Lecture Notes in Networks and Systems 172

Paola Magnaghi-Delfino
Giampiero Mele
Tullia Norando *Editors*

Faces of Geometry

II Edition

Springer

Lecture Notes in Networks and Systems

Volume 172

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Faces of Geometry

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ISSN 2367-3370 ISSN 2367-3389 (electronic)
Lecture Notes in Networks and Systems
ISBN 978-3-030-63701-9 ISBN 978-3-030-63702-6 (eBook)
https://doi.org/10.1007/978-3-030-63702-6

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

This book is dedicated to the memory of our colleague and friend Paola Magnaghi Delfino.

Paola has been working for many years in our Department, employing the best of her efforts in planning and carrying out many initiatives. In particular, as a member of the FDS didactic laboratory, she was actively engaged in several projects dedicated to high school students, aiming at the improvement of their scientific knowledge and, in particular, at the study of mathematics through art. In particular, she took active part in the projects "Progettiamo con la Matematica" (Planning with Math), "In Action with Math" and "Matemartiamo" (Math & Art). All these initiatives were attended by hundreds of students, together with their teachers, throughout the years.

Paola was interested in all the fields in which Mathematics plays a role; another example is her collaboration with the company "Pacta dei.Teatri" for a series of theatre shows, inspired by mathematical concepts.

Yet another field in which Paola was engaged is the valorisation of the contributions made

by women to scientific research in Mathematics. In particular, Paola was deeply involved in the conception and organization of the international conference "Faces of Geometry, from Agnesi to Mirzakhani". The Proceedings of the second edition of this conference are published here.

The whole staff at the Department of Mathematics of the Politecnico Milano, including of course myself, is deeply grateful to Paola for all the work she carried out with enthusiasm and competence. I hope that reading these proceedings can act as an incentive to the youngest researchers to undertake the difficult but important field of dissemination of Mathematical Knowledge in multidisciplinary contexts. And I am sure, that Paola would share this intent with us.

—Giulio Magli, Head of the Department of Mathematics, Politecnico Milano

Dear Paola,

We began to work together by chance, then we continued by choice. Sharing the same passion for Mathematics, we have decided to engage in experimenting with innovative forms of students training and dissemination. Paola, do you remember when I pushed you to go on stage to explain the mathematical arguments of theatre shows? You were frightened but you faced it with great courage. Not to mention the time when you arrived by car to set up the panels for the exhibition saying: "If the students can't come to the Politecnico, let's go to them!". And what about the time when we decided that art was the key to bringing scientific thinking into

the hearts of students, and you were telling me about how your father was talking to you about Architecture. That was the point when we connected, and you let me into that same world where you were putting all your heart in, and all your enthusiasm, determination, and will to overcome any obstacles.

A few years ago, during a conference in Bratislava, we met Giampiero with whom we immediately bonded. Giampiero, an architect like your father, immediately became part of our group. We shared new ideas that linked Mathematics to Architecture first, and then to Art. A collaboration that quickly turned into friendship. True Giampiero?

Of course Tullia! You and Paola for me were true guardian angels, when I was in trouble you were there ready to support me. I shared many great moments with you. Mathematics and Architecture and then Art were like the glues for true spontaneous collaborations in which, each of us with our own specificity, entered to complete each other's work. In a few years we have worked a lot together and we have had and shared ideas and passions. The proceedings of this second Faces of Geometry conference are a proof of this. When I think about this time, I don't remember the fatigue but the pleasure of spending time together. Unfortunately, Paola left us suddenly and we didn't have time to do everything we had planned. Paola were a volcano of both sympathy and goodness. In her name and honour we will try to carry out all the projects we had created together even though we miss her. Paola, up there observing us, will enjoy seeing and hearing

our lucubration and will take us back as your usual.

Dear Friend, we only ask you to intercede for us as only you know how to do when you want to get something.

You will always be with us.

—Tullia and Giampiero

Preface

Some introductory remarks about the reasons that motivated the choice of the topics of the conference Faces of Geometry. From Agnesi to Mirzakhani. We have two purposes, equally important. First, we have the intent of promoting interdisciplinary discussions and connections between theoretical researches and practical studies on geometric structures and its applications in architecture, arts, design, education, engineering and mathematics. Indeed, we believe that these related fields of study might enrich each other and renew common interests on these topics through networks of common inspirations. We invite researchers, teachers and students to share their ideas, to discuss their scientific opinions in teaching these disciplines, in order to enhance the quality of geometry and graphics education. Second, but not less important. We are sure that the scientific community and mathematics, in particular, need the contribution of women. Women have made significant contributions to science from the earliest times. Historians with an interest in gender and science have illuminated the scientific endeavours and accomplishments of women, the barriers they have faced, and the strategies they have implemented to have their work peer-reviewed and accepted in major scientific journals and other publications. The historical, critical and sociological study of these issues has become an academic discipline in its own right. In 2018, we celebrated, in Politecnico di Milano, the anniversary of Maria Gaetana Agnesi, Milanese mathematician, the first woman to write the first vernacular handbook of mathematics for learners.

In 2019, we celebrated the first Women in Mathematics Day, dedicated to Maryam Mirzakhani, the first woman who won the Fields Medal. The Turkish mathematician Betul Tanbay, in her tribute to Mirzakhani, recalled the classic geometric problem, called illumination problem, and compared Maryam Mirzakhani to candlelighting the path for others to follow. Quoting, she said "Maryam showed forever that excellence is not a matter of gender or geography. Maths is a universal truth that is available to us all". During the conference, we commemorate Giuseppina Biggiogero, the first woman who taught Descriptive Geometry in the Faculty of Architecture at Politecnico di Milano.

The Organizing Board of the Conference announced the birth of The International Association in Mathematics and Art—Italy (IAMAI), promoted by Italian scholars from various academic, disciplinary and cultural backgrounds. The mission

of the association is the promotion of researches and the dissemination of results in the various application fields, in reference to national and international contexts, enhancing the plots and convergences between areas that link mathematics to art, opened to forms of collaboration and involvement of other subjects, institutions and organizations. Mathematics is the fruit of the thought both creative and logical, inspired and deeply linked to the beauty, recognizable in various expressions of art, from architecture to design and fashion, from painting to sculpture, from music to dance and theatre, including their digital and virtual expressions. For centuries, Italy has been a land of promotion and encounter between art and science and our country is full of signs of the Italian Cultural Heritage. The aim of the association is to give the maximum sharing to these witness through the appropriate communication and publishing channels.

The second edition of the conference Faces of Geometry was scheduled for 11 and 12 May 2020, the world day of Women in Science. Due to the situation generated by COVID-19 in large part of Europe, after careful consideration of the Organizing Committee, the congress has been postponed.

The speakers' response to our request of preparing their presentation for new scheduled dates has been in large part positive and for that we thank them.

Milan, Italy — Paola Magnaghi-Delfino
Novedrate, Italy — Giampiero Mele
Milan, Italy — Tullia Norando

Organizing Board

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The Siege of Florence Between Chorography and Scenography

Maria Teresa Bartoli

Abstract The "Siege of Florence", painted by Vasari to celebrate the event that restored Medicean power in Florence and made Cosimo's lordship possible, shows a perspective image of Florence depicted in a vast landscape, from the eastern hills to the western Arno valley. The painting shows the marvelous natural basin in which Florence is situated, and its treasured buildings, castles and convents scattered over the hills occupied by the enemy. In his *Ragionamenti*, Vasari describes the steps he took to achieve his perspective: (1) a map of the surroundings of Florence, (2) the observation from the highest southern viewpoint. Perspective technique involves first drawing the plan and the profile and then intersecting the projecting rays. This would have enabled him to place "20 miles of territory" (about 35 km) in "a space of 6 arms" (3 m). Are these explanations enough? Is the image adequate? How accurate it is when compared with the actual geometric reality of the places? How realistic is the image of the River Arno? The answer to these questions requires a more detailed investigation of the actual geometric strategies implemented by the artist.

Keywords History of perspectives · History of science · Renaissance · History of land surveying

1 Introduction

At the end of the fourteenth century, the rediscovery of Ptolemy Almagest and Geography gave rise to a new area of research divided into three branches. Two of them, cosmography and geography, rested entirely on the sciences (mathematics and geometry, and therefore drawing). A third, chorography, namely the description of a region, depended on the first two and, to a significant degree, on the figurative, requiring the imitation of nature, familiarity with proportions and colours, and knowledge of the rules of architecture. In the 1400s and 1500s in Europe this manifested in the genre of views, often of a city, produced in the form of a printed

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P. Magnaghi-Delfino et al. (eds.), *Faces of Geometry*, Lecture Notes in Networks and Systems 172, https://doi.org/10.1007/978-3-030-63702-6_1

Fig. 1 *The Siege of Florence* by G. Vasari, Palazzo Vecchio, Florence

drawing, which could be published in numerous editions. This figurative typology alluded to two scientific practices: geometric surveys and perspective [1]. Florence also took part in these activities: the first representations of the city were produced in the last quarter of the fifteenth century: they are the famous "Carta del Massaio" and the "Veduta della Catena," the former a sort of orthogonal projection, the latter a pseudo-perspective. There was an additional application similar to this type of action but more geared towards the scientific field: the survey of fortresses and territories for military purposes, used to obtain plans also by applying the *inverse procedure* to perspective drawings (obtaining plans from perspectives [2]). In Cosimo dei Medici's Florence, all these research areas existed and were pursued by men of science and action, in conjunction with their counterparts in the courts of Europe.

Vasari's *Siege of Florence*, in Palazzo Vecchio (painted between 1556 and 1562, in the Sala di Clemente VII in Palazzo Vecchio, Fig. 1), fits into this figurative trend, with some specific characteristics: (1) it is not a map, but rather a fresco that celebrates a historical event; (2) it represents a war story[1] which involved not only the city but also the nearby mountains and the Arno valley. A vast landscape following the river for around 35 km between Florence and Pistoia is depicted in a panoramic scene impossible to take in from a single viewpoint. The fresco is 4.72 m long and 2.07 m high. In it men are shown only as miniatures, participants in the events of a story full of martial and chivalric episodes, scattered about in an image dominated by the vastness of the scene.

[1] The siege depicted in the painting was brought upon Florence between 1529 and 1530 by the army of Charles V, who had made a pledge to the Medici Pope Clement VII to restore the family's power in the city, from which it had been banished a few years earlier. The siege ended after about a year with the famous defeat of Francesco Ferrucci in Gavinana.

2 The Urban Survey in Florence in the Sixteenth Century

The city within the walls, dominated by the dome, is drawn with accurate representations of the main monuments, major churches, squares and roads leading to the walls. In the "Fourth of his *Ragionamenti* of the second day" [3, IV], in which he describes the problems of the fresco, Vasari explains that he "portrayed it in a natural and measured way that does not stray far from the truth". He says nothing about this "way" in his *Ragionamenti*, almost as if he assumed the image was known.

The verisimilitude of the surrounding landscape is the technical innovation considered, as it represented the challenge the author had to overcome. A map was needed. By the time the siege occurred, the necessities of war had already driven Pope Clement VII to make a peculiar request to a highly skilled *maestro di levar piante* (plans surveyor) Benvenuto da Volpaia [3, IV, 61], a mathematician and astronomer: a survey of the city and the nearby hills, for one mile outside of the walls. The Pope's aim was to study the terrain in view of the impending siege. Benvenuto requested the help of a man of art versed in surveying, Tribolo; over the course of several months, working at night, they produced the survey in the form of a wooden model, as required by orography. The story of the survey is known to us from Vasari's account of it in the Life of Tribolo. The model was sent to the Pope who was very happy with it; it was returned to Florence after the Pope's death, but there is no other information about it.

Cosimo, who was extremely interested in knowledge of the territories to be conquered or defended, summoned to his court mathematicians, architects and engineers who refined the land survey and topography. In 1556 he commissioned Vasari to produce a perspective plan of Florence. Vasari responded by asking for clarification, but the outcome is unknown to us [2, note 15]. In the fourth *Ragionamento*, in describing the problems with the fresco, he mentions a map of the places around the city within one mile of the walls, which he produced using a compass. In 1559 Cosimo Bartoli, a humanist with wide-ranging interests at the court of the Duke, wrote "How to measure distances, surfaces, bodies, maps, provinces and all other earthly things" published in Venice in 1567. In the fourth book of the work, on provinces, he describes the construction of a compass (a circle with a diameter of 19.5 cm), necessary to survey territories. The circle of the compass is divided into 360°, each degree is divided into 5 min. A magnetic needle points North; a pointer identifies the position of the places to be surveyed with the help of a second vertical compass, integral with the pointer of the first (Fig. 2a). An illustration shows the drawing in relation to a practical example he had made: the survey of Florentine places from Giotto's bell tower (Palazzo Pitti, Torre degli Alessandri in Vincigliata, Fiesole, Villa la Petraia, Bellosguardo, Belvedere, Monte Uliveto, Bastione di San Giorgio, Mercato Nuovo, Fig. 2b): it seems to be the start of a survey of the city and its surroundings.

In around 1557 Cosimo hired the military architect Baldassarre Lanci to fortify the territory of Siena.

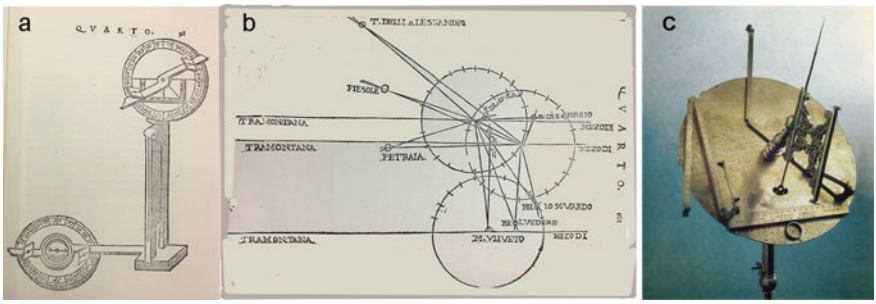

Fig. 2 **a** Cosimo Bartoli, the compass for surveying; **b** Cosimo Bartoli, example of topographical survey; **c** distances meter of Baldassarre Lanci. Museo Galileo - Istituto e Museo di Storia della Scienza, Florence

Lanci set about surveying buildings and territories of military interest and invented a tool to survey them (Fig. 2c). The instrument developed in 1557 is suitable for representing both landscapes in orthogonal projections and buildings in central projections, obtaining plans and elevations from them [2]. He also invented a perspective tool with a circular framework, which was praised by Daniele Barbaro in The Practice of Perspective (Venice 1568) and instead torn apart as a source of representation errors by the mathematician Danti (who also frequented Cosimo's court from 1567) in Vignola's *Two Rules of Practical Perspective* (Rome 1583), edited by him (Fig. 3). The instrument projects the view onto part of a cylinder, thereby transferring the images of real points onto it by means of a pointer coaxial to a finder placed on an axis in the centre of a portion of the cylindrical surface, which is then straightened on the plane. The device reduces the line of vision of an open view of 120° on the cylinder to an opening of 90° on the plane with the result that what would escape a motionless eye can be drawn as visible. Our vision, though, even if the head remains

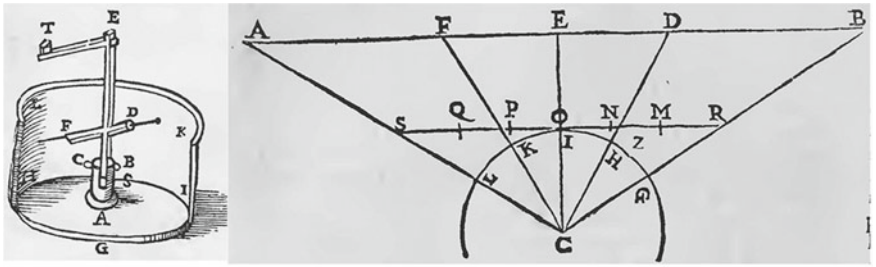

Fig. 3 The perspectograph of Baldasarre Lanci, drawn by Danti in Vignola Treatise

Fig. 4 The course of the Arno river between Florence and Empoli, according to Google Earth

in one position, allows us to turn our gaze to take in a wide scene. Lanci's image may be more similar to the view than Danti's, assembling the views of many main points. Lanci faced a new problem, the unitary and compact drawing of wide-ranging views. The great difficulty faced by Vasari exceeded the topics on the agenda. On Google Earth we see 20 miles of territory (Fig. 4). It was impossible to see them all from the same viewpoint. The image is not a panorama in a technical sense; however, it is unitary and persuasive. Vasari had only surveyed the territory within one mile of the walls. Do the representation and what is represented correspond?

3 Geometric Analysis of the Fresco

The analysis was conducted with reference to what Vasari did not have, namely a credible plan, constructing a perspective diagram of the places in his drawing. The viewpoint was located at the highest point of Pian dei Giullari, at the top of Via di Monteripaldi (height of 200 m), and at a higher height, which is not real because the perspective is a bird's eye view. The reference plan was the Florence Contour Map in Fantozzi's Guide to Florence, of 1848 (see Fig. 5, on which the radius circle one mile outside the walls and the graphic plan for the perspective construction were also marked). It was thought that the schematic perspective obtained (Fig. 6) provided better insights about what Vasari intended to communicate with his figures. The

Fig. 5 Detail from the Fantozzi's map of the surroundings of Florence. Bleu, the circle one mile outside the walls; red and green the plan drawing for the perspective scheme

horizon has been imagined above Fiesole, just above the haze of the Mugello region, because at that height the depth of perspective of Florence is compatible with that of the fresco.

The diagram only takes into consideration the salient points of the fresco, which have been associated with the elevations provided by Google, having deducted that of Florence. When superimposed on the fresco, it shows that the line of the walls, even if it differs from the real line, is acceptable in its extension. The inside of the city is described convincingly and is clearly the protagonist of the painting. The places on raised ground (all recognizable), compared with the results of the diagram, are a considerable distance from the geometric reality of the line of vision, with shortened or elongated distances and heights in order to emphasize, bring closer, move and group the many places portrayed within the space of the painting, always however managing to maintain a recognizable topological relationship between them.

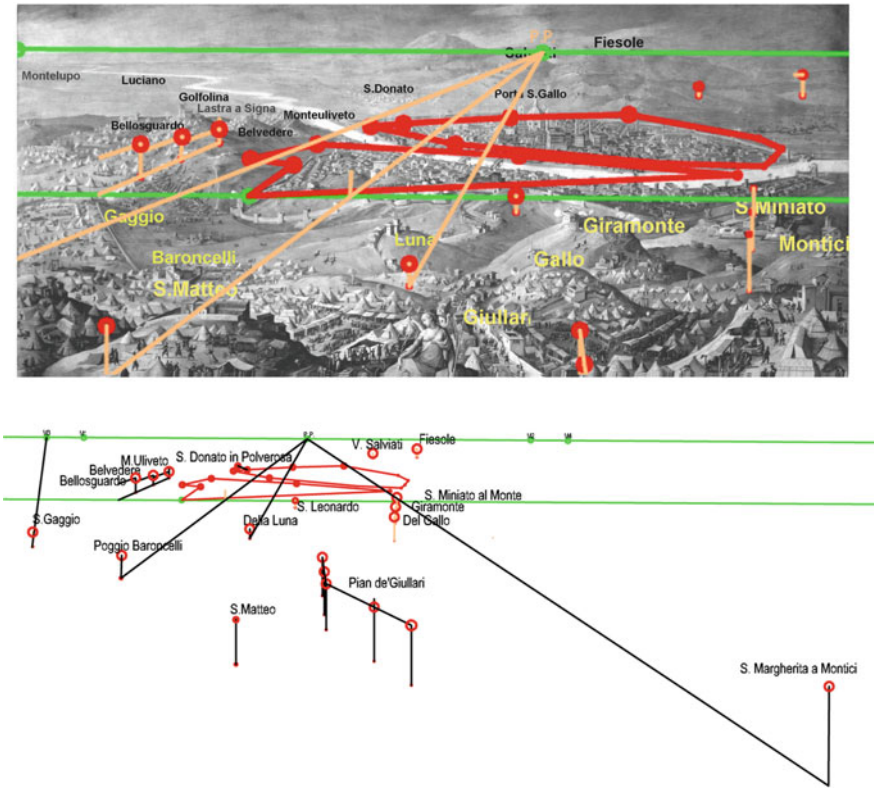

Fig. 6 Low, perspective of the scheme; high, the perspective put on the fresco

A comparison between the painting and the diagram therefore shows that in the image the horizon of the hills to the south is lower than that of the mountains to the north; the perspective cone towards Torre del Gallo, Giramonte and San Miniato (all closer to the city) would place the Tower higher, blocking our view of the others, while they are shown as visible in the painting, as if the vertex of the cone had been moved further east. The three hills in sequence in the southwest (Bellosguardo, Belvedere and Monte Uliveto) are brought closer to the city and pulled in towards the river, as if the eye had been moved to the west, while the convent of San Salvi in the northeast would like the eye on the hills to be moved east. The hills of Arcetri and Pian dei Giullari would be outside the fresco field and S. Margherita a Montici would be almost behind the eye's gaze. Fiesole and the hills to its right are moved to the west and brought closer to the city's most important monuments; the Mugnone stream rightly starts from Porta San Gallo, but the Porta is shown much further west than it actually is and placed strangely between the Bell Tower and the Dome of the Cathedral. Monte Morello, represented as an isolated cone, is seen from the east. In summary, the surroundings of the city are represented as closing in on it and ordered so as to make them visible in a small space, with a horizon that changes in height.

San Gaggio, on the left, deserves special observation. In the painting it is placed on Via Senese which exits Porta Romana, on a plane parallel to the painting. In order see it as positioned in the fresco, the viewpoint above Pian de'Giullari should rotate the gaze left, in an orthogonal direction to a plane passing through San Gaggio and the Gate: Landi's instrument would make this view compatible.

4 Towards a Chorography

The Arno leaves the walls with winding movements that would demand an even higher horizon. The river, however, would be seen as little more than an incision in the ground. How was it known? It was drawn in the early sixteenth century by Leonardo da Vinci [4]. A sheet of the Madrid codex traces its course between Florence and Pisa (Fig. 7). The names of the places written along its path allow a comparison with the real river and show that the drawing highlights all its meanders. Vasari's image is perhaps similar (smaller but emphasized) to the shape and proportions of its movements (Fig. 4). In the absence of images of urban settlements along the meanders, we can only attempt to associate them with toponyms: Lastra a Signa, Golfolina, Montelupo, Empoli; in the background, the mountains of Pistoia to the north descend towards the river and meet with the hills to the south behind which the river disappears. Cosimo brought renewed attention to the system of rivers, assigning to them two of the ten Captains of the Party, established in 1545 [5].

Later on, a team of master-builders skilled in surveying were appointed, whose maps that escaped the fires are conserved in the Piante di popoli e strade (A.S. Fi).

Fig. 7 The Arno river from Florence to Empoli by Leonardo da Vinci. Madrid codex, Biblioteca Nacional de España, Madrid

Fig. 8 The Arno in front of Signa

Many of them describe the course of the Arno; only the map that illustrates the meander below Signa remains (Fig. 8), of the stretch between the city and Empoli. It may however have been possible to create a model, whose perspective image was juxtaposed with that of the city. The urban settlements of Sesto, Prato and Pistoia are reported as between the river and the mountains to the north. The panorama was testimony of the ducal government's attention to the territory.

5 The Conclusion, a Scenography

Florence dominates the painting. The monuments, squares and streets within the walls are distributed in a plausible way and suggest that there was a map or that one was being worked on. But the perimeter of the city to the north of the Arno, outlined

by the walls and the right bank of the river, has the strange shape of an almond within two circular arches and does not resemble the image of a correct perspective. In the sixteenth century, Baldassarre Peruzzi (died in 1536) produced a drawing of the walls of Florence (Gallerie degli Uffizi, Florence) very similar to those in the fresco (Fig. 9)

The city to the north of the river is contained between two circular arches with accentuated concavity (like the drawing of the Carta della Catena, Fig. 10); the gates of the north walls are distributed on the upper arch like those of Vasari. The city depicted in the fresco would seem to be drawn within those walls. In the Life of Peruzzi [6, III] Vasari praises his work as both an architect and a painter famous for creating scenic apparatus and perspectives, passionate about mathematics, and an expert in fortresses (between 1527 and 1529 he was engaged in fortifying Siena). In 1529, Pope Clement VII proposed that he help organize the camp for the siege of Florence; he refused for moral reasons, but the request reveals his worth as a land surveyor. The scene he designed for La Calandria, a famous comedy written by Cardinal Bibbiena (and an acclaimed literary work), was well-known in his time. Vasari's description enthusiastically summarizes its most noteworthy characteristics:

> Nor could one imagine how he, in such a narrow place, made room for so many streets, palaces, and various temples, balconies and cornices so well made that they seemed not imitations but very real, and the piazza not painted and little but real and very large. [6, III, 284]

Fitting many beautiful architectural works into a very narrow space reveals a special rhetorical meaning in the drawing and the scenographer's educated participation in the scholar's work (Fig. 11).

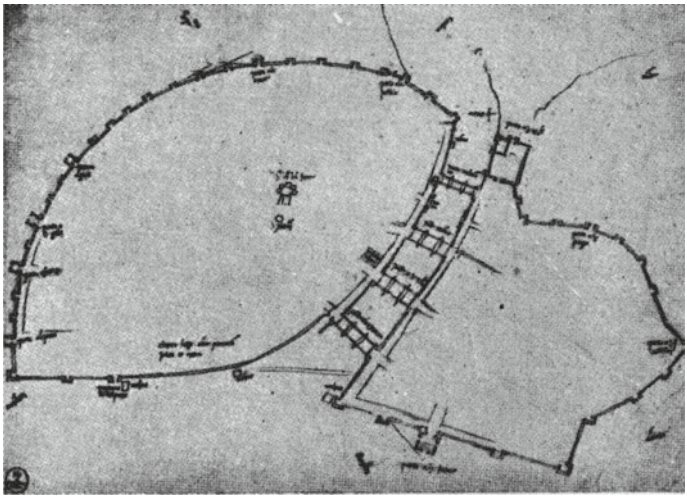

Fig. 9 Peruzzi, Pianta di Firenze, G.D.S. Uffizi (A.S. Fi, 595)

Fig. 10 View of Florence called "Carta della Catena", engraving attributed to Francesco Rosselli, 1470, Kupferstichkabinett Berlin

Fig. 11 Detail of *The Siege of Florence* by G. Vasari, Palazzo Vecchio, Florence

The drawing of a theatre scene (G.D.S. Uffizi U298A), which belonged to Vasari and was attributed by him to Peruzzi, shows the characteristics of the drawing (Fig. 12): it compresses into a small space, with meticulous scenic texture, a rich series of references that make the place recognizable (a square in Rome), highlighting them to the full [7]. Such was the spirit with which Vasari produced his painting on the Siege, free of the accuracy of the right angle but committed to showing, through consistent references, the well-known qualities of the Florentine landscape, the nobility of man-made constructions and the elegant lines of the narrow mountains around the city, followed by the undulating flow of the river in the distance. He did not seek an exact perspective reflecting reality, but rather a correct sequence of

Fig. 12 Drawing of Theatral scene, attributed from Vasari to Peruzzi. Gallerie degli Uffizi, Florence

images which in short best describe the work of nature and man, coming together to craft a scene in which the events of a tragic war are depicted, in small miniatures, in the places where they took place, "so that little strays far from the truth" [8, VI, 221] and so that every observer familiar with the place could recognize each site and each war event that took place there.

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