, such as the revised simplex method, duality theory, the decomposition principle, and post-optimality analysis, are discussed in Chapter 4. The extension of linear programming to solve quadratic programming problems is also considered in Chapter 4.

Chapters 5–7 deal with the solution of nonlinear programming problems. In Chapter 5, numerical methods of finding the optimum solution of a function of a single variable are given. Chapter 6 deals with the methods of unconstrained optimization. The algorithms for various zeroth-, first-, and second-order techniques are discussed along with their computational aspects. Chapter 7 is concerned with the solution of nonlinear optimization problems in the presence of inequality and equality constraints. Both the direct and indirect methods of optimization are discussed. The methods presented in this chapter can be treated as the most general techniques for the solution of any optimization problem.

Chapter 8 presents the techniques of geometric programming. The solution techniques for problems of mixed inequality constraints and complementary geometric programming are also considered. In Chapter 9, computational procedures for solving discrete and continuous dynamic programming problems are presented. The problem of dimensionality is also discussed. Chapter 10 introduces integer programming and gives several algorithms for solving integer and discrete linear and nonlinear optimization problems. Chapter 11 reviews the basic probability theory and presents techniques of stochastic linear, nonlinear, and geometric programming. The theory and applications of calculus of variations, optimal control theory, and optimality criteria methods are discussed briefly in Chapter 12. Chapter 13 presents several modern methods of optimization including genetic algorithms, simulated annealing, particle swarm optimization, ant colony optimization, neural-network-based methods, and fuzzy system optimization. Chapter 14 deals with metaheuristic optimization algorithms and introduces nearly 20 algorithms with emphasis on Crow search, Firefly, Harmony search, Teaching-Learning and Honey bee swarm optimization algorithms. The practical aspects of optimization, including reduction of size of problem, fast reanalysis techniques and sensitivity of optimum solutions are discussed in Chapter 15. The multilevel and multiobjective optimization methods are covered in Chapter 16. Finally, Chapter 17 presents the solution of different types of optimization problems using the MATLAB software.

Appendix A presents the definitions and properties of convex and concave functions. A brief discussion of the computational aspects and some of the commercial optimization programs is given in Appendix B. Finally, Appendix C presents a brief introduction to Matlab, optimization toolbox, and use of MATLAB programs for the solution of optimization problems. Answers to selected problems are given after Appendix C.

Acknowledgment

I would like to acknowledge that the thousands of hours spent in writing/revising this book were actually the hours that would otherwise have been spent with my wife Kamala and other family members. My heartfelt gratitude is for Kamala's patience and sacrifices which could never be measured or quantified.

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About the Author

Dr. S.S. Rao is a Professor in the Department of Mechanical and Aerospace Engineering at University of Miami, Coral Gables, Florida. He was the Chairman of the Mechanical and Aerospace Engineering Department during 1998–2011 at University of Miami. Prior to that, he was a Professor in the School of Mechanical Engineering at Purdue University, West Lafayette, Indiana; Professor of Mechanical Engineering at San Diego State University, San Diego, California; and Indian Institute of Technology, Kanpur, India. He was a visiting research scientist for two years at NASA Langley Research Center, Hampton, Virginia.

Professor Rao is the author of eight textbooks: *The Finite Element Method in Engineering, Engineering Optimization, Mechanical Vibrations, Reliability-Based Design, Vibration of Continuous Systems, Reliability Engineering, Applied Numerical Methods for Engineers and Scientists, Optimization Methods: Theory and Applications*. He coedited a three-volume *Encyclopedia of Vibration*. He edited four volumes of *Proceedings of the ASME Design Automation and Vibration Conferences*. He has published over 200 journal papers in the areas of multiobjective optimization, structural dynamics and vibration, structural control, uncertainty modeling, analysis, design and optimization using probability, fuzzy, interval, evidence and grey system theories. Under his supervision, 34 PhD students have received their degrees. In addition, 12 Post-Doctoral researchers and scholars have conducted research under the guidance of Dr. Rao.

Professor Rao has received numerous awards for academic and research achievements. He was awarded the *Vepa Krishnamurti Gold Medal for University First Rank* in all the five years of the BE (Bachelor of Engineering) program among students of all branches of engineering in all the Engineering Colleges of Andhra University. He was awarded the *Lazarus Prize for University First Rank* among students of Mechanical Engineering in all the Engineering Colleges of Andhra University. He received the First Prize in James F. Lincoln Design Contest open for all MS and PhD students in USA and Canada for a paper he wrote on *Automated Optimization of Aircraft Wing Structures* from his PhD dissertation. He received the *Eliahu I. Jury Award for Excellence in Research* from the College of Engineering, University of Miami in 2002; was awarded the *Distinguished Probabilistic Methods Educator Award* from the Society of Automotive Engineers (SAE) International for *Demonstrated Excellence in Research Contributions in the Application of Probabilistic Methods to Diversified Fields, Including Aircraft Structures, Building Structures, Machine Tools, Airconditioning and Refrigeration Systems, and Mechanisms* in 1999; received the American Society of Mechanical Engineers *(ASME) Design Automation Award* for *Pioneering Contributions to Design Automation, particularly in Multiobjective Optimization, and Uncertainty Modeling, Analysis and Design Using Probability, Fuzzy, Interval, and Evidence Theories* in 2012; and was awarded the *ASME Worcester Reed Warner Medal* in 2013 for *Outstanding Contributions to the Permanent Literature of Engi-*

neering, particularly for his Many Highly Popular Books and Numerous Trendsetting Research Papers. Dr. Rao received the *Albert Nelson Marquis Lifetime Achievement Award* for demonstrated unwavering excellence in the field of Mechanical Engineering in 2018. In 2019, the American Society of Mechanical Engineers presented him the J.P. Den Hartog Award for his Lifetime Achievements in research, teaching and practice of Vibration Engineering.

1

Introduction to Optimization

1.1 INTRODUCTION

Optimization is the act of obtaining the best result under given circumstances. In design, construction, and maintenance of any engineering system, engineers have to take many technological and managerial decisions at several stages. The ultimate goal of all such decisions is either to minimize the effort required or to maximize the desired benefit. Since the effort required or the benefit desired in any practical situation can be expressed as a function of certain decision variables, *optimization* can be defined as the process of finding the conditions that give the maximum or minimum value of a function. It can be seen from Figure 1.1 that if a point x^* corresponds to the minimum value of function $f(x)$, the same point also corresponds to the maximum value of the negative of the function, $-f(x)$. Thus, without loss of generality, optimization can be taken to mean minimization, since the maximum of a function can be found by seeking the minimum of the negative of the same function.

In addition, the following operations on the objective function will not change the optimum solution x^* (see Figure 1.2):

1. Multiplication (or division) of $f(x)$ by a positive constant c.
2. Addition (or subtraction) of a positive constant c to (or from) $f(x)$.

There is no single method available for solving all optimization problems efficiently. Hence a number of optimization methods have been developed for solving different types of optimization problems. The optimum seeking methods are also known as *mathematical programming techniques* and are generally studied as a part of operations research. *Operations research* is a branch of mathematics concerned with the application of scientific methods and techniques to decision-making problems and with establishing the best or optimal solutions. The beginnings of the subject of operations research can be traced to the early period of World War II. During the war, the British military faced the problem of allocating very scarce and limited resources (such as fighter airplanes, radar, and submarines) to several activities (deployment to numerous targets and destinations). Because there were no systematic methods available to solve resource allocation problems, the military called upon a team of mathematicians to develop methods for solving the problem in a scientific manner. The methods developed by the team were instrumental in the winning of the Air Battle by Britain. These methods, such as linear programming (LP), which were developed as a result of research on (military) operations, subsequently became known as the methods of operations research.

In recent years several new optimization methods that do not fall in the area of traditional mathematical programming have been and are being developed. Most of

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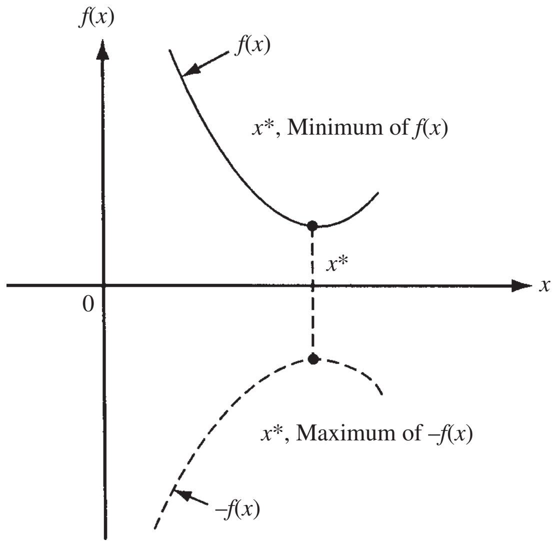

Figure 1.1 Minimum of $f(x)$ is same as maximum of $-f(x)$.

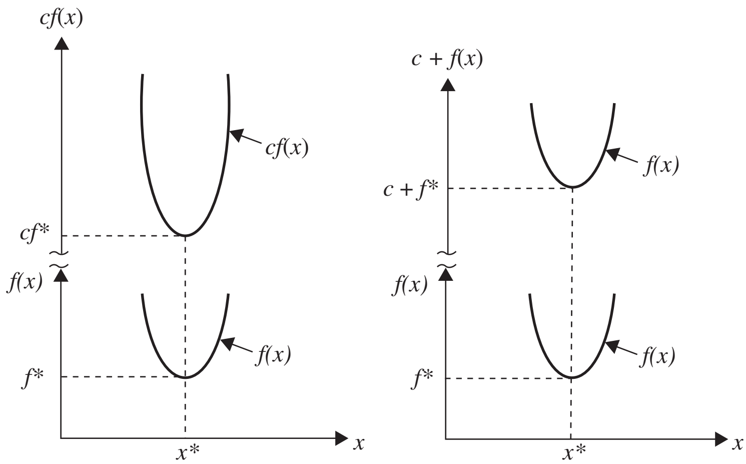

Figure 1.2 Optimum solution of $cf(x)$ or $c + f(x)$ same as that of $f(x)$.

these new methods can be labeled as metaheuristic optimization methods. All the metaheuristic optimization methods have the following features: (i) they use stochastic or probabilistic ideas in various steps; (ii) they are intuitive or trial and error based, or heuristic in nature; (iii) they all use strategies that imitate the behavior or characteristics of some species such as bees, bats, birds, cuckoos, and fireflies; (iv) they all tend to find the global optimum solution; and (v) they are most likely to find an optimum solution, but not necessarily all the time.

Table 1.1 lists various mathematical programming techniques together with other well-defined areas of operations research, including the new class of methods termed metaheuristic optimization methods. The classification given in Table 1.1 is not unique; it is given mainly for convenience.

Mathematical programming techniques are useful in finding the minimum of a function of several variables under a prescribed set of constraints. Stochastic process techniques can be used to analyze problems described by a set of random variables

Table 1.1 Methods of Operations Research.

Mathematical programming or optimization techniques	Stochastic process techniques	Statistical methods
Calculus methods	Statistical decision theory	Regression analysis
Calculus of variations	Markov processes	Cluster analysis, pattern recognition
Nonlinear programming	Queueing theory	Design of experiments
Geometric programming	Renewal theory	Discriminate analysis (factor analysis)
Quadratic programming	Simulation methods	
Linear programming	Reliability theory	
Dynamic programming		
Integer programming		
Stochastic programming		
Separable programming		
Multiobjective programming		
Network methods: Critical Path Method (CPM) and Program (Project) Management and Review Technique (PERT)		
Game theory		
Modern or nontraditional optimization techniques (including Metaheuristic optimization methods)		
Genetic algorithms	Bat algorithm	Salp swarm algorithm
Simulated annealing	Honey Bee algorithm	Cuckoo algorithm
Ant colony optimization	Crow search algorithm	Water evaporation algorithm
Particle swarm optimization	Firefly algorithm	Passing vehicle search algorithm
Tabu search method	Harmony search algorithm	Runner-root algorithm
	Teaching-learning algorithm	Artificial immune system algorithm
	Fruitfly algorithm	Neural network-based optimization
		Fuzzy optimization

having known probability distributions. Statistical methods enable one to analyze the experimental data and build empirical models to obtain the most accurate representation of the physical situation. This book deals with the theory and application of mathematical programming techniques suitable for the solution of engineering design problems. A separate chapter is devoted to the metaheuristic optimization methods.

1.2 HISTORICAL DEVELOPMENT

The existence of optimization methods can be traced to the days of Newton, Lagrange, and Cauchy. The development of differential calculus methods of optimization was possible because of the contributions of Newton and Leibnitz to calculus. The foundations of calculus of variations, which deals with the minimization of functionals, were laid by Bernoulli, Euler, Lagrange, and Weierstrass. The method of optimization for constrained problems, which involves the addition of unknown multipliers, became known by the name of its inventor, Lagrange. Cauchy made the first application of the steepest descent method to solve unconstrained minimization problems. Despite these early contributions, very little progress was made until the