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Contents

Multi-scale Simulation of Elastic Waves Containing Higher Harmonics ........................................ 1
Ambuj Sharma, Sandeep Kumar and Amit Tyagi

Effect of Skewness on Random Frequency Responses of Sandwich Plates .................................. 13
R. R. Kumar, Vaishali, K. M. Pandey and S. Dey

Design and Simulation of 3-DoF Strain Gauge Force Transducer ........ 21
Ankur Jaiswal, H. P. Jawale and Kshitij Shrivastava

A Micromechanical Study of Fibre-Reinforced Composites with Uncertainty Quantification and Statically Equivalent Random Fibre Distribution ........................................... 37
S. Koley, P. M. Mohite and C. S. Upadhyay

Free Vibration Analysis of Laminated Composite Plates and Shells Subjected to Concentrated Mass at the Centre ............... 49
Arpita Mandal, Chaitali Ray and Salil Haldar

Buckling Analysis of Thick Isotropic Shear Deformable Beams ....... 59
Kedar S. Pakhare, Rameshchandra P. Shimpi and P. J. Guruprasad

Spectral Finite Element for Dynamic Analysis of Piezoelectric Laminated Composite Beams ....................... 67
Namita Nanda

Determination of Interlaminar Stress Components in a Pretwisted Composite Strip by VAM ............. 81
Santosh B. Salunkhe and P. J. Guruprasad

A Study on Wrinkling Characteristics of NBR Material .......... 109
Vaibhav S. Pawar, Rajkumar S. Pant and P. J. Guruprasad
<table>
<thead>
<tr>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Ply Failure Study of Laminated Composite Conoidal Shells</td>
<td>119</td>
</tr>
<tr>
<td>Using Geometrically Nonlinear Formulation</td>
<td></td>
</tr>
<tr>
<td>Kaustav Bakshi and Dipankar Chakravorty</td>
<td></td>
</tr>
<tr>
<td>Analysis of Transformed Sixth-Order Polynomial for the Contraction</td>
<td>133</td>
</tr>
<tr>
<td>Wall Profile by Using OpenFOAM</td>
<td></td>
</tr>
<tr>
<td>R. Lakshman and Ranjan Basak</td>
<td></td>
</tr>
<tr>
<td>Fatigue Life Assessment of an Existing Railway Bridge in India</td>
<td>145</td>
</tr>
<tr>
<td>Incorporating Uncertainty</td>
<td></td>
</tr>
<tr>
<td>Mrinal Chanda, Kishore Chandra Misra and Soumya Bhattacharjya</td>
<td></td>
</tr>
<tr>
<td>Numerical Simulation of Acoustic Emission Waveforms Generated</td>
<td>155</td>
</tr>
<tr>
<td>by Tension and Shear Cracks in RCC Beams</td>
<td></td>
</tr>
<tr>
<td>Arun Roy, Paresh Mirgal and Sauvik Banerjee</td>
<td></td>
</tr>
<tr>
<td>Applicability of Tricycle Modelling in the Simulation of Aircraft</td>
<td>171</td>
</tr>
<tr>
<td>Steering System</td>
<td></td>
</tr>
<tr>
<td>S. Sathish, L. Suryanarayanan, J. Jaidev Vyas and G. Balamurugan</td>
<td></td>
</tr>
<tr>
<td>Free Vibration and Stress Analysis of Laminated Box Beam</td>
<td>185</td>
</tr>
<tr>
<td>with and Without Cut-Off</td>
<td></td>
</tr>
<tr>
<td>Raj B. Bharati, Prashanta K. Mahato, E. Carrera, M. Filippi and A. Pagani</td>
<td></td>
</tr>
<tr>
<td>Free Vibration Analysis of the Functionally Graded Porous Circular</td>
<td>197</td>
</tr>
<tr>
<td>Arches in the Thermal Environment</td>
<td></td>
</tr>
<tr>
<td>Mohammad Amir and Mohammad Talha</td>
<td></td>
</tr>
<tr>
<td>Vibration Response of Shear Deformable Gradient Plate</td>
<td>209</td>
</tr>
<tr>
<td>with Geometric Imperfection</td>
<td></td>
</tr>
<tr>
<td>Ankit Gupta and Mohammad Talha</td>
<td></td>
</tr>
<tr>
<td>Characterization of 2D Nanomaterials for Energy Storage</td>
<td>221</td>
</tr>
<tr>
<td>Akarsh Verma and Avinash Parashar</td>
<td></td>
</tr>
<tr>
<td>Cold Expansion of Elongated Hole: A Realistic Finite Element</td>
<td>227</td>
</tr>
<tr>
<td>Simulation</td>
<td></td>
</tr>
<tr>
<td>S. Anil Kumar and N. C. Mahendra Babu</td>
<td></td>
</tr>
<tr>
<td>Effect of Module on Wear Reduction in High Contact Ratio Spur Gears Drive Through Optimized Fillet Stress</td>
<td>239</td>
</tr>
<tr>
<td>Gears Drive Through Optimized Fillet Stress</td>
<td></td>
</tr>
<tr>
<td>R. Ravivarman, K. Palaniradja and R. Prabhu Sekar</td>
<td></td>
</tr>
<tr>
<td>Force Estimation on a Clamped Plate Using a Deterministic–Stochastic Approach</td>
<td>251</td>
</tr>
<tr>
<td>Akash Shrivastava and Amiya R. Mohanty</td>
<td></td>
</tr>
</tbody>
</table>
Dynamic Analysis of Composite Cylinders Using 3-D Degenerated Shell Elements ............................................ 261
Pratik Tiwari, Dipak Kumar Maiti and Damodar Maity

A New Hybrid Unified Particle Swarm Optimization Technique for Damage Assessment from Changes of Vibration Responses ........ 277
Swarup K. Barman, Dipak Kumar Maiti and Damodar Maity

Semi-active Control of a Three-Storey Building Structure ............ 297
P. Chaudhuri, Damodar Maity and Dipak Kumar Maiti

A Direction-Based Exponential Crossover Operator for Real-Coded Genetic Algorithm ................................................. 311
Amit Kumar Das and Dilip Kumar Pratihar

Axial Deformation Characteristics of Graphene-Sonicated Vinyl Ester Nanocomposites Subjected to High Rate of Loading ................ 325
B. Pramanik, P. R. Mantena and A. M. Rajendran

State Estimation Using Filtering Methods Applied for Aircraft Landing Maneuver ......................................................... 339
P. S. Suresh, Niranjan K. Sura and K. Shankar

Numerical Solution of Steady Incompressible Flow in a Lid-Driven Cavity Using Alternating Direction Implicit Method ............. 353
Banamali Dalai and Manas Kumar Laha

Stagnation and Static Property Correlations for Equilibrium Flows ................................................................. 365
Shubham Maurya and Aravind Vaidyanathan

CFD Simulation of Hypersonic Shock Tunnel Nozzle .................. 381
Jigarkumar Sura

A DNS Study of Bulk Flow Characteristics of a Transient Diabatic Plume that Simulates Cloud Flow .................................. 387
Samrat Rao, G. R. Vybhav, P. Prasanth, S. M. Deshpande and R. Narasimha

Transverse-Only Vibrations of a Rigid Square Cylinder ............... 397
Subhankar Sen

Steady Flow Past Two Square Cylinders in Tandem ..................... 407
Deepak Kumar, Kumar Sourav and Subhankar Sen

A Robust and Accurate Convective-Pressure-Split Approximate Riemann Solver for Computation of Compressible High Speed Flows ................................................................. 415
Sangeeth Simon and J. C. Mandal
Numerical Investigation of Flow Through a Rotating, Annular, Variable-Area Duct ........................................ 425
Palak Saini, Sagar Saroha, Shrish Shukla and Sawan S. Sinha

Development of M–DSMC Numerical Algorithm for Hypersonic Flows ........................................ 437
G. Malaikannan and Rakesh Kumar

Fluid–Structure Interaction Dynamics of a Flexible Foil in Low Reynolds Number Flows ....................... 449
Chandan Bose, Sunetra Sarkar and Sayan Gupta
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Multi-scale Simulation of Elastic Waves Containing Higher Harmonics

Ambuj Sharma, Sandeep Kumar and Amit Tyagi

Abstract  Numerical simulation of wave propagation is essential to understand the physical phenomenon of the wide variety of practical problems. However, the requirement of minimum grid point density per wavelength limits the computational stability, convergence, and accuracy of simulation of engineering application by numerical method. The purpose of this paper is to provide an improved framework for simulation of linear and nonlinear elastic wave propagation and guided-wave-based damage identification techniques feasible in the context of online structural health monitoring (SHM). Nonstandard wavelet-based multi-scale operator developed by using finite element discretization is used to represent waves. The proposed masking eliminates the requirement of a very large number of nodes in finite element method necessary for the propagation of such waves. The method is also useful in the situation where higher harmonics of propagating waves are ignored due to very high computational cost. The wavelet-based finite element scheme achieves an excellent numerical simulation and expresses an applicability for the guided waves’ study.

Keywords  Nonstandard wavelet operator · Structural health monitoring · Multi-scale simulation · Higher harmonics · Lamb waves

1 Introduction

Wave propagation can be characterized by the localized region of the sharp gradient of field variable which changes its locations in space with time. This gives permission to recognize the unusual nature that could be suitable for ultrasonic nondestructive
testing techniques. Guided-wave-based nondestructive techniques offer to evaluate the integrity of critical structures and to find out damage position, shape, and size [1, 2]. Several numerical techniques have been proposed to analyze the wave equations. Due to their relatively simple mathematical expressions and the possibility to be applied to the very large class of engineering problems, finite difference [3], boundary element [4], and finite element [2] based methods have been used by many authors for the simulation of guided Lamb waves. Finite element method (FEM) [2, 5] is a widespread numerical method used to simulate elastic guided wave propagation problem. Finite difference method (FDM) has also been used for the study of wave simulation and damage interaction by several researchers. Although FDM schemes are well situated for wave propagation in homogeneous media, however, the major limitation of the FDM schemes is that stiffness jumps due to continuously changing physical properties cause stability problems [6]. Furthermore, boundaries as well as discontinuities between different types of media lead to fairly accurate solutions and can generate severe errors [7]. With this in mind, more recently, Delsanto et al. have proposed the local interaction simulation approach (LISA) in combination with the sharp interface model [7]. Recently, customized elements and geometric multi-scale finite element method have been introduced to analyze various types of wave propagation problems [8]. The finite element method, which has been preferred for elastic wave propagation, is not suitable to simulate nonlinear waves or higher harmonics of propagating waves. A drastic increase of nodes for the simulation of nonlinear wave problems demands some necessary alteration in FEM which must be numerically efficient and straightforward.

In recent years, wavelet-based numerical methods gain much attention for solving partial differential equations. The major advantage of this approach allows one to examine a problem in different resolutions, simultaneously. In addition, wavelet-based schemes are efficient in problems comprising singularities and sharp transitions in solutions for limited zones of a computation domain. Initially, Beylkin et al. [9] employed the study of numerical computation based on wavelet. Several mathematicians and scientists have established the superiority of wavelet-based methods for solving elliptic partial differential equations [10, 11]. The adaptivity of wavelets is one of the leading advantages for the implementation of wavelets in numerical analysis [12, 13]. Liandrat and Tchamitchian have solved regularized 1D Burgers’ equation by using spatial wavelet approximation [14]. Later, Beylkin and Keiser [15], Vasilyev and Bowman [16], and Kumar and Mehra [17] have developed different wavelets-based algorithms and tested on 1D Burgers’ and advection–diffusion equations. Researchers have increased the usage of wavelets for solution of partial differential equations (PDEs) after the development of the lifting scheme by Swelden [17] and stable completion by Carnicer et al. [18]. A review of wavelet techniques for the solution of PDEs has been presented by Dahmen [19]. However, a very few researchers have applied the wavelet-based method for analyzing wave propagation problem. Hong and Kennett proposed wavelet-based method for the numerical modeling and simulation of elastic wave propagation in 2D media [20]. Chen and Yang et al. presented the wave motion analysis of short wave in one-dimensional structures [21]. Mitra and Gopalakrishnan proposed wavelet-based
spectral finite element method (WSFEM) for simulation of elastic wave propagation in one- and two-dimensional situations [22]. In the literature, some researchers have used wavelets as basis function to solve PDEs but most researchers have applied the wavelet-based adaptive technique in finite difference schemes. These papers have presented the adaptive method for propagation of a single wave, but there is a need for different algorithms for more than one waves propagating with different velocities. Generation of higher harmonics due to material nonlinearity is not addressed in these papers.

Multi-scale modeling is one possible solution for higher harmonics in wave propagation simulation. Wavelet-based multi-scale method leads to fast and locally adaptive algorithms. The compactly supported refinable basis functions are main potential advantage of the wavelet [10, 11]. However, these methods are unable to compete with conventional finite element method. In this paper, proposed technique is inspired by the interesting paper by Krysl et al. [23].

This paper presents multi-scale adaptive approach for solving the wave propagation problem. In the proposed wavelet-based technique, FEM is preferred due to its capability to handle complex boundary and loading conditions instead of any other methods. This multi-scale transformation hierarchically filters out the less significant frequencies and offers an operative framework to retain the necessary frequencies of the wave. In this procedure, the finest level of the coefficient matrix is calculated once for the whole domain while the adaptively compressed coefficient matrix, which is very small compared to complete coefficient matrix, is used in every marching step of the solution. This paper is presenting wavelet-based nonstandard operator to improve finite element simulation of linear and nonlinear wave propagation in a large structure. We use nonstandard operator because it is more efficient than standard operator [24]. This will not only be useful to the structural health monitoring, but it can also be used where waves with higher harmonics move at different group velocities. A simple description of the nonstandard operator along with necessary algorithm and mathematical comments is provided to remove an execution headache connected with adaptive grid techniques. The algorithm is applied to 2D plane strain problem, but it is general and independent of domain dimensions.

2 Mathematical Formulation

2.1 Lamb Waves

In an elastic medium, elastic waves are defined as propagating disturbances that transport energy without any material transfer. Elastic waves of plane strain that exist in free plates are called Lamb waves. For an orthotropic and symmetrical plate, particle motion is often outlined through the elemental elastodynamic differential equation of wave
\[ \partial_t (C_{klmn} \partial_n w_m) = \rho \partial^2_t w_k, \quad (k, l, m, n = 1, 2). \]  

(1)

Substituting stress relation in governing equations and Lamb wave can be expressed as

\[
C_L^2 \frac{\partial^2 u}{\partial x^2} + (C_L^2 - C_T^2) \frac{\partial^2 v}{\partial x \partial y} + C_T^2 \frac{\partial^2 u}{\partial y^2} + f_x = \frac{\partial^2 u}{\partial t^2},
\]

(2(a))

\[
C_L^2 \frac{\partial^2 v}{\partial y^2} + (C_L^2 - C_T^2) \frac{\partial^2 u}{\partial x \partial y} + C_T^2 \frac{\partial^2 v}{\partial x^2} + f_y = \frac{\partial^2 v}{\partial t^2}.
\]

(2(b))

\[ C_L^2 = \frac{\lambda + 2\mu}{\rho} \text{ and } C_T^2 = \frac{\mu}{\rho} \] are longitudinal velocity and shear velocity, respectively, where \( \lambda = \frac{E \nu}{(1+\nu)(1-2\nu)} \) and \( \mu = \frac{E}{2(1+\nu)} \) are Lamé constants, \( E \) is Young’s modulus, and \( \nu \) is Poisson ratio. The 2D plane strain problem is discretized into the set of finite element equations as

\[
[K][u] + [M][\ddot{u}] = 0,
\]

(3)

where \([u]\) and \([\ddot{u}]\) are unknown coefficient vectors. \([K]\) and \([M]\) are global stiffness and mass matrix, respectively.

### 2.2 Multi-scaling Using Wavelets

The idea of multi-scale exploration is to interpolate an unknown field at a coarse level with the assistance of supposed scaling functions. Any enhancement to initial approximation is accomplished by adding “details” rendered by basis functions referred to as wavelets. A multi-scaling analysis forms a sequence of closed subspaces to satisfy certain self-similarity relations as well as completeness and regularity relations. The basis functions in \( W_j \) are called wavelet functions. Wavelet functions are symbolized by \( \psi_{j,k} \). These scaling and wavelet functions are employed for wavelet-based multi-scaling. A function \( f \in L^2(\mathbb{R}) \) is approximated by its projection \( P^j f \) onto the space \( V_j \) and the projection of \( f \) on \( W_j \) as \( Q^j f \), we have

\[
P^j f = P^{j-1} f + Q^{j-1} f.
\]

(4)

If the coefficient vector of \( P^j f \) (or coefficients of scaling functions) is \( C_j = \{C_{j,0}, \ldots, C_{j,v(j)}\}^T \) and coefficient vector of \( Q^j f \) (or coefficients of some wavelets) is \( d_j = \{d_{j,0}, \ldots, d_{j,w(j)}\}^T \), then we can write wavelet transform as

\[
C_j = [T_j] \begin{bmatrix} C_{j-1} \\ d_{j-1} \end{bmatrix}.
\]

(5)
The matrix $[T_j]$ is used to achieve next higher level by transforming scaling and detail coefficients of $V_{j-1}$ and $W_{j-1}$ spaces, respectively. In this paper, B-spline wavelet and Daubechies (D4) wavelet [25] are used for wave propagation.

### 2.3 Nonstandard Multi-scale Decomposition of Finite Element Matrix

Two observations can be made while solving some PDEs using the wavelet bases: (i) In theoretical terms, most of the available wavelet methods have stable Riesz basis and better condition number than FEM or FDM. (ii) But in practical applications, wavelet methods are not yet ready to compete with the traditional FEM approach. One important reason is while the FEM can always produce a sparse matrix with more regular sparsity patterns, use of wavelet bases does not produce such sparse matrices. But the combination of wavelets with other methods, such as FDM, FEM, and recently SEM [22], show good results. Here, we have used FEM discretization to derive a sparse matrix, as the FEM remains the most versatile tool to solve PDEs.

Let us consider a continuous wave field $u(x, y)$ and $v(x, y)$ for a source of excitation over 2D homogeneous medium. The approximation of the continuous wave field on the discrete domain is denoted by $u_j$ and $v_j$. It represents the discrete wave field that is obtained with a classic time–space finite element method for a sufficiently fine discretization of $V_j \subset R^2$. The 2D wavelet transform cascades projections of the discrete wave field over different approximation grids $V_1, V_2, V_3, \ldots, V_j$ of increasing resolution.

In this multi-scale algorithm, we used NS operator proposed by Beylkin [26]. To the best of authors’ knowledge, no one researcher has used NS operator in wavelet–FEM coupling or wavelet–FDM coupling. It has been proved by Beylkin [26, 27] that NS operator is more efficient than the standard form of operator used by most of the researchers. In this paper, we have used NS operator in two-dimensional wavelet–finite element coupling technique. The finite element equations for the transient problem, Eq. 3, can be expressed in the expanded form as

$$
\begin{bmatrix}
  k_{uu}^j & k_{uv}^j \\
  k_{vu}^j & k_{vv}^j
\end{bmatrix}
\begin{bmatrix}
  u^j \\
  v^j
\end{bmatrix} =
\begin{bmatrix}
  f_u^j \\
  f_v^j
\end{bmatrix}.
$$

We can apply the wavelet transformation on the field variables of both the directions:
\[
\begin{bmatrix}
[T^T] & [0] \\
[0] & [T^T]
\end{bmatrix}
\begin{bmatrix}
k_{uu}^j \\
k_{uv}^j \\
k_{vu}^j \\
k_{vv}^j
\end{bmatrix}
\begin{bmatrix}
[T] & [0] \\
[0] & [T]
\end{bmatrix}
\begin{bmatrix}
d^1 \\
u^1 \\
e^1 \\
v^1
\end{bmatrix}
= 
\begin{bmatrix}
[T^T] & [0] \\
[0] & [T^T]
\end{bmatrix}
\begin{bmatrix}
f_u^j \\
f_v^j
\end{bmatrix}.
\]
(7)

The organization of the matrix after three-level transform of nonstandard form can be extended and expressed in the new notations as [24]

\[
\begin{align*}
K_{s}^{j-1} & \quad K_{b}^{j-1} \\
K_{c}^{j-1} & \quad & \\
K_{s}^{j-2} & \quad K_{b}^{j-2} \\
& \quad & \\
K_{c}^{j-2} & \quad & \\
& \quad & \\
K_{s}^{j-3} & \quad K_{b}^{j-3} \\
K_{c}^{j-3} & \quad & \\
& \quad & \\
D_{j+1} & \quad & U_{j+1} \\
& \quad & \\
D_{j+2} & \quad & U_{j+2} \\
& \quad & \\
D_{j+3} & \quad & U_{j+3}
\end{align*}
= 
\begin{align*}
G_{j-1} \\
F_{j-2} + G_{j-2} \\
F_{j-2} - G_{j-2} \\
F_{j-3} + G_{j-3} - F_{j-3} \\
G_{j-3} - G_{j-3} \\
F_{j-3} - F_{j-3}
\end{align*}
\]
(8)

In order to solve it, Gines et al. [24] proposed nonstandard LU decomposition.

3 Results and Discussion

Elastic waves have been employed for identification of damage in the thin wall structures such as plates and pipes [5]. Guided Lamb waves are excited in the structures through narrowband burst signals. In order to evaluate the performance of wavelet-based multi-scale method. We considered an example in which a 50 × 50 mm\(^2\) homogeneous, isotropic aluminum plate with a density of 2700 kg/m\(^3\). The simulation of Lamb wave in this plate with 400 kHz central frequency is presented in Fig. 1. Contour plots of the displacement in the x-direction at three different time instants are well depicted in this figure.

Nonlinear Lamb wave is more sensitive to small-scale damage identification. However, the investigations of the higher harmonics in propagating Lamb waves
Multi-scale Simulation of Elastic Waves Containing …

were ignored due to high computational cost. To see the efficiency of the wavelet-based method, higher harmonics are added in the Lamb wave and propagation of waves is observed. This study uses the following actuation function with 400 kHz central frequency:

$$E(t) = \begin{cases} f_o \sin(\Omega t) \ast (\sin(0.1 \Omega t))^2 + 0.1 f_o \sin(2 * 5 \Omega t) \ast (\sin(\Omega t))^2, & t < 10\pi / \Omega \\ 0, & \text{otherwise} \end{cases}$$

where $\Omega$ is the frequency of excitation and $E_o$ is the maximum amplitude. The excitation signal on a plate with a higher harmonics is shown in Fig. 2. An aluminum plate of 200 mm length and 2 mm thickness is used in the analysis. Poisson’s ratio =

**Fig. 1** Contour plots of the displacement in the $x$-direction at three different time frames for isotropic plate.
0.3, density $\rho = 2700$ kg/m$^3$, and Young’s modulus $E = 69$ GPa are assumed as material properties. The Lamb wave in this material has longitudinal velocity $C_L = 5299$ m/s and transverse velocity $C_T = 3135$ m/s. The waves are actuated by employing pin forces applied to the left boundary of the plate. The excitation forces are parallel to the longitudinal (propagating) direction. In-phase pin forces are applied to the top and bottom edge nodes of the plate for excitation of fundamental symmetric ($S_0$) modes, and the antisymmetric modes are propagated by imposing out-of-phase pin forces. In this paper, we considered the cases in which the pure $S_0$ mode is excited.

Ten cycles Hanning-window actuation is given through excitation function to deliver a limited cycle sinusoidal tone burst.

Higher frequency wave propagation problems demand enormous computer resources because of very large number of time integration steps and highly dense mesh. Generally, in the case of Lamb wave, 20 elements per wavelength are required but this is not sufficient for higher harmonic simulation. Figure 3 depicts the measured nodal displacement response of time-domain signals obtained using FEM simulation of the plate with 40, 80, and 120 elements per wavelength. It can be observed that higher harmonics are not properly visible in the response of the plate with 40 elements per wavelet. On the other hand, as shown in the same figure, higher harmonics are visible for 80 elements per wavelength.
In the present analysis, B-spline and Daubechies (D4) wavelet are used to establish robustness and sensitivity of wavelet-based wave propagation method. To capture higher harmonics in the plates, FEM uses 17,080 uniformly distributed nodes while half of the FEM nodes are required after application of one level of wavelet transform. Nodal displacement response of plate received from B-spline and D4 wavelet transform at level 1 along with FEM results is demonstrated in Fig. 4. It establishes good agreement between conventional finite element and proposed wavelet-based method. It can be observed that B-spline wavelet produces response close to FEM results, while there is some deviation in the results of D4 wavelet. Further, we examined wavelet-based method at various levels of wavelet transform to find the level up to which this method can work efficiently. These results show some attenuation but wavelets are not eliminating higher frequency components of waves which are important in many analyses.

Fig. 3 Comparison of response of plate for 40, 80, and 120 elements per wavelength
4 Conclusion

The spatial derivative operators in the wave equations are handled using multi-resolution transforms in a physical domain. We presented a wavelet-based framework to reduce the size of global stiffness matrix of finite element analysis which is becoming too large in the case of nonlinear wave propagation problem. Wavelet-based method is not only to develop the compressed stiffness matrix, but also to propagate higher harmonics of waves using least number of nodes and able to reduce the computational cost significantly. Without disturbing the programming advantages of FE regarding the implementation of boundary conditions and efficient numerical integration of interpolation functions, wavelet-based methods are able to reduce the size of matrix as much as one by a sixteenth of original FE matrix. These fundamental characteristics show that the wavelet-based method can be utilized for more complex wave propagation problems.
References

Effect of Skewness on Random Frequency Responses of Sandwich Plates

R. R. Kumar, Vaishali, K. M. Pandey and S. Dey

Abstract This study presents the effect of skewness in natural frequency responses of sandwich plates. The free vibration analysis is carried out by using higher order zigzag theory (HOZT) considering random input parameters. It satisfies the transverse shear stress continuity condition and the transverse flexibility effect. The in-plane displacement throughout the thickness is assumed to vary cubically while transverse displacement is considered to vary quadratically within the core and constant at top and bottom plates. An efficient $C_0$ stochastic finite element approach is developed for the implementation of proposed plate theory in the random variable surrounding. Compound stochastic effect of all input parameters is presented for the different degrees of skewness in sandwich plates. Intensive Monte Carlo simulation (MCS) is employed for solving the stochastic-free vibration equations and statistical analysis is conducted for illustration of the results. The present algorithm for sandwich plate is validated with previous literatures and it is found to be in good agreement.

Keywords Monte Carlo simulation · Sandwich plate · Natural frequency · Higher order zigzag theory · Skewness

1 Introduction

A sandwich plate is a multilayered plate having two face sheets and a core embedded in between them through adhesive. The face sheets are relatively thin but of high strength and stiffness material, whereas the core is made up of relatively thick and lower density material. The high specific strength and stiffness of sandwich structures make them suitable for crucial engineering applications like automobile, civil construction, aerospace, and marine industries. Sandwich plates are widely used in design and construction of aerospace craft. In such application, these materials are subjected to wide environmental changes such as pressure, temperature, density, and
humidity. This inevitable change in surrounding affects the vibrational response of the structure. Therefore, it is essential to include the actual operating condition in order to get the changes in vibration characteristic of sandwich plate. The vibration response of the aerospace craft is usually carried out in atmospheric condition rather than actual unevenly varying condition for the cause of convenience. Thus, it is essential to consider the material and geometric uncertainty in order to accommodate above-mentioned environmental changes as well as other inaccuracies occurring during design and fabrication of the sandwich plate. The cost-effective sandwich panel requires sandwich core of low-cost material which exhibits better weight sensitivity as well. The development and automation in production processes make possible the production of low-cost sandwich panel. The sandwich panel is not considered for low-cost application due to insufficient knowledge about their cost-saving potential. The manufacturing of such sandwich structure always experiences spatial variability due to manufacturing imperfections and other inaccuracies. Moreover, dynamic behavior of sandwich structure possesses high statistical variation due to unavoidable skewness occurring during complex fabrication processes. Various interdependent parameters influencing the properties are core thickness, number of face sheet layer, face sheet and core material properties, and topology of core. Because of these parameters, uncertain responses can be seen and the system properties become inevitably random in nature. So to have a safe and realistic design, we must not neglect these inherent uncertainties. This cannot be obtained through usual deterministic approach. So, to incorporate the source uncertainties in design and analysis of the mechanical system, it is required to quantify the present uncertainties.

Recently, Grover et al. [1] worked on sandwich plate and studied the parametric uncertainties influencing the deflection statics and after that they also ensured the validity by comparing it with that of Monte Carlo simulation having finite element solution. Earlier, Aguib et al. [2] worked on magnetorheological elastomer core sandwich beam. The proposed structure was directly applied to civil engineering. Nayak et al. [3] studied the free vibration response on sandwich plates in damped random environment. Jin et al. [4] studied the natural frequency response by considering a viscoelastic core sandwich beam. For honeycomb sandwich beams, Debruyn et al. [5] analyzed the design parameters’ variability. The compressibility effect in transverse direction is studied using laminate mechanics by Plagianakos and Papadopoulos [6] and Liu [7] carried out the analytical study on sensitivity analysis for natural frequencies and their mode shapes. SFEM was furthermore studied by Gadade et al. [8]. The vibration characteristic was studied by Scarpa and Tomlinson [9] on regular hexagonal honeycomb cells and re-entrant auxetic honeycomb cells. Later, spectral finite element method was used by Ruzzene [10]. By using this method, we can accurately evaluate the acoustic properties of honeycomb. An optimized study of truss-core sandwich panel was done by Denli and Sun [11]. A similar study was also presented by Franco et al. [12]. With recent advancement in finite element software, for example, ABAQUS and ANSYS, the efficiency and accuracy have greatly increased. The study of stochastic natural frequency including the effect of noise was done by Dey
et al. [13]. Recently, stochastic analysis is carried out by Kumar et al. [14–19], Karsh et al. [20–26], and Mukhopadhyay et al. [27, 28]. Most of the research is carried out by using deterministic approach, whereas few researchers focused on stochastic approach.

Here, the effect of skewness (Fig. 1) on natural frequency response, having taken into consideration the compound variation of all input parameters, is studied. Thereafter, this paper is presented as: Theoretical formulation is described in Sect. 2, result and discussion are illustrated in Sect. 3, whereas conclusion and future scope are presented in Sect. 4.

2 Theoretical Formulation

The strain–displacement equation [29] can be shown as

$$\{\bar{\varepsilon}(\omega)\} = \left[ \begin{array}{c} \frac{\partial u(\omega)}{\partial x} \frac{\partial v(\omega)}{\partial y} \frac{\partial w(\omega)}{\partial z} + \frac{\partial v(\omega)}{\partial y} \frac{\partial u(\omega)}{\partial z} \\ + \frac{\partial w(\omega)}{\partial x} \frac{\partial v(\omega)}{\partial z} + \frac{\partial w(\omega)}{\partial x} \end{array} \right] \{a(\omega)\},$$  \hspace{1cm} (1)

i.e.,  \(\{\psi(\omega)\} = [a(\omega)]\{\psi(\omega)\}\),  

where \([a]\) is unit step function. The equation for generalized displacement vector is given as

$$\{s(\omega)\} = \sum_{k=1}^{n} \zeta_i(\omega)s_i(\omega),$$  \hspace{1cm} (2)

where \(\{s\} = \{U_0V_0W_0\theta_x\theta_yU_uW_uU_lV_l\}^T\). Equation (1) is used to give strain vector equation

$$\{\psi(\omega)\} = [a(\omega)]\{s(\omega)\}.$$  \hspace{1cm} (3)
The strain–displacement matrix can be represented as \([a]\). The dynamic equilibrium equation for natural frequency analysis is written by using Hamilton’s principle as
\[
[r(\varpi)]\dd s = \lambda^2 [m(\varpi)]s,
\]  
(4)

where \([r(\varpi)]\) is the random natural frequency. The global mass matrix \([m(\varpi)]\) is
\[
[m(\varpi)] = \sum_{k=1}^{n_u+n_l} \iiint \rho_k(\varpi)[j]^T[n][j]dx dy dz = \iiint [n]^T[k(\varpi)][n]dx dy,
\]  
(5)

where \(\rho_k(\varpi)\) is stochastic mass density of \(k\)th order, \([j]\) is of the order of 3X11, and \([n]\) is the shape function matrix. The equation for stiffness matrix \([k(\varpi)]\) is given as
\[
[k(\varpi)] = \sum_{k_l}^{n_u+n_l} \rho_k(\varpi)[j]^T[j]dz.
\]  
(6)

For storing the global stiffness in one array, we have used the skyline technique. For getting static solution, Gaussian decomposition scheme is used and for free vibration analysis simultaneous iteration technique is used.

### 3 Results and Discussion

Here, HOZT is applied to a sandwich plate (simply supported boundary condition) of length \((l) = 10\) cm, width \((b) = 10\) cm, and thickness \((t) = 1\) cm to demonstrate the proposed finite element (FE) model. The present model is having eight-layer symmetric cross-ply laminate having core thickness of 0.8 and face sheet thickness of 0.1 with equal layers on both sides of core. Here, the first, second, and third natural frequencies without any skewness are compared with 15°, 30°, 45°, and 60° skewed plates. The material properties considered for the present analysis are shown in Table 1.

Based on the present model, the natural frequencies for the first mode obtained for 30° and 45° skew angles and the results of Wang et al. [30] and Kulkarni and Kapuria [31] are tabulated in Table 2. Such a small deviation between various results obtained for natural frequencies is shown in Table 2, which can justify the accuracy and applicability of HOZT.

It is evident from Fig. 2 that with increase in skew angle, fundamental and third natural frequencies initially increase for \((\phi) = 15°\) and 30°, decrease for \((\phi) = 45°\), and then again increase up to maximum for \((\phi) = 60°\), whereas second natural frequency initially increases for \((\phi) = 15°\) and 30° and decreases for \((\phi) = 45°\) and 60°. The mean value of second natural frequency for \((\phi) = 45°\) and 60° remains...
Table 1  Material properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Core</th>
<th>Face sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$ (GPa)</td>
<td>0.5</td>
<td>38.6</td>
</tr>
<tr>
<td>$E_2 = E_3$ (GPa)</td>
<td>0.5</td>
<td>8.27</td>
</tr>
<tr>
<td>$G_{12} = G_{13}$ (GPa)</td>
<td>0.4</td>
<td>4.14</td>
</tr>
<tr>
<td>$G_{23}$ (GPa)</td>
<td>0.2</td>
<td>1.656</td>
</tr>
<tr>
<td>$\nu_{12} = \nu_{13} = \nu_{23} = \nu_{32}$</td>
<td>0.27</td>
<td>0.26</td>
</tr>
<tr>
<td>$\nu_{21} = \nu_{31}$</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td>$P$ (kg/m³)</td>
<td>1000</td>
<td>2600</td>
</tr>
</tbody>
</table>

Table 2  Natural frequency of (0°/90°/0°/90°) sandwich plate

<table>
<thead>
<tr>
<th>Skew angle ($\phi$) (°)</th>
<th>Present study</th>
<th>Wang et al. [30]</th>
<th>Kulkarni and Kapuria [31]</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>1.8889</td>
<td>1.9410</td>
<td>1.9209</td>
</tr>
<tr>
<td>45</td>
<td>2.5806</td>
<td>2.6652</td>
<td>2.6391</td>
</tr>
</tbody>
</table>

This almost same which lies in between $\phi = 15^\circ$ and $\phi = 30^\circ$. This corroborates the fact obtained vide probability density function (PDF) plots.

4 Conclusions

Based on higher order zigzag theory (HOZT), the accuracy and applicability of the proposed finite element model for free vibration analysis of sandwich plates are studied. The novelty of the present study includes the skewness effect on free vibration of sandwich plates. The natural frequency of sandwich plate is compared with that of skewed sandwich plate by means of probability density function (PDF) plots. The first, second, and third natural frequencies of unskewed sandwich plates are compared with plates having skewness of 15°, 30°, 45°, and 60°. It is observed that the unavoidable source uncertainties cause significant deviation of natural frequency from the mean deterministic value. Therefore, it is of utmost importance to consider the effect of skewness and source uncertainty in design and analysis of sandwich plate and other complex structures for safe and realistic design. Based on these observations, the present work can be extended to deal with more complex structures.
Fig. 2 Random natural frequency (rad/s) of sandwich plates for a first, b second, and c third natural frequencies with skew angle, $(\phi) = 0^\circ, 15^\circ, 30^\circ, 45^\circ,$ and $60^\circ$
Effect of Skewness on Random Frequency Responses …

Acknowledgements  The first and second authors would like to acknowledge the financial support received from MHRD GOI during this research work.

References


