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José Manoel Balthazar *Editor*

Vibration Engineering and Technology of Machinery

Proceedings of VETOMAC XV 2019

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Volume 95

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Preface

Machines of all kinds are used in nearly every aspect of our daily lives from the vacuum cleaner and washing machine we use at home to the industrial machinery used to manufacture nearly every product we use on a daily basis. On other hand, the research development on nonlinear dynamics, nowadays, continuously reveals that nonlinear phenomena can bring many amazing and advantageous effects in every practical machinery engineering problem, such as vibration control, energy harvesting, structure health monitoring, micro/nano-electro-mechanical systems, and so on. Recent trends in machinery vibration and technology including nonlinear systems and phenomena (cases studies) are the main strength of this book of interest to both researchers and practicing engineers.

Following the scientific tradition of the conference VETOMAC, its 15th edition is internationally recognized as a central forum for discussing scientific achievements and is intended to provide a widely selected forum among scientists and engineers to exchange methods, techniques, and ideas related to Vibration Engineering and Technology of Machinery problems

So, this book presents the most significant contributions to the VETOMAC 2019 Conference held in Hotel Nacional Inn, Curitiba, Paraná, Brazil from November 10 to 15, 2019, covering a range of Vibration Engineering and Technology of Machinery problems to provide insights into recent trends and advances in a broad variety of fields in dynamics and control.

VETOMAC 2019 was promoted by The Vibration Institute of India through its International Steering of JVET (Journal of Vibration Engineering & Technologies) and Local Committees at Curitiba, Brazil.

All the papers gathered here will be of interest to all researchers, graduate students, and engineering professionals working in the fields of Vibration Engineering and Technology of Machinery problems and related areas around the globe.

It should be emphasized that all the chapters have been reviewed by two independent referrers and authors are responsible for their opinions expressed in their work.

This book comprises 29 contributions from different countries.

The main keywords are vibration engineering, technology of machinery problems, nonlinear phenomena, and control design.

The chapters will be subdivided into four main areas:

- Concepts and methods in dynamics, containing seven sub-chapters,
- Dynamics of mechanical and structural system, containing five sub-chapters,
- Dynamics and control containing seven sub-chapters, and
- Recent and emergent trends in dynamics and control containing ten sub-chapters.

I would like to thank the authors, presenters, and session chairs for their participation. Special gratitude must be extended to several individuals whose invaluable help enabled the organization of the VETOMAC XV 2019 Conference in Hotel Nacional Inn, Curitiba, Paraná, Brazil.

Sincere gratitude is also expressed to the Steering Committee:
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I would like to give a special thanks to Ms. Nathalie Jacobs for her help and encouragements to the publication of this volume.

Finally, the history of VETOMAC Series of Conferences (in collaboration with the Vibration Institute of India) can be summarized as follows:

- VETOMAC I, 2000: Indian Institute of Science, Bangalore, India;
- VETOMAC II, 2002: Bhabha Atomic Research Centre, Mumbai, India;
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- VETOMAC IV, 2007: University College of Engineering, Osmania University, Hyderabad, India, Bharat Heavy Electricals Limited, Hyderabad, India;
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- VETOMAC XII, 2016, Organized by Institute of Fluid Flow Machinery, Polish Academy of Sciences, Gdansk and Air Force Institute of Technology, Warsaw, Poland;
- VETOMAC XIII, 2017, Organization 12th World Congress on Engineering ASSET Management and 13th International Conference on Vibration Engineering and Technology of Machinery Brisbane, Queensland, Australia;
- VETOMAC XIV, 2018, organized by Faculdade de Ciências e Tecnologia of Universidade Nova de Lisboa (DEC/FCT/UNL) and IDMEC—Institute of Engineering Mechanics of Instituto Superior Técnico of University of Lisbon (IDMEC/IST/UL); and
- VETOMAC XV, 2019, organized by Universidade Tecnológica Federal do Paraná, Câmpus Ponta Grossa, Paraná/Brazil.

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Dynamics and Control

Concepts and Methods in Dynamics

Stabilization of Chaos Via Strong Nonlinearities: The Lorenz-Malkus Wheel Under Coulomb and Hystersis Frictions

Mikhail E. Semenov, Evgeny A. Karpov, Sergey G. Tikhomirov, Peter A. Meleshenko, and Margarita Teplyakova

Abstract In this chapter we consider the modified Lorenz-Malkus water wheel model. Within a novel approach we take into account friction features on the rim of the water wheel formalized by strong nonlinearities. Namely, the dry friction (within the Coulomb model) and hysteresis friction (within the Bouc-Wen model and the Dahl model) are considered. The dynamic characteristics such as fixed points, Lyapunov characteristic exponents, bifurcation diagrams, are presented and discussed. Detailed analysis of a 2-dimensional Lorenz-Malkus system (where the third coordinate is supposed to be constant) is also presented and discussed. Namely we show the bifurcation process where two saddles and stable node birth from a saddle. It is shown that the static friction (formalized within the Coulomb model) leads to stabilization of the system at the origin independent on the value of the friction coef-

This chapter is an extension of the work "Chaos vs Hysteresis: water wheel under dry friction" presented on VETOMAC-2019, Curitiba, Brazil.

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ficient. At the same time we show that using certain parameters within the Bouc-Wen and Dahl models, chaotic behaviour can be controlled. A way to use the modified Lorenz-Malkus system with certain parameters as a “natural” pseudo-random numbers generator is also discussed.

Keywords Chaos · Lorenz system · Coulomb friction · Hysteresis · Bouc-wen model · Dahl model · Lyapunov characteristic exponents · Bifurcation diagrams

1 Introduction

1.1 The Lorenz System

In 1963 American researcher Edward Lorenz engaged with weather forecasting problems and published famous paper 3 [1] “Deterministic Nonperiodic Flow” in the Journal of the Atmospheric Sciences where he showed that relatively simple system of three ordinary differential equations (which is well-known now as the “Lorenz system”), which was obtained during the analysis of the convection process in a fluid layer, demonstrated unexpected behaviour. This research was a starting point for such a modern field as the chaos theory.

The Lorenz system

$$\begin{cases} \dot{x} = \sigma(y - x), \\ \dot{y} = x(r - z) - y, \\ \dot{z} = xy - bz, \end{cases} \tag{1}$$

includes variables x, y and z that have the following sense (in terms of the convection problem):

- x is proportional to the rate of convection;
- y is proportional to the horizontal temperature variation;
- z is proportional to the vertical temperature variation.

Parameters σ, r and b are proportional to the Prandtl number, the Rayleigh number, and the coefficient corresponding to the geometry of the convective region, respectively.

During numerical simulations, Lorenz obtained some unexpected feature of system (1). Such a feature plays a special role in the following investigation of the system and reflects the high sensitivity of the solution to small deviations of initial conditions. Using the standard iterative procedure it was shown that the sensitivity to small deviations of initial conditions leads to the divergence of the initially close solutions. In our days this feature[1] is called the dependence of the solution on the initial conditions (Fig. 1).

[1]Note, that it can be served as a main sign of the chaotic behaviour.

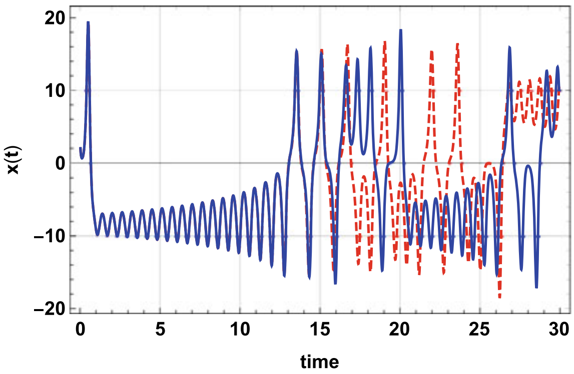

Fig. 1 A solution to system (1). Dependence of variable x on the time for two systems with close initial conditions: initial condition for red curve is $(2.01, -2, 2)$ and for blue curve is $(2, -2, 2)$

For many years researchers detected chaotic behavior in different physical models such as lasers [2], dissipative oscillator with inertial excitation [3], chemical reactions [4], and others. Models of these systems can be described in terms of the Lorenz system (1). Recent works [5, 6] are dedicated to modifications of the Lorenz system where ordinary derivatives were replaced by fractional derivatives (the so-called fractional Lorenz system). Particularly, in [5] athors reported a new algorithm of calculus for fractional derivatives, moreover they obtained that chaos in the fractional-order system exists with order lower than 2.97. Novel results in chaos control and system synchronization together with the corresponding analytical results were presented and disccused in [6], where the Malkus water wheel model (which is also described by the Lorenz system) was considered in terms of the fractional calculus. Attracting regions of the Lorenz system are also important charcteristics of chaotic systems and have to be examined in details as was done in [9]. Particularly, authors investigated the form of attractors and their behavior by expanding the space $\mathbb{R}$ into space $\mathbb{C}$. Results of [9] are proved that the obtained attractors are not chaotic. Another important chracteristics within the nonlinear dynamics are the so-called homoclinic orbits. In [7, 8] the main objects for analysis are homoclinic orbits of the Lorenz system as well as their construction.

One of the major questions in the chaos theory (from both fundamental and applied points of view) is chaos control. Nowadays, a wide range of techniques to reduce chaos has invented. Firstly, let us note a well-known method to control chaos which is based on feedback principles. For example, in [13] authors investigated changes in dynamics of the Lorentz system under feedback control. Particularly, it was shown that the trajectory of the Lorenz system is located at one stable point. In [11], the global attracting set of the simplified Lorenz model was investigated within the Lyapunov stability theory. It was shown that the method of constructing the Lyapunov function applied to classical systems with chaos, does not work for a simplified model of the Lorenz system. In [12], the dynamics of unidirectionally connected chaotic Lorenz systems were investigated. It was shown that chaos is observed in the system independently on the generalized synchronization. To demonstrate the lack of synchronization, the Lyapunov exponents were used. In some cases, this result can be applied in the field of the weather forecasting. In [10] the Lorenz system was considered as a model of atmospheric disturbances and as a part of the so-called LDWNN (Lorenz Distribution Wavelet Neural Network) model. It was established that the Lorenz system gives more accurate results of the wind speed prediction comparing to the WNN (Wavelet Neural Network) model.

In recent years an interest to the Lorenz system is growing up in the field of cryptography (signal generated by the Lorenz system can be considered as a pseudo-random sequence). For instance, in [14], a method for generating sequences of pseudo-random numbers based on the generalized Lorenz system is proposed (in this paper the Lorenz system used to generate a binary sequence). It was shown that this method shows an effective crypto-robustness. Also, a comparison between the presented method and other methods for generating of pseudo-random sequences is considered. In [15] authors propose a novel asymmetric watermarking mechanism using the Lorenz system. This feature added nonlinear properties to this mechanism, as well as expanded the space of generated keys. Thus, the Lorenz system remains relevant even after more than fifty years of its discovering and gives promising and rich ideas to researchers in various fields of modern science. In this chapter we take a fresh look to the Lorenz system under various strongly nonlinear control (Coulomb friction and hysteresis friction) and discuss some features occurring in this case.

1.2 *Chaos: Analysis and Control*

Analysis of chaotic dynamics in nonlinear systems usually reduces to two separate problems: diagnosis of the system's behaviour (analysis) and control of the dynamics (usually, by means of an appropriate excitation). At the same time, two main methods usually implement for chaos control: program control (by means of an initially defined time function) and feedback control principles.

Diagnosis allows to investigate behaviour of the system at different values of the system parameters or, for example, depending on the initial conditions, etc. From both fundamental and applied points of view it is very important to analyze dynamics of the system depending on its parameters. An excellent example is a changing of

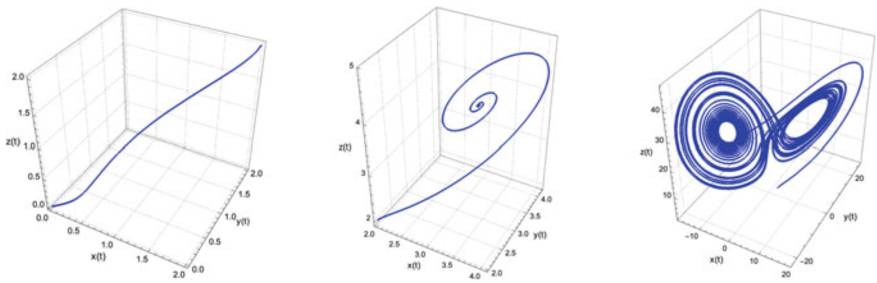

Fig. 2 Phase portraits of system (1) with parameters $\sigma = 10$, $b = 8/3$ and varying parameter r: left panel—$r = 0.5$, middle panel—$r = 5$, right panel—$r = 28$

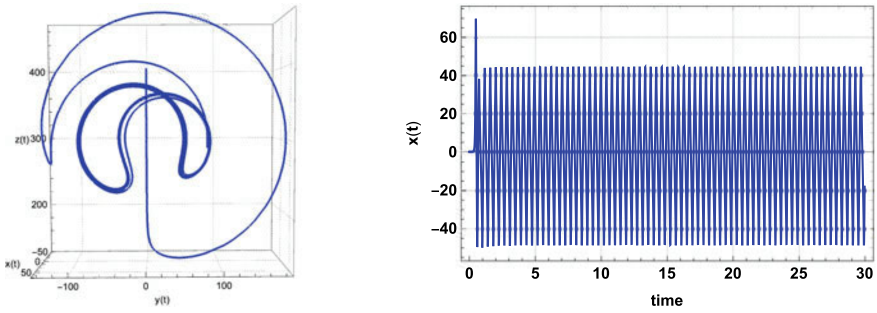

Fig. 3 Left panel—Phase portrait of system (1) with the corresponding parameters: $\sigma = 10$, $r = 300$, $b = 8/3$. Right panel—the x-coordinate versus time

the Rayleigh number in the Lorenz system (see Fig. 2). Varying this parameter a critical value when system (1) loss stability was found. Determining the dependence of solutions on the initial conditions is also an important task in the analysis of chaotic systems. Particularly, this task makes it possible to determine the attraction regions, as well as to identify the type of behaviour as it was done by Lorenz [1].

One of the well-known approaches to chaos control is based on the parameters varying. Indeed, by increasing parameter r in the classical Lorenz system (1), we can achieve completely regular dynamics (for $r = 300$ an absolutely stable limit cycle is observed, see Fig. 3).

Another implementation of chaos control is based on using the control function in one (or more) equation of the Lorenz system (modified Lorenz system):

$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = x(r - z) - y + u \\ \dot{z} = xy - bz \end{cases}, \tag{2}$$

where u is the control input. Additional variable introduced in order to control dynamics of the system. Depending on how it acts, dynamics of the system will also change.

2 Modified Lorenz System: Water Wheel Under Coulomb Friction

2.1 Water Wheel Model

One of the real physical model described by Lorenz equations is a water wheel model. This model has been constructed by Willem Malkus and his colleagues in1970s. A water wheel includes some leaking cups on the rim of the wheel, liquid flows from the

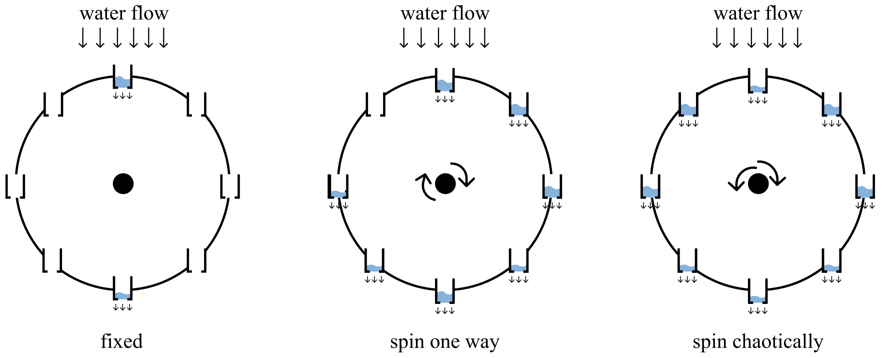

Fig. 4 A water wheel model with different rates of water flow. Left panel—low rate and the water wheel remains motionless; middle panel—moderate rate and the water wheel moves with a steady speed; right panel—high rate and the water wheel moves chaotically

top and each cup leaks from the bottom. Under different input flow rates, the wheel will fixed, spin one way, or spin chaotically (direction is changing unpredictable) (see Fig. 4).

2.2 Main Equations

Usually, authors consider an ideal model [16–19] and do not take into account the Coulomb friction on the rim [20] which, obviously, exists in a real physical model. In this chapter we pay attention to this fact and modify the classical Lorenz-Malkus water wheel model. Following the derivation of equations describing the Lorenz-Malkus model [16], ω is an angular velocity of the water wheel. It is clear, that the dry friction (the Coulomb model) affects only the speed of the wheel:

$$N_{df} = m \, \mathrm{sign}(\omega), \tag{3}$$

where m is the coefficient of the dry friction, $\mathrm{sign}(\cdot)$ is a standard signum function. Following the second Newton's law (as it was done in [17] with one remark: in our consideration the water wheel is placed in the vertical plane relative to the Earth surface), the rate of change of the angular momentum equals to the torque which is the sum of N_g (gravity component), N_{df} (a component describing by equation (3)), N_ν (viscous friction component), N_w (a component corresponding to bringing the incoming water flow up to the speed of the cup into which it falls):

$$I \frac{d\omega}{dt} = N_g + N_w + N_\mu + N_{df}, \tag{4}$$

and rewriting each of these components in an explicit form, equation (4) take the form:

$$\frac{d\omega}{dt} = \frac{Mg}{I}y - \frac{\alpha}{I}\omega + \lambda\frac{I_w}{I}\omega - \frac{m}{I}\text{sign}(\omega), \tag{5}$$

where g is the garvitational acceleration, α is a coefficient of the viscous friction, m is a coefficient of the dry friction, I_w is the moment of inertia of the water, λ is the cup leakage parameter, M is the total mass of the water. Equations for y and z can be represented as (following to [16]):

$$\frac{dy}{dt} = \omega z - \lambda y, \tag{6}$$

$$\frac{dz}{dt} = -\omega y + \lambda(R - z), \tag{7}$$

where R is a radius of the water wheel. Finally, bringing equations and parameters in (5–7) to the dimensionless form (in this case ω turns to x), the modified Lorenz-Malkus water wheel model reads:

$$\begin{cases} \dot{x} = \sigma(y - x) - M\text{sign}(x), \\ \dot{y} = x(r - z) - y, \\ \dot{z} = xy - bz, \end{cases} \tag{8}$$

where M_{df} is a coefficient of the dry friction moment. In this model variables have the following meaning:

- x is an angular velocity of the water wheel;
- y and z are coordinates of center of mass of the water.

2.3 *Stationary Points*

It is known that investigation of any nonlinear system described by differential equations starts from analysis of stationary points. These points play a special role in system behaviour. Therefore, we start from identification of stationary points for the modified Lorenz-Malkus system (8). Following the standard approach, $\dot{x}$, $\dot{y}$ and $\dot{z}$ are supposed to be zero, then system (8) reads:

$$\begin{cases} \sigma(y - x) - M_{df}\text{sign}(x) = 0 \\ x(r - z) - y = 0 \\ xy - bz = 0 \end{cases} . \tag{9}$$

A solution to algebraic equations (9) determines a set of stationary points. Additional term M_{df} sign(x) complicates the process of finding stationary points unlike for the classical Lorenz system and an explicit form of the obtained points has a quite complicated for analysis form (see Appendix). However, we show numerical values of these points for the system with parameters $\sigma = 10$, $r = 28$, $b = 8/3$, $M_{df} = 3$:

$$\begin{aligned} P_0 &= (0, 0, 0) \\ P_1 &= (8.642059263, 8.34205926, 27.03471396) \\ P_2 &= (-8.330947618, -8.630947618, 26.9639896) \\ P_3 &= (-0.011111644, -0.311111644, 0.001296360) \\ P_4 &= (0.011111644, 0.311111644, 0.001296360) \end{aligned} \quad . \tag{10}$$

A set of stationary points is denoted by $P_i, i = 0 \dots 4$. As can be seen from Eq. (10) we obtain two more stationary points (points P_3 and P_4) comparing to the classical system, however other three points have similar values. An analysis of the system behaviour in the neighborhood of these points was carried out using the first Lyapunov method.

An analysis of the trajectories behaviour near point P_0 starts from the identification of type of this point. For that reason, the standard linearization procedure is implemented. A solution to system (8) can be written in the form:

$$\mathbf{A} = \mathbf{A_0} + \widetilde{\mathbf{A}}, \tag{11}$$

where $\mathbf{A} = (x(t), y(t), z(t))^{\mathrm{T}}$ is a column vector of the time-dependent variables, $\mathbf{A_0} = (x_0, y_0, z_0)^{\mathrm{T}}$ is a column vector of coordinates of stationary point (in our case this vector is (0, 0, 0)), and $\widetilde{\mathbf{A}} = (\widetilde{x}(t), \widetilde{y}(t), \widetilde{z}(t))^{\mathrm{T}}$ is a column vector of small additives. Further, keeping only the main terms in right-hand sides of the linearised system, we obtain:

$$\begin{cases} \dot{x} = \sigma(\widetilde{y} - \widetilde{x}) - M_{df}\,\mathrm{sign}(\widetilde{x}), \\ \dot{y} = r\widetilde{x} - \widetilde{y} - x_0\widetilde{z} - \widetilde{x}z_0, \\ \dot{z} = -b\widetilde{z} + x_0\widetilde{y} + \widetilde{x}y_0, \end{cases} \tag{12}$$

where terms of zero-order have removed, because (x_0, y_0, z_0) is a stationary point, and term $M_{df}\,\mathrm{sign}(\widetilde{x} + x_0)$ is replaced by[2]: $M_{df}(\mathrm{sign}(\widetilde{x}) + \mathrm{sign}(x_0))$.

Note, that linearization of the right-hand side of the first equation in (12) is impossible due to the non-smoothness of the dry friction term. However, assuming that time dependence of the perturbation of small additives is exponential and keeping only term $-M_{df}\,\mathrm{sign}(\widetilde{x})$ in the first equation, we obtain the eigenvalue problem. The results of numerical simulation show that all eigenvalues are real and negative independent on the M_{df} value (in our case only positive values of this parameter has an

[2]Using the following obvious relation: $\mathrm{sign}(x) + \mathrm{sign}(y) - 1 \le \mathrm{sign}(x + y) \le \mathrm{sign}(x) + \mathrm{sign}(y) + 1$.

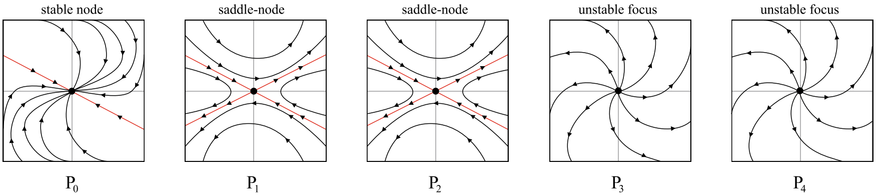

Fig. 5 Trajectories behaviour in a small neighborhood of stationary points for system (8)

interest), therefore point P_0 is a stable node. An illustration of the behaviour of phase trajectories near P_0 and other four points[3] are presented in Fig. 5.

2.4 Dynamical Features

This section discusses dynamical features of the modified Lorenz-Malkus system. First of all, it seems interesting to consider the system dynamics depending on the parameter corresponding to the moment of dry friction M_{df}, and obviously this new parameter must change dynamics.

Firstly, note that dependence of the time during which the system exhibit chaotic dynamics on the dry friction parameter is essentially nonlinear. Qualitatively this fact is shown in Fig. 6. For example, at $M_{df} = 3$ the system is staying in chaotic regime during a longer period of time as compared to the case when $M_{df} = 2$. Systems were solved with same initial conditions, however, this property holds for any initial conditions. At the same time, a period during which the system is in chaotic regime for various values of M_{df} depends on the initial conditions. This feature is due to the fact that additional term $-M_{df}\,\text{sign}(x)$ leads to an additional parametric dependence described by a non-smooth function, which, in turn, complicates the structure of the phase space. Also, the time when the system trajectory comes to a stable zero point directly depends on how soon it enters the attraction basin.

An additional contribution to understanding the system dynamics gives an evolution of the x-coordinate depending on the parameter M_{df}. As can be seen in Fig. 7 for time instant $t = 10$ the behaviour of function $x(M_{df})$ for small values of the dry friction parameter has a completely deterministic structure. However, starting from a certain value of M_{df}, $x(M_{df})$ exhibits chaotic behaviour and tends to zero while M_{df} increases (this is clearly seen from the right panel in Fig. 7). That means that the solution trajectory entered in an attraction basin and rapidly came to stable point P_0. Moreover, we also note (as follows from the results presented in the left panel of Fig. 7) that for any fixed time instant with a non-zero value of M_{df} the system inevitably comes to a small neighborhood of the origin. The modelling results

[3]During numerical simulations it was established that P_1 and P_2 are saddle-node points, and P_3 and P_4 are unstable focuses.

were obtained for a fixed time instant (200 units of model time), initial conditions are (1, 1, 1), and parameters of the system are $\sigma = 10$, $r = 28$, $b = 8/3$.

2.5 Lyapunov Characteristic Exponents

The presence or absence of chaotic behaviour is closely related to instability. One of the most important methods that allow to identify the stability of the system is the Lyapunov method. This method characterizes the behaviour of a given trajectory depending on the behaviour of trajectories located in its small neighborhood. An informative characteristic of this behaviour is the spectrum of Lyapunov characteristic exponents.[4] Chaotic behavior is fully determined by the largest Lyapunov exponent. Its value uniquely determines the type of dynamical regime. However, a well-known fact that following the fundamental Kolmogorov-Arnold-Moser the-

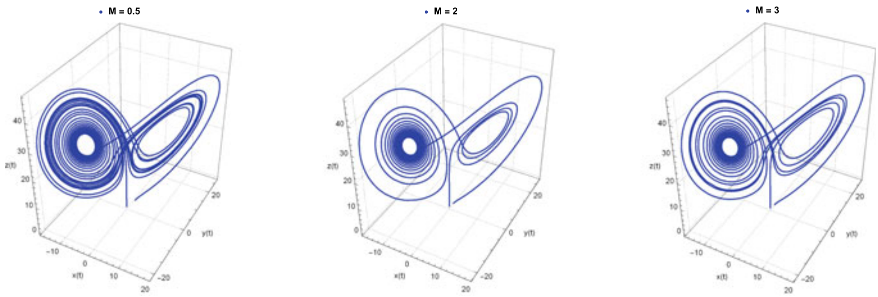

Fig. 6 Phase portraits of the modified Lorenz-Malkus system (8) with parameters $\sigma = 10, r = 28$, $b = 8/3$ and varying parameter M_{df}: left panel – $M_{df} = 0.5$, middle panel – $M_{df} = 2$, right panel – $M_{df} = 3$

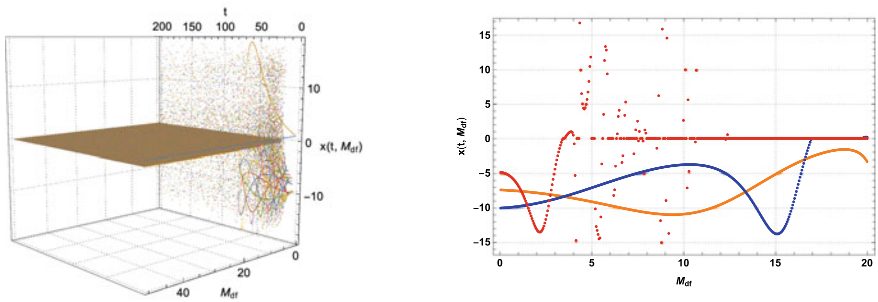

Fig. 7 Left panel – coordinate x versus M_{df} and time. Right panel – the x-coordinate versus M_{df} at different time instants: $t = 3$ – orange curve, $t = 5$ – blue curve, $t = 10$ – red curve

[4]Recall, that number of Lyapunov exponents is equal to the dimension of the system's phase space.

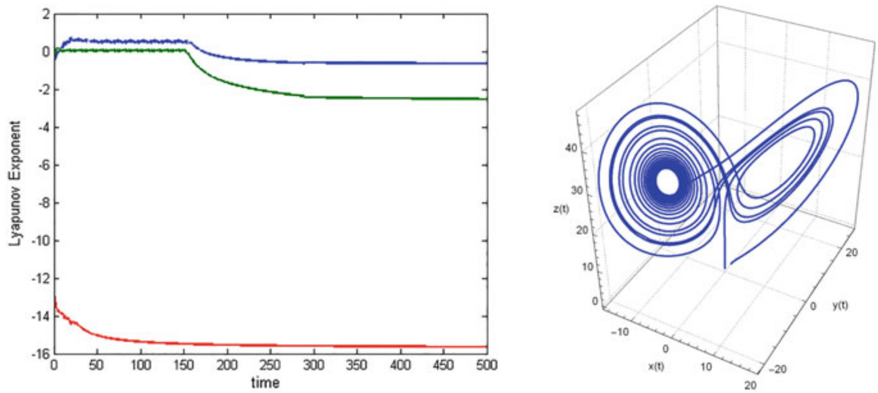

Fig. 8 Left panel – The Lyapunov spectrum versus time (the largest Lyapunov exponent is indicated by blue curve); Right panel – phase portrait of the corresponding modified Lorenz-Malkus system. Modelling parameter are: $\sigma = 10$, $r = 28$, $b = 8/3$, $M_{df} = 3$. Initial conditions are $(1, 1, 1)$

orem [3], to obtain a spectrum of Lyapunov characteristic exponents, it is enough to consider an evolution of the perturbation only for the individual solution of the Lorenz system (Fig. 8).

In this work the spectrum of Lyapunov exponents were calculated using the standard Gram-Schmidt orthogonalization procedure together with the Wolf algorithm (for details, see [21]). An analysis of the spectrum of Lyapunov exponents for system (8) with the corresponding parameters $\sigma = 10$, $r = 28$, $b = 8/3$, $M_{df} = 3$ showed that on the initial time period (around 150 units of model time) the largest Lyapunov exponent is positive, therefore the solution is chaotic. However, after that, the largest Lyapunov exponent monotonically decreases down to the region of negative values, which indicates the stabilization of the system in a stable fixed point.[5]

Assumption 1 Stabilization of the modified Lorenz-Malkus system occurs at **any** non-zero value of the dry friction parameter M_{df}.

In this work we also analyze the dependence of the dry friction parameter M_{df} on parameter r in a situation when the largest Lyapunov exponent is equal to zero. Note, that dependence $M_{df}(r)$, obtained during numerical simulations, has a very irregular structure as can be seen in Fig. 9. Based on the features presented above, it can be concluded that there is an attractive manifold, in which the solution trajectory inevitably ends up in an arbitrarily small neighborhood of a stable stationary point P_0.

[5]This point is P_0 with coordinates $(0, 0, 0)$.

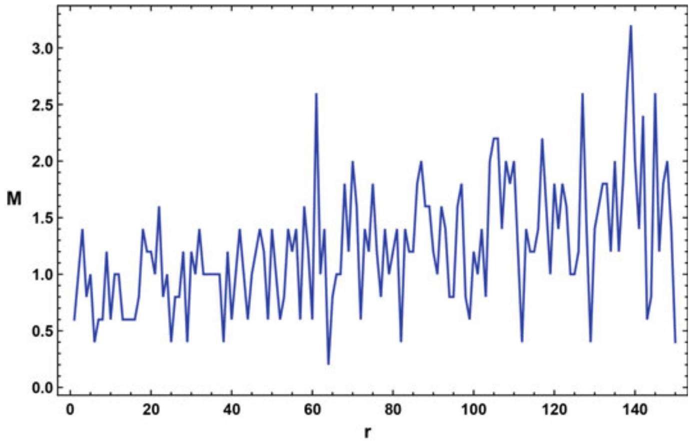

Fig. 9 The dependence of parameter M_{df} on r, corresponding to zero value of the largest Lyapunov exponent. Numerical simulations were carried out for initial conditions (10, 10, 10) and calculated during 500 units of model time

2.6 Stochastic Features

This section discusses stochastic properties of the modified Lorenz-Malkus system. Particularly, we investigate the time during which the system is in chaotic regime depending on the initial conditions. Figure 10 shows histograms of the distribution of time during which the system is in chaotic regime for different values of the dry friction moment. To simulate the corresponding histograms in a random way,[6] initial conditions were selected from the cube with a side of 20 units centered at the origin (ten thousand initial values were generated). Next, we considered a quantity corresponding to the time during which the solution trajectory was not in a neighborhood of the origin. It is important to note, that obtained histograms are stable enough for various sets of initial conditions. The shape of histograms obviously depends on the parameter M_{df}, however, in this work, we do not identify the universal distribution law which describes histograms for various values of the dry friction parameter. At the same time, it was found that for some values of M_{df} (particularly, for $M_{df} = 10$), the obtained histograms can be approximated with a good accuracy by the Gamma distribution which has the form:

$$f(\alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta} \left(\frac{x}{\beta}\right)^{\alpha-1} \mathrm{e}^{-x/\beta}, \qquad (13)$$

where α and β are parameters of the Gamma distribution, and $\Gamma(\cdot)$ is a standard Gamma function. In the right panel of Fig. 10 we show approximation of the histogram by the Gamma distribution (13) with parameters $\alpha = 4.9$, $\beta = 1.98$.

[6]We supposed the uniform distribution of initial conditions.

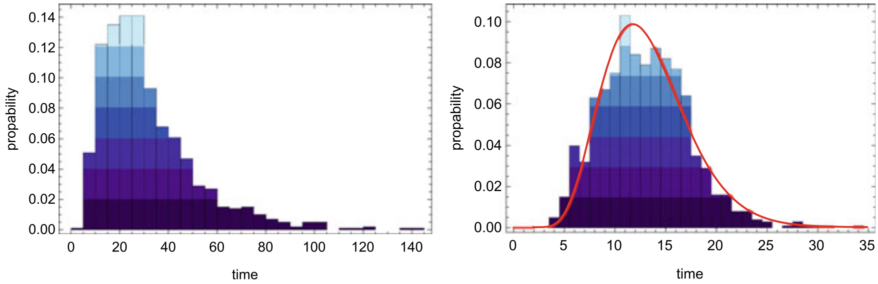

Fig. 10 Histograms describing how much time needs to solution to achieve the origin. Numerical simulations was carried out with following parameters: $\sigma = 10$,$r = 28$, $b = 8/3$ and $M_{df} = 3$ (left panel), $M_{df} = 10$ (right panel). Red curve on the right panel corresponds to the Gamma distribution with parameters $\alpha = 4.9$, $\beta = 1.98$

The obtained result can be used in Bayesian statistics, biology, medicine and many other fields where the Gamma distribution is traditionally used. In other words, for some values of M_{df}, the modified Lorenz system may serves as a “natural” pseudo-random number generator due to stochastic properties satisfying the Gamma distribution law.

2.7 Bifurcation Analysis

It is well-known, that a bifurcation analysis and the corresponding bifurcation diagrams are powerful tools in analysis of systems with chaotic regimes. Most of these systems have internal parameters which determine in general the behaviour of systems. During a monotonic slight change of these parameters, a transition from one mode to another can occur. A set of events that happen during these changes are called bifurcations.

In this work for the modified Lorentz-Malkus system we construct the bifurcation diagram in not a common sense, but merging a set of bifurcation diagrams because, during the numerical simulation, it has to be taken into account that when the trajectory hit the attracting region of the origin, a solution to the system is localized in the zero-dynamics region.[7] Therefore, in presented figures there are many points in the domain of zero values. As can be seen from the left top and bottom panels in Fig. 11 for the modified system (8), transition to chaos becomes a bit later comparing to the classical Lorenz system. For large values of the Rayleigh parameter, transition to chaos occurs through a cascade of period-doubling, which can be seen from the right top and bottom panels in Fig. 11.

[7]Note, that zero is absolutely stable regardless of the value of parameter r.

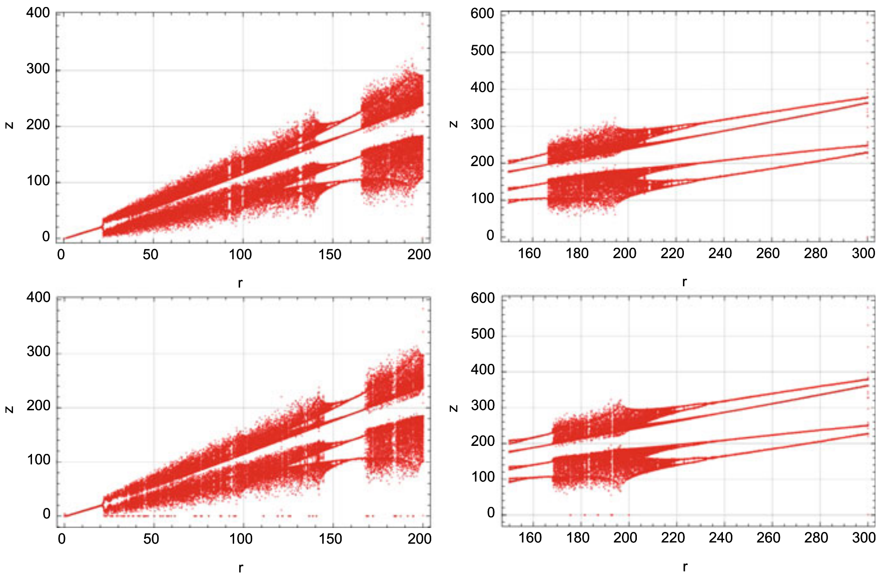

Fig. 11 Bifurcation diagrams depending on the parameter r. Top panels correspond to the classical Lorenz system; Bottom panels correspond to the modified Lorenz-Malkus system (8) with parameter $M_{df} = 3$

2.8 Analysis of a "flat" System

Analysis of trajectories near stationary points is important for studying the dynamic features of the system, and also gives some ideas about the structure of the attracting manifold. To simplify an analysis of the modified Lorenz-Malkus system (8), we consider a situation when $z = const$. Further, consider a two-dimensional system:

$$\begin{cases} \dot{x} = \sigma(y - x) - M_{df}\text{sign}(x) = \Phi(x, y), \\ \dot{y} = x(r - \dfrac{1}{b}xy) - y = \Psi(x, y). \end{cases} \tag{14}$$

In this section, we will consider vector fields of the "flat" system. This will help us to understand the trajectories behaviour, especially near stationary points. One of the most important characteristics in the analysis of vector fields is an index of the stationary point, which, first of all, gives an opportunity to establish the type of the singular point, namely, whether it belongs to the saddle type or not. The calculation of the index of the stationary point is carried out by finding a sign of the determinant which includes partial derivatives of the right-hand sides of equations (14):

$$\text{sign} \begin{vmatrix} \Phi(x, y)_x & \Phi(x, y)_y \\ \Psi(x, y)_x & \Psi(x, y)_y \end{vmatrix}, \tag{15}$$

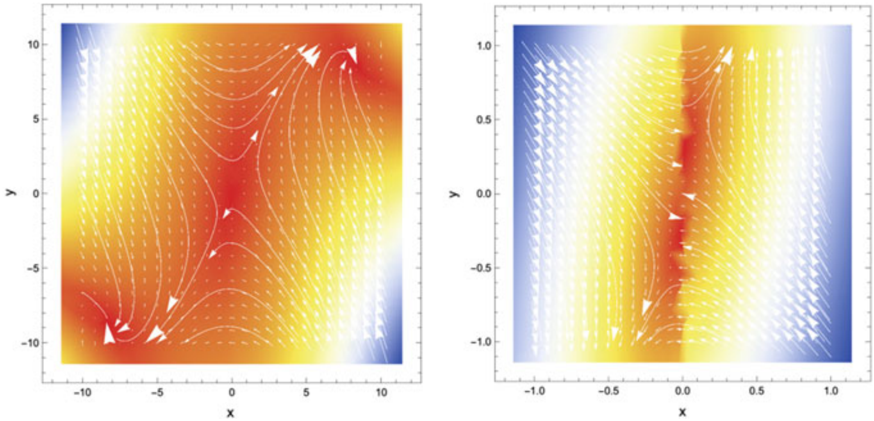

Fig. 12 Attraction manifold for trajectories of the "flat" modified Lorenz-Malkus system

where lower indexes denote partial derivatives. If the obtained value equals to -1, then the investigated point is a saddle-type point, otherwise when the value is 1, then this point is either center, or node, or focus.

For system (14) determinant (15) can be written as:

$$\begin{vmatrix} -\sigma - 2M_{df}\delta(0) & \sigma \\ r & -1 \end{vmatrix}. \tag{16}$$

Obviously, the determinant (16) is $\sigma(1-r) + 2M_{df}\delta(0)$ (here $\delta(\cdot)$ is the Dirac delta-function), and it is clear that $2M_{df}\delta(0) \gg \sigma(1-r)$.

Based on the obtained results the following conclusion can be formulated: zero in the modified Lorenz system is not a saddle regardless of the value r, which is clearly seen from the resulting expression. Let us analyze the behaviour of trajectories near stationary point P_0. The vector field of the modified flat system sufficiently differs from the vector field of the classical system, particularly, because P_0 is a stable node. Points P_1 and P_2 also bring changes to the vector space near zero since they are saddle points.

As can be seen from the right panel in Fig. 12 a set of all paths is separated into three subsets that correspond to three attractive points (two focuses and one stable node). Note, that dry friction parameter M_{df} is a bifurcation parameter. When parameter M_{df} becomes positive, then from the saddle point, two symmetric saddle points and a stable node at the origin are born (see Fig. 13).

It is clear that the size of the region depends on the value of parameter M_{df} and is determined by positions of two saddle points P_1 and P_2. The larger attraction area corresponds to the larger value of parameter M_{df}. Moreover, as can be seen from

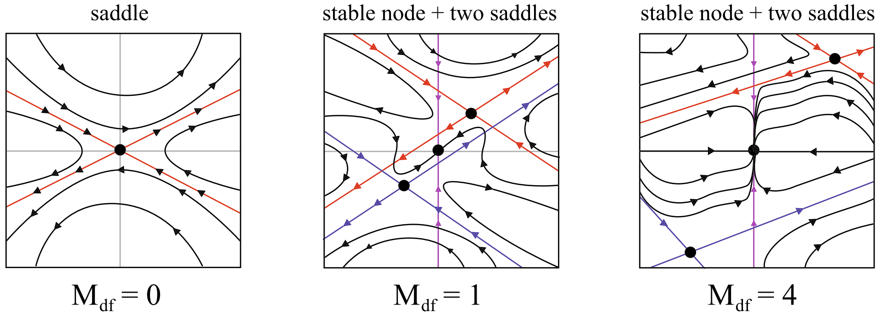

Fig. 13 Bifurcation process from a saddle-node to two saddle-nodes and a stable node during varying parameter M_{df} from zero to positive value

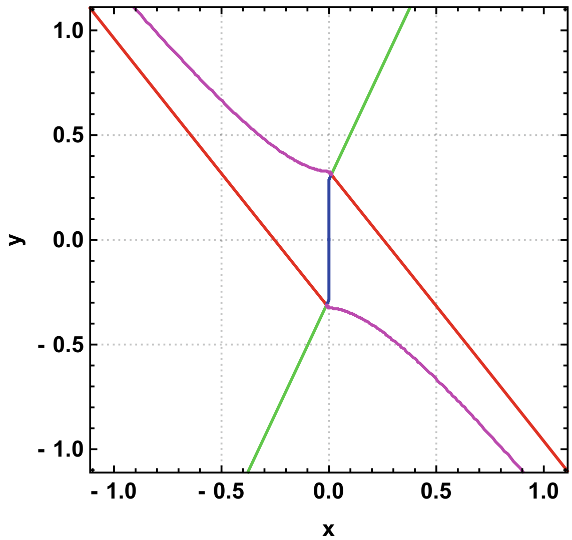

Fig. 14 Separatrices of two saddle-nodes. Blue curve corresponds to eigenvectors of the linearized matrix of system (14)

Fig. 14 the structure of the attracting region has a complex structure even in a two-dimensional case. An additional fact: the tangent to the boundary of the attracting set in a small neighborhood of saddle points obviously coincides with eigenvectors of the matrix of the linearized system in the neighborhood of the corresponding points (see the Fig. 14). Separatrices are shown in Fig. 14 and were constructed by solving the system in the reverse time.

3 Modified Lorenz-Malkus System Under Hysteresis Friction: The Bouc-Wen Model

3.1 Some Preliminaries to the Bouc-Wen Model

One of the famous approaches that allow describing many real-life physical systems and processes with memory is based on the model of hysteresis phenomena. It is caused, on the one hand, by design features of such systems, and, on the other hand, by features of external conditions that directly effect the system. Hysteresis phenomena are quite spread within physical, chemical, biological, and other processes in which states of the object are ambiguously dependent on external conditions. One of the main examples of a hysteresis relation is the dependence of the magnetization on the external field strength. Note that this dependence is caused not only by current state of the object (system) but also depends on its history. Historically, one of the first references to hysteresis dependencies was presented in the work of J. Ewing [22].

From a mathematical point of view, a hysteresis operator can be considered as a "black box", which respond to an external action (input) with a certain reaction (output). At the same time, the output value depends on the state of the object at the initial time instant. In this work, the carrier of hysteresis properties is understood as operator W (depending on its initial state as a parameter), which corresponds to continuous output $u(t)$ and receive continuous input signal $x(t)$ (see Fig. 15). The parametric relationship between input and output of such an operator is called as a hysteresis loop. Note that hysteresis curves are usually identical for periodic input signals with different frequencies, in other words, properties of operator W do not depend on the time scale. In this case such an operator is called static. The shape (in particular, the area of the loop) significantly effects the dynamics of systems containing hysteresis term. This feature makes it possible to introduce hysteresis operators in the control tasks. In recent years many hysteresis models have been obtained to solve various problems in a wide field of engineering tasks. First of all, let us note work [25], which has made a huge contribution to hysteresis analysis. Particularly, this work is a base to the so-called design approach to hysteresis phenomena.

One of the most frequently used phenomenological models of hysteresis is the Bouc-Wen model. This model was first proposed by Bouc [23] in 1971 and generalized by Wen in 1976 [24]. The Bouc-Wen model is represented as the first-order nonlinear differential equation of the following form:

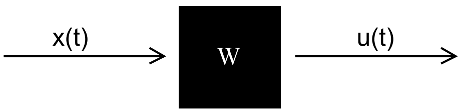

Fig. 15 The "black-box" approach. Here $x(t)$ is an input signal, $u(t)$ is an output signal, W is a hysteresis operator