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Fourth Edition

Quantitative Investment Analysis Workbook

WILEY

QUANTITATIVE INVESTMENT ANALYSIS

Fourth Edition

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PART I

LEARNING OBJECTIVES, SUMMARY OVERVIEW, AND PROBLEMS

CHAPTER 1

THE TIME VALUE OF MONEY

LEARNING OUTCOMES

The candidate should be able to:

- interpret interest rates as required rates of return, discount rates, or opportunity costs;
- explain an interest rate as the sum of a real risk-free rate and premiums that compensate investors for bearing distinct types of risk;
- calculate and interpret the effective annual rate, given the stated annual interest rate and the frequency of compounding;
- solve time value of money problems for different frequencies of compounding;
- calculate and interpret the future value (FV) and present value (PV) of a single sum of money, an ordinary annuity, an annuity due, a perpetuity (PV only), and a series of unequal cash flows;
- demonstrate the use of a time line in modeling and solving time value of money problems.

SUMMARY

In this chapter, we have explored a foundation topic in investment mathematics, the time value of money. We have developed and reviewed the following concepts for use in financial applications:

- The interest rate, r, is the required rate of return; r is also called the discount rate or opportunity cost.
- An interest rate can be viewed as the sum of the real risk-free interest rate and a set of premiums that compensate lenders for risk: an inflation premium, a default risk premium, a liquidity premium, and a maturity premium.
- The future value, FV, is the present value, PV, times the future value factor, $(1 + r)^N$.
- The interest rate, r, makes current and future currency amounts equivalent based on their time value.
- The stated annual interest rate is a quoted interest rate that does not account for compounding within the year.

- The periodic rate is the quoted interest rate per period; it equals the stated annual interest rate divided by the number of compounding periods per year.
- The effective annual rate is the amount by which a unit of currency will grow in a year with interest on interest included.
- An annuity is a finite set of level sequential cash flows.
- There are two types of annuities, the annuity due and the ordinary annuity. The annuity due has a first cash flow that occurs immediately; the ordinary annuity has a first cash flow that occurs one period from the present (indexed at $t = 1$).
- On a time line, we can index the present as 0 and then display equally spaced hash marks to represent a number of periods into the future. This representation allows us to index how many periods away each cash flow will be paid.
- Annuities may be handled in a similar approach as single payments if we use annuity factors rather than single-payment factors.
- The present value, PV, is the future value, FV, times the present value factor, $(1 + r)^{-N}$.
- The present value of a perpetuity is A/r, where A is the periodic payment to be received forever.
- It is possible to calculate an unknown variable, given the other relevant variables in time value of money problems.
- The cash flow additivity principle can be used to solve problems with uneven cash flows by combining single payments and annuities.

PRACTICE PROBLEMS

1. The table below gives current information on the interest rates for two two-year and two eight-year maturity investments. The table also gives the maturity, liquidity, and default risk characteristics of a new investment possibility (Investment 3). All investments promise only a single payment (a payment at maturity). Assume that premiums relating to inflation, liquidity, and default risk are constant across all time horizons.

Investment	Maturity (in Years)	Liquidity	Default Risk	Interest Rate (%)
1	2	High	Low	2.0
2	2	Low	Low	2.5
3	7	Low	Low	r_3
4	8	High	Low	4.0
5	8	Low	High	6.5

   Based on the information in the above table, address the following:
   A. Explain the difference between the interest rates on Investment 1 and Investment 2.
   B. Estimate the default risk premium.
   C. Calculate upper and lower limits for the interest rate on Investment 3, r_3.
2. A couple plans to set aside $20,000 per year in a conservative portfolio projected to earn 7 percent a year. If they make their first savings contribution one year from now, how much will they have at the end of 20 years?
3. Two years from now, a client will receive the first of three annual payments of $20,000 from a small business project. If she can earn 9 percent annually on her investments and

plans to retire in six years, how much will the three business project payments be worth at the time of her retirement?

4. To cover the first year's total college tuition payments for his two children, a father will make a $75,000 payment five years from now. How much will he need to invest today to meet his first tuition goal if the investment earns 6 percent annually?
5. A client can choose between receiving 10 annual $100,000 retirement payments, starting one year from today, or receiving a lump sum today. Knowing that he can invest at a rate of 5 percent annually, he has decided to take the lump sum. What lump sum today will be equivalent to the future annual payments?
6. You are considering investing in two different instruments. The first instrument will pay nothing for three years, but then it will pay $20,000 per year for four years. The second instrument will pay $20,000 for three years and $30,000 in the fourth year. All payments are made at year-end. If your required rate of return on these investments is 8 percent annually, what should you be willing to pay for:
   A. The first instrument?
   B. The second instrument (use the formula for a four-year annuity)?
7. Suppose you plan to send your daughter to college in three years. You expect her to earn two-thirds of her tuition payment in scholarship money, so you estimate that your payments will be $10,000 a year for four years. To estimate whether you have set aside enough money, you ignore possible inflation in tuition payments and assume that you can earn 8 percent annually on your investments. How much should you set aside now to cover these payments?
8. A client plans to send a child to college for four years starting 18 years from now. Having set aside money for tuition, she decides to plan for room and board also. She estimates these costs at $20,000 per year, payable at the beginning of each year, by the time her child goes to college. If she starts next year and makes 17 payments into a savings account paying 5 percent annually, what annual payments must she make?
9. A couple plans to pay their childs college tuition for 4 years starting 18 years from now. The current annual cost of college is C$7,000, and they expect this cost to rise at an annual rate of 5 percent. In their planning, they assume that they can earn 6 percent annually. How much must they put aside each year, starting next year, if they plan to make 17 equal payments?
10. The nominal risk-free rate is *best* described as the sum of the real risk-free rate and a premium for:
    A. maturity.
    B. liquidity.
    C. expected inflation.
11. Which of the following risk premiums is most relevant in explaining the difference in yields between 30-year bonds issued by the US Treasury and 30-year bonds issued by a small private issuer?
    A. Inflation
    B. Maturity
    C. Liquidity

12. A bank quotes a stated annual interest rate of 4.00%. If that rate is equal to an effective annual rate of 4.08%, then the bank is compounding interest:
    A. daily.
    B. quarterly.
    C. semiannually.
13. The value in six years of $75,000 invested today at a stated annual interest rate of 7% compounded quarterly is *closest* to:
    A. $112,555.
    B. $113,330.
    C. $113,733.
14. A client requires £100,000 one year from now. If the stated annual rate is 2.50% compounded weekly, the deposit needed today is *closest* to:
    A. £97,500.
    B. £97,532.
    C. £97,561.
15. For a lump sum investment of ¥250,000 invested at a stated annual rate of 3% compounded daily, the number of months needed to grow the sum to ¥1,000,000 is *closest* to:
    A. 555.
    B. 563.
    C. 576.
16. Given a €1,000,000 investment for four years with a stated annual rate of 3% compounded continuously, the difference in its interest earnings compared with the same investment compounded daily is *closest* to:
    A. €1.
    B. €6.
    C. €455.
17. An investment pays €300 annually for five years, with the first payment occurring today. The present value (PV) of the investment discounted at a 4% annual rate is *closest* to:
    A. €1,336.
    B. €1,389.
    C. €1,625.
18. A perpetual preferred stock makes its first quarterly dividend payment of $2.00 in five quarters. If the required annual rate of return is 6% compounded quarterly, the stock's present value is *closest* to:
    A. $31.
    B. $126.
    C. $133.
19. A saver deposits the following amounts in an account paying a stated annual rate of 4%, compounded semiannually:

Year	End-of-Year Deposits ($)
1	4,000
2	8,000
3	7,000
4	10,000

At the end of Year 4, the value of the account is *closest* to:
A. $30,432
B. $30,447
C. $31,677

20. An investment of €500,000 today that grows to €800,000 after six years has a stated annual interest rate *closest* to:
A. 7.5% compounded continuously.
B. 7.7% compounded daily.
C. 8.0% compounded semiannually.

21. A sweepstakes winner may select either a perpetuity of £2,000 a month beginning with the first payment in one month or an immediate lump sum payment of £350,000. If the annual discount rate is 6% compounded monthly, the present value of the perpetuity is:
A. less than the lump sum.
B. equal to the lump sum.
C. greater than the lump sum.

22. At a 5% interest rate per year compounded annually, the present value (PV) of a 10-year ordinary annuity with annual payments of $2,000 is $15,443.47. The PV of a 10-year annuity due with the same interest rate and payments is *closest* to:
A. $14,708.
B. $16,216.
C. $17,443.

23. Grandparents are funding a newborn's future university tuition costs, estimated at $50,000/year for four years, with the first payment due as a lump sum in 18 years. Assuming a 6% effective annual rate, the required deposit today is *closest* to:
A. $60,699.
B. $64,341.
C. $68,201.

24. The present value (PV) of an investment with the following year-end cash flows (CF) and a 12% required annual rate of return is *closest* to:

Year	**Cash Flow (€)**
1	100,000
2	150,000
5	–10,000

A. €201,747.
B. €203,191.
C. €227,573.

25. A sports car, purchased for £200,000, is financed for five years at an annual rate of 6% compounded monthly. If the first payment is due in one month, the monthly payment is *closest* to:
A. £3,847.
B. £3,867.
C. £3,957.

26. Given a stated annual interest rate of 6% compounded quarterly, the level amount that, deposited quarterly, will grow to £25,000 at the end of 10 years is *closest* to:
    A. £461.
    B. £474.
    C. £836.
27. Given the following time line and a discount rate of 4% a year compounded annually, the present value (PV), as of the end of Year 5 (PV_5), of the cash flow received at the end of Year 20 is *closest* to:
    A. $22,819.
    B. $27,763.
    C. $28,873.
28. A client invests €20,000 in a four-year certificate of deposit (CD) that annually pays interest of 3.5%. The annual CD interest payments are automatically reinvested in a separate savings account at a stated annual interest rate of 2% compounded monthly. At maturity, the value of the combined asset is *closest* to:
    A. €21,670.
    B. €22,890.
    C. €22,950.

CHAPTER 2

ORGANIZING, VISUALIZING, AND DESCRIBING DATA

LEARNING OUTCOMES

The candidate should be able to:

- Identify and compare data types;
- Describe how data are organized for quantitative analysis;
- Interpret frequency and related distributions;
- Interpret a contingency table;
- Describe ways that data may be visualized and evaluate uses of specific visualizations;
- Describe how to select among visualization types;
- Calculate and interpret measures of central tendency;
- Select among alternative definitions of mean to address an investment problem;
- Calculate quantiles and interpret related visualizations;
- Calculate and interpret measures of dispersion;
- Calculate and interpret target downside deviation;
- Interpret skewness;
- Interpret kurtosis;
- Interpret correlation between two variables.

SUMMARY

In this chapter, we have presented tools and techniques for organizing, visualizing, and describing data that permit us to convert raw data into useful information for investment analysis.

- Data can be defined as a collection of numbers, characters, words, and text—as well as images, audio, and video—in a raw or organized format to represent facts or information.

- From a statistical perspective, data can be classified as numerical data and categorical data. Numerical data (also called quantitative data) are values that represent measured or counted quantities as a number. Categorical data (also called qualitative data) are values that describe a quality or characteristic of a group of observations and usually take only a limited number of values that are mutually exclusive.
- Numerical data can be further split into two types: continuous data and discrete data. Continuous data can be measured and can take on any numerical value in a specified range of values. Discrete data are numerical values that result from a counting process and therefore are limited to a finite number of values.
- Categorical data can be further classified into two types: nominal data and ordinal data. Nominal data are categorical values that are not amenable to being organized in a logical order, while ordinal data are categorical values that can be logically ordered or ranked.
- Based on how they are collected, data can be categorized into three types: cross-sectional, time series, and panel. Time-series data are a sequence of observations for a single observational unit on a specific variable collected over time and at discrete and typically equally spaced intervals of time. Cross-sectional data are a list of the observations of a specific variable from multiple observational units at a given point in time. Panel data are a mix of time-series and cross-sectional data that consists of observations through time on one or more variables for multiple observational units.
- Based on whether or not data are in a highly organized form, they can be classified into structured and unstructured types. Structured data are highly organized in a pre-defined manner, usually with repeating patterns. Unstructured data do not follow any conventionally organized forms; they are typically alternative data as they are usually collected from unconventional sources.
- Raw data are typically organized into either a one-dimensional array or a two-dimensional rectangular array (also called a data table) for quantitative analysis.
- A frequency distribution is a tabular display of data constructed either by counting the observations of a variable by distinct values or groups or by tallying the values of a numerical variable into a set of numerically ordered bins. Frequency distributions permit us to evaluate how data are distributed.
- The relative frequency of observations in a bin (interval or bucket) is the number of observations in the bin divided by the total number of observations. The cumulative relative frequency cumulates (adds up) the relative frequencies as we move from the first bin to the last, thus giving the fraction of the observations that are less than the upper limit of each bin.
- A contingency table is a tabular format that displays the frequency distributions of two or more categorical variables simultaneously. One application of contingency tables is for evaluating the performance of a classification model (using a confusion matrix). Another application of contingency tables is to investigate a potential association between two categorical variables by performing a chi-square test of independence.
- Visualization is the presentation of data in a pictorial or graphical format for the purpose of increasing understanding and for gaining insights into the data.
- A histogram is a bar chart of data that have been grouped into a frequency distribution. A frequency polygon is a graph of frequency distributions obtained by drawing straight lines joining successive midpoints of bars representing the class frequencies.

- A bar chart is used to plot the frequency distribution of categorical data, with each bar representing a distinct category and the bar's height (or length) proportional to the frequency of the corresponding category. Grouped bar charts or stacked bar charts can present the frequency distribution of multiple categorical variables simultaneously.
- A tree-map is a graphical tool to display categorical data. It consists of a set of colored rectangles to represent distinct groups, and the area of each rectangle is proportional to the value of the corresponding group. Additional dimensions of categorical data can be displayed by nested rectangles.
- A word cloud is a visual device for representing textual data, with the size of each distinct word being proportional to the frequency with which it appears in the given text.
- A line chart is a type of graph used to visualize ordered observations and often to display the change of data series over time. A bubble line chart is a special type of line chart that uses varying-sized bubbles as data points to represent an additional dimension of data.
- A scatter plot is a type of graph for visualizing the joint variation in two numerical variables. It is constructed by drawing dots to indicate the values of the two variables plotted against the corresponding axes. A scatter plot matrix organizes scatter plots between pairs of variables into a matrix format to inspect all pairwise relationships between more than two variables in one combined visual.
- A heat map is a type of graphic that organizes and summarizes data in a tabular format and represents it using a color spectrum. It is often used in displaying frequency distributions or visualizing the degree of correlation among different variables.
- The key consideration when selecting among chart types is the intended purpose of visualizing data (i.e., whether it is for exploring/presenting distributions or relationships or for making comparisons).
- A population is defined as all members of a specified group. A sample is a subset of a population.
- A parameter is any descriptive measure of a population. A sample statistic (statistic, for short) is a quantity computed from or used to describe a sample.
- Sample statistics—such as measures of central tendency, measures of dispersion, skewness, and kurtosis—help with investment analysis, particularly in making probabilistic statements about returns.
- Measures of central tendency specify where data are centered and include the mean, median, and mode (i.e., the most frequently occurring value).
- The arithmetic mean is the sum of the observations divided by the number of observations. It is the most frequently used measure of central tendency.
- The median is the value of the middle item (or the mean of the values of the two middle items) when the items in a set are sorted into ascending or descending order. The median is not influenced by extreme values and is most useful in the case of skewed distributions.
- The mode is the most frequently observed value and is the only measure of central tendency that can be used with nominal data. A distribution may be unimodal (one mode), bimodal (two modes), trimodal (three modes), or have even more modes.
- A portfolio's return is a weighted mean return computed from the returns on the individual assets, where the weight applied to each asset's return is the fraction of the portfolio invested in that asset.

- The geometric mean, $\overline{X}_G$, of a set of observations $X_1, X_2, \ldots, X_n$, is $\overline{X}_G = \sqrt[n]{X_1 X_2 X_3 \ldots X_n}$, with $X_i \geq 0$ for $i = 1, 2, \ldots, n$. The geometric mean is especially important in reporting compound growth rates for time-series data. The geometric mean will always be less than an arithmetic mean whenever there is variance in the observations.
- The harmonic mean, $\overline{X}_H$, is a type of weighted mean in which an observation's weight is inversely proportional to its magnitude.
- Quantiles—such as the median, quartiles, quintiles, deciles, and percentiles—are location parameters that divide a distribution into halves, quarters, fifths, tenths, and hundredths, respectively.
- A box and whiskers plot illustrates the interquartile range (the "box") as well as a range outside of the box that is based on the interquartile range, indicated by the "whiskers."
- Dispersion measures—such as the range, mean absolute deviation (MAD), variance, standard deviation, target downside deviation, and coefficient of variation—describe the variability of outcomes around the arithmetic mean.
- The range is the difference between the maximum value and the minimum value of the dataset. The range has only a limited usefulness because it uses information from only two observations.
- The MAD for a sample is the average of the absolute deviations of observations from the mean, $\frac{\sum_{i=1}^{n} |X_i - \overline{X}|}{n}$, where $\overline{X}$ is the sample mean and n is the number of observations in the sample.
- The variance is the average of the squared deviations around the mean, and the standard deviation is the positive square root of variance. In computing sample variance (s^2) and sample standard deviation (s), the average squared deviation is computed using a divisor equal to the sample size minus 1.
- The target downside deviation, or target semideviation, is a measure of the risk of being below a given target. It is calculated as the square root of the average squared deviations from the target, but it includes only those observations below the target (B), or $\sqrt{\sum_{\text{for all } X_i \leq B}^{n} \frac{(X_i - B)^2}{n - 1}}$.
- The coefficient of variation, CV, is the ratio of the standard deviation of a set of observations to their mean value. By expressing the magnitude of variation among observations relative to their average size, the CV permits direct comparisons of dispersion across different datasets. Reflecting the correction for scale, the CV is a scale-free measure (i.e., it has no units of measurement).
- Skew or skewness describes the degree to which a distribution is asymmetric about its mean. A return distribution with positive skewness has frequent small losses and a few extreme gains compared to a normal distribution. A return distribution with negative skewness has frequent small gains and a few extreme losses compared to a normal distribution. Zero skewness indicates a symmetric distribution of returns.
- Kurtosis measures the combined weight of the tails of a distribution relative to the rest of the distribution. A distribution with fatter tails than the normal distribution is referred to as fat-tailed (leptokurtic); a distribution with thinner tails than the normal distribution is referred to as thin-tailed (platykurtic). Excess kurtosis is kurtosis minus 3, since 3 is the value of kurtosis for all normal distributions.

- The correlation coefficient is a statistic that measures the association between two variables. It is the ratio of covariance to the product of the two variables' standard deviations. A positive correlation coefficient indicates that the two variables tend to move together, whereas a negative coefficient indicates that the two variables tend to move in opposite directions. Correlation does not imply causation, simply association. Issues that arise in evaluating correlation include the presence of outliers and spurious correlation.

PRACTICE PROBLEMS

1. Published ratings on stocks ranging from 1 (strong sell) to 5 (strong buy) are examples of which measurement scale?
   A. Ordinal
   B. Continuous
   C. Nominal

2. Data values that are categorical and not amenable to being organized in a logical order are *most likely* to be characterized as:
   A. ordinal data.
   B. discrete data.
   C. nominal data.

3. Which of the following data types would be classified as being categorical?
   A. Discrete
   B. Nominal
   C. Continuous

4. A fixed-income analyst uses a proprietary model to estimate bankruptcy probabilities for a group of firms. The model generates probabilities that can take any value between 0 and 1. The resulting set of estimated probabilities would *most likely* be characterized as:
   A. ordinal data.
   B. discrete data.
   C. continuous data.

5. An analyst uses a software program to analyze unstructured data—specifically, management's earnings call transcript for one of the companies in her research coverage. The program scans the words in each sentence of the transcript and then classifies the sentences as having negative, neutral, or positive sentiment. The resulting set of sentiment data would *most likely* be characterized as:
   A. ordinal data.
   B. discrete data.
   C. nominal data.

Use the following information to answer Questions 6 and 7.

An equity analyst gathers total returns for three country equity indexes over the past four years. The data are presented below.

Time Period	Index A	Index B	Index C
Year *t*–3	15.56%	11.84%	–4.34%
Year *t*–2	–4.12%	–6.96%	9.32%
Year *t*–1	11.19%	10.29%	–12.72%
Year *t*	8.98%	6.32%	21.44%

6. Each individual column of data in the table can be *best* characterized as:
   A. panel data.
   B. time-series data.
   C. cross-sectional data.

7. Each individual row of data in the table can be *best* characterized as:
   A. panel data.
   B. time-series data.
   C. cross-sectional data.

8. A two-dimensional rectangular array would be most suitable for organizing a collection of raw:
   A. panel data.
   B. time-series data.
   C. cross-sectional data.

9. In a frequency distribution, the absolute frequency measure:
   A. represents the percentages of each unique value of the variable.
   B. represents the actual number of observations counted for each unique value of the variable.
   C. allows for comparisons between datasets with different numbers of total observations.

10. An investment fund has the return frequency distribution shown in the following exhibit.

Return Interval (%)	Absolute Frequency
–10.0 to –7.0	3
–7.0 to –4.0	7
–4.0 to –1.0	10
–1.0 to +2.0	12
+2.0 to +5.0	23
+5.0 to +8.0	5

Which of the following statements is correct?
A. The relative frequency of the bin "–1.0 to +2.0" is 20%.
B. The relative frequency of the bin "+2.0 to +5.0" is 23%.
C. The cumulative relative frequency of the bin "+5.0 to +8.0" is 91.7%.

11. An analyst is using the data in the following exhibit to prepare a statistical report.

Portfolio's Deviations from Benchmark Return for a 12-Year Period (%)

Year 1	2.48	Year 7	–9.19
Year 2	–2.59	Year 8	–5.11
Year 3	9.47	Year 9	1.33
Year 4	–0.55	Year 10	6.84
Year 5	–1.69	Year 11	3.04
Year 6	–0.89	Year 12	4.72

The cumulative relative frequency for the bin $-1.71\% \leq x < 2.03\%$ is *closest* to:
A. 0.250.
B. 0.333.
C. 0.583.

Use the following information to answer Questions 12 and 13.

A fixed-income portfolio manager creates a contingency table of the number of bonds held in her portfolio by sector and bond rating. The contingency table is presented here:

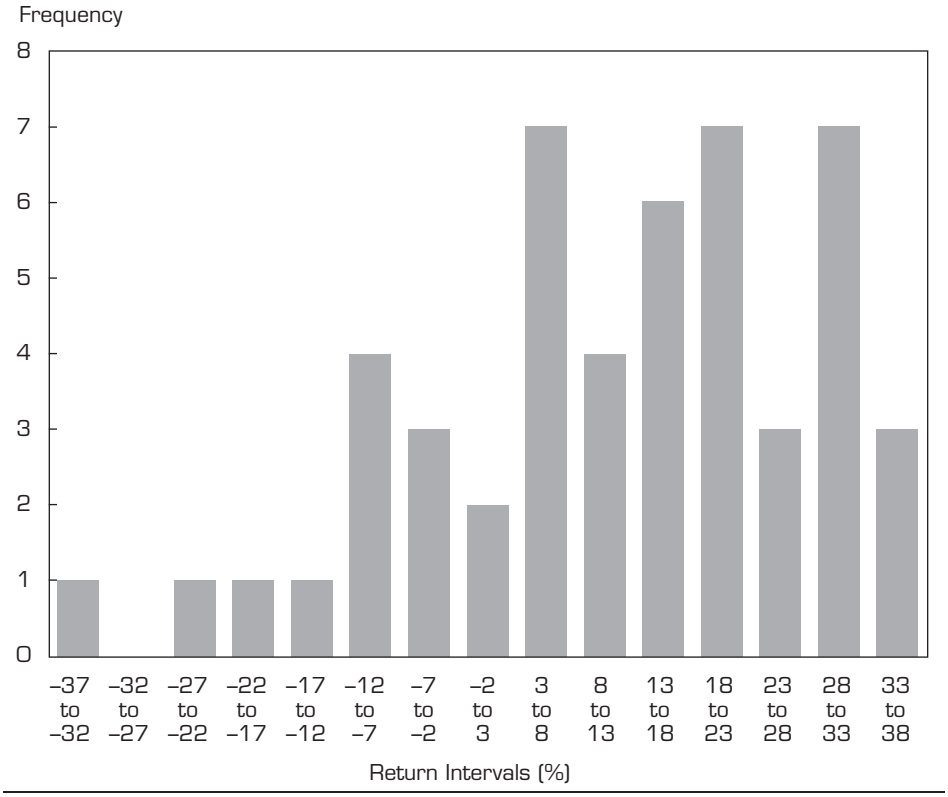
	Bond Rating		
Sector	**A**	**AA**	**AAA**
Communication Services	25	32	27
Consumer Staples	30	25	25
Energy	100	85	30
Health Care	200	100	63
Utilities	22	28	14

12. The marginal frequency of energy sector bonds is *closest* to:
A. 27.
B. 85.
C. 215.

13. The relative frequency of AA rated energy bonds, based on the total count, is *closest* to:
A. 10.5%.
B. 31.5%.
C. 39.5%.

The following information relates to Questions 14–15

The following histogram shows a distribution of the S&P 500 Index annual returns for a 50-year period:

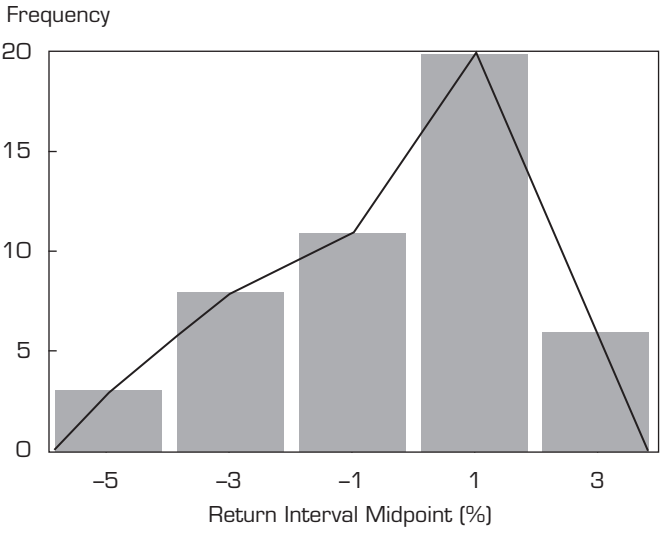


14. The bin containing the median return is:
    A. 3% to 8%.
    B. 8% to 13%.
    C. 13% to 18%.

15. Based on the previous histogram, the distribution is *best* described as being:
    A. unimodal.
    B. bimodal.
    C. trimodal.

16. The following is a frequency polygon of monthly exchange rate changes in the US dollar/Japanese yen spot exchange rate for a four-year period. A positive change represents yen appreciation (the yen buys more dollars), and a negative change represents yen depreciation (the yen buys fewer dollars).

Monthly Changes in the US Dollar/Japanese Yen Spot Exchange Rate



Based on the chart, yen appreciation:
A. occurred more than 50% of the time.
B. was less frequent than yen depreciation.
C. in the 0.0 to 2.0 interval occurred 20% of the time.

17. A bar chart that orders categories by frequency in descending order and includes a line displaying cumulative relative frequency is referred to as a:
A. Pareto Chart.
B. grouped bar chart.
C. frequency polygon.

18. Which visualization tool works *best* to represent unstructured, textual data?
A. Tree-Map
B. Scatter plot
C. Word cloud

19. A tree-map is best suited to illustrate:
A. underlying trends over time.
B. joint variations in two variables.
C. value differences of categorical groups.

20. A line chart with two variables—for example, revenues and earnings per share—is best suited for visualizing:
A. the joint variation in the variables.
B. underlying trends in the variables over time.
C. the degree of correlation between the variables.

21. A heat map is best suited for visualizing the:
    A. frequency of textual data.
    B. degree of correlation between different variables.
    C. shape, center, and spread of the distribution of numerical data.

22. Which valuation tool is recommended to be used if the goal is to make comparisons of three or more variables over time?
    A. Heat map
    B. Bubble line chart
    C. Scatter plot matrix

23. The annual returns for three portfolios are shown in the following exhibit. Portfolios P and R were created in Year 1, Portfolio Q in Year 2.

	Annual Portfolio Returns (%)				
	Year 1	**Year 2**	**Year 3**	**Year 4**	**Year 5**
Portfolio P	–3.0	4.0	5.0	3.0	7.0
Portfolio Q		–3.0	6.0	4.0	8.0
Portfolio R	1.0	–1.0	4.0	4.0	3.0

The median annual return from portfolio creation to Year 5 for:
A. Portfolio P is 4.5%.
B. Portfolio Q is 4.0%.
C. Portfolio R is higher than its arithmetic mean annual return.

24. At the beginning of Year X, an investor allocated his retirement savings in the asset classes shown in the following exhibit and earned a return for Year X as also shown.

Asset Class	**Asset Allocation (%)**	**Asset Class Return for Year X (%)**
Large-cap US equities	20.0	8.0
Small-cap US equities	40.0	12.0
Emerging market equities	25.0	–3.0
High-yield bonds	15.0	4.0

The portfolio return for Year X is *closest to*:
A. 5.1%.
B. 5.3%.
C. 6.3%.

25. The following exhibit shows the annual returns for Fund Y.

	Fund Y (%)
Year 1	19.5
Year 2	–1.9
Year 3	19.7
Year 4	35.0
Year 5	5.7

The geometric mean return for Fund Y is *closest* to:
A. 14.9%.
B. 15.6%.
C. 19.5%.

26. A portfolio manager invests €5,000 annually in a security for four years at the prices shown in the following exhibit.

	Purchase Price of Security (€ per unit)
Year 1	62.00
Year 2	76.00
Year 3	84.00
Year 4	90.00

The average price is *best* represented as the:
A. harmonic mean of €76.48.
B. geometric mean of €77.26.
C. arithmetic average of €78.00.

The following information relates to Questions 27–28.

The following exhibit shows the annual MSCI World Index total returns for a 10-year period.

Year 1	15.25%	Year 6	30.79%
Year 2	10.02%	Year 7	12.34%
Year 3	20.65%	Year 8	–5.02%
Year 4	9.57%	Year 9	16.54%
Year 5	–40.33%	Year 10	27.37%

27. The fourth quintile return for the MSCI World Index is *closest* to:
A. 20.65%.
B. 26.03%.
C. 27.37%.

28. For Year 6–Year 10, the mean absolute deviation of the MSCI World Index total returns is *closest* to:
A. 10.20%.
B. 12.74%.
C. 16.40%.

29. Annual returns and summary statistics for three funds are listed in the following exhibit:

	Annual Returns (%)		
Year	**Fund ABC**	**Fund XYZ**	**Fund PQR**
Year 1	–20.0	–33.0	–14.0
Year 2	23.0	–12.0	–18.0
Year 3	–14.0	–12.0	6.0
Year 4	5.0	–8.0	–2.0
Year 5	–14.0	11.0	3.0
Mean	–4.0	–10.8	–5.0
Standard deviation	17.8	15.6	10.5

The fund with the highest absolute dispersion is:

A. Fund PQR if the measure of dispersion is the range.
B. Fund XYZ if the measure of dispersion is the variance.
C. Fund ABC if the measure of dispersion is the mean absolute deviation.

30. The mean monthly return and the standard deviation for three industry sectors are shown in the following exhibit.

Sector	**Mean Monthly Return (%)**	**Standard Deviation of Return (%)**
Utilities (UTIL)	2.10	1.23
Materials (MATR)	1.25	1.35
Industrials (INDU)	3.01	1.52

Based on the coefficient of variation, the riskiest sector is:

A. utilities.
B. materials.
C. industrials.

31. The average return for Portfolio A over the past twelve months is 3%, with a standard deviation of 4%. The average return for Portfolio B over this same period is also 3%, but with a standard deviation of 6%. The geometric mean return of Portfolio A is 2.85%. The geometric mean return of Portfolio B is:

A. less than 2.85%.
B. equal to 2.85%.
C. greater than 2.85%.

32. An analyst calculated the excess kurtosis of a stock's returns as –0.75. From this information, we conclude that the distribution of returns is:

A. normally distributed.
B. thin-tailed compared to the normal distribution.
C. fat-tailed compared to the normal distribution.

33. When analyzing investment returns, which of the following statements is correct?
    A. The geometric mean will exceed the arithmetic mean for a series with non-zero variance.
    B. The geometric mean measures an investment's compound rate of growth over multiple periods.
    C. The arithmetic mean measures an investment's terminal value over multiple periods.

The following information relates to Questions 34–38

A fund had the following experience over the past 10 years:

Year	**Return**
1	4.5%
2	6.0%
3	1.5%
4	–2.0%
5	0.0%
6	4.5%
7	3.5%
8	2.5%
9	5.5%
10	4.0%

34. The arithmetic mean return over the 10 years is *closest* to:
    A. 2.97%.
    B. 3.00%.
    C. 3.33%.

35. The geometric mean return over the 10 years is *closest* to:
    A. 2.94%.
    B. 2.97%.
    C. 3.00%.

36. The harmonic mean return over the 10 years is *closest* to:
    A. 2.94%.
    B. 2.97%.
    C. 3.00%.

37. The standard deviation of the 10 years of returns is *closest* to:
    A. 2.40%.
    B. 2.53%.
    C. 7.58%.