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Cathleen Heil

The Impact of Scale on Children's Spatial Thought

A Quantitative Study for Two Settings in Geometry Education

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The Impact of Scale on Children's Spatial Thought

A Quantitative Study for Two Settings in Geometry Education

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… my methods of navigation have their advantage. I may not have gone where I intended to go, but I think I have ended up where I needed to be.

—Douglas Adams

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Mein besonderer Dank gilt meiner Doktormutter Prof. Dr. Silke Ruwisch für ihre kontinuierliche fachliche wie auch persönliche Unterstützung während meiner Promotionszeit. Sie half mir mit sehr viel Geduld und Vertrauen, den richtigen Weg für mein Vorhaben und mich zu finden, ohne das Steuer an sich zu reißen. Das ist nicht selbstverständlich und gab mir die Freiheit, der es für meine Navigation durch den (wissenschaftlichen) Ideendschungel bedurfte.

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Abstract

Children's thought about space is influenced by their abilities to perceive, encode, and mentally manipulate spatial relations they experience and explore in everyday life. Geometry education in primary school aims to support children as they organize those experiences at an abstract level and develop cognitive abilities to consciously manipulate spatial information in different spatial settings, that is, their spatial abilities. Many studies have investigated children's abilities to mentally manipulate spatial relations in tabletop settings but not those required when the self is located or moving in real space. Addressing this gap in the literature, this study proposes map-based spatial tasks in real space and examines the relations of individual differences in the corresponding underlying cognitive abilities used to solve spatial tasks at both scales of space, small-scale and large-scale spatial abilities, in greater detail.

Using a correlational study design, this study investigates the relation between performances of 240 fourth graders on a mid-sized German university completing paper-based tasks in a classroom setting and map-based orientation tasks in a real space setting. The former test consisted of a subset of tasks that required the children to mentally manipulate object-based transformations and another asking the children to transform the imagined self. The latter test mimicked the practical use of maps such as indicating the direction toward unseen locations, finding one's position and viewing direction on the map, or navigating toward a predefined goal. The test also included tasks without a map that required the children to make inferences on directions to landmarks from survey knowledge acquired during movement in space.

Descriptive results revealed that paper-and-pencil tasks requiring multistep mental transformations of abstract and complex spatial information were appropriate means to measure individual differences in children's performances reflecting small-scale spatial abilities. Moreover, maps were found to be potentially powerful cognitive tools for teachers and researchers to stimulate and measure children's spatial thought in real space. By comparing different models in confirmatory factor analyses, the study showed that at both scales of space, spatial abilities should not be treated as an undifferentiated construct but rather be understood as multidimensional.

The results suggested that a two-factor model distinguishing between object manipulation and perspective transformation abilities might be an option to model small-scale spatial abilities. They also confirmed a three-factor model distinguishing between the abilities to make inferences on directions from survey knowledge and two subclasses of map use, namely the abilities to transform information from the map to the referent space, comprehension abilities, and the ones to use information from the referent space to reason about spatial locations on the map, production abilities.

The results of multivariate statistical analyses at the manifest and latent level indicated that children's spatial abilities at both scales of space are partially but not fully related. These results specify the degree of overlap between subclasses of small-scale and subclasses of large-scale spatial abilities, clarify the role of visuospatial working memory as a mediator when it comes to relations with abilities to use a map in real space, and emphasize the predictive role of particular spatial tasks. The results provide new insights regarding the similarities and differences between both classes of spatial abilities. The findings of this study contribute to the literature in the study of spatial thought in mathematics education and provide empirical evidence for the development of pedagogical interventions both in geometry education and beyond.

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Introduction

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"Tracks," said Piglet. "Paw-marks." He gave a little squeak of excitement. "Oh, Pooh! Do you think it's a – a – a Woozle?" "It may be," said Pooh. "Sometimes it is, and sometimes it isn't. You never can tell with paw-marks." With these few words he went on tracking, and Piglet, after watching him for a minute or two, ran after him. Winnie-the-Pooh had come to a sudden stop, and was bending over the tracks in a puzzled sort of way.
"What's the matter?" asked Piglet.
"It's a very funny thing," said Bear, "but there seem to be two *animals now. This – whatever-it-was – has been joined by another – whatever-it-is – and the two of them are now proceeding in company. Would you mind coming with me, Piglet, in case they turn out to be Hostile Animals?"*

(Winnie-the-Pooh by Milne & Rojahn-Deyk 1988, pp. 46–48)

Children and Space

Space plays an important role in children's everyday life and behavior since it is subject to their experiences, observations, and activities. Starting in toddlerhood, children explore and interact with the space that surrounds them, first by crawling, then by touching objects of different sizes and shapes and observing their positions and locations in space. Later on, children begin to understand that these objects exist beyond their direct experience by reasoning about them and their visual-spatial characteristics. In other words, they develop a conception of space.

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Seeking to understand the space they interact with, children commonly face two types of cognitive challenges: first, the perception of forms, colors, and structures and second, the handling of spatial relations. The first challenge typically refers to characteristics of objects that are mostly time-invariant. Successfully dealing with the second challenge, however, allows children to gain a deeper understanding of the space they explore, since spatial relations may change over time whenever an object is moved or the child moves. In the latter case, merely perceiving spatial relations is not sufficient, because children also have to cognitively engage with the spatial situation by mentally encoding and transforming it to keep track of or to anticipate changing spatial relations (D. H. CLEMENTS 1999). Hereby, children have to understand that relations between various objects and the self are organized in different spatial frames of reference, which constantly change over time (HERSHKOWITZ, PARZYSZ, & VAN DORMOLEN 1996).

Children's cognitive abilities of knowing where they are and where objects are located with respect to their surrounding environment and of drawing purposeful spatial inferences from those configurations, *spatial abilities*, are indispensable practical life abilities and are developed from toddlerhood to early adulthood (e. g., NEWCOMBE 1982). When children solve *spatial tasks*, spatial abilities enable *spatial thinking* during solving these, which can be described as reasoning about space by mentally manipulating a broad class of task-specific spatial information, that is, reasoning about relations between locations and configurations of objects and the self and how they can vary over time (NEWCOMBE & SHIPLEY 2015). Spatial tasks may differ in terms of the spatial information available, but reasoning about these pieces of information usually involves the construction of mental representations and subsequent inference processes in transformations of those (HEGARTY & WALLER 2005).

Whenever spatial abilities are developed only to a little extent, situations that require the safe handling of varying spatial relations may, as in the case of Winnie-the-Pooh and Piglet, become challenging. Indeed, spatial challenges located in real space such as the one experienced by Pooh and Piglet, seem to be particularly difficult since they require the spatio-temporal integration of information that is only partly perceived from different viewpoints while moving through space. This might not be the case for settings involving very limited spatial information such as written material, where all spatial relations can be observed from a single vantage point. Solving spatial tasks in both settings can be assumed to be enabled by spatial abilities, but it is not clear whether there are indeed different classes of spatial abilities required to master them or not. Gaining a deeper understanding of this relation is the primary goal of the present study.

Spatial Abilities in Conceptions of Geometry Education

> What is geometry? […] There can be no doubt what I should then answer—geometry is *grasping space*. And since it is about the education of children, it is grasping that space in which the child lives, breathes and moves. The space that the child must learn to know, explore, conquer, in order to live, breathe and move better in it.
> (FREUDENTHAL 1973, pp. 402–403, emphasis added)

Having had a range of spatial challenges in everyday experiences from infancy to toddlerhood, children have acquired an early spatial knowledge that they bring to school. The fact that spatial thinking is pervasive in the children's everyday life does, however, neither imply that it is mastered without efforts nor that there is no need for spatial education at school. Geometry education in primary school typically focuses on organizing and structuring this implicit experience-based knowledge by fostering cognitive abilities that allow children to perform mental inferences on spatial information in various contexts of their everyday and school life (P. BRYANT 2009). School geometry can therefore be a means to help children using spatial objects and sucessfully navigate spatial situations encountered in an informal way, to help them to develop the abilities to analyze observed relationships and transformations in a formalized, mathematical manner, and ultimately, to develop a clear conception of space.

Undoubtedly, these ideas of geometry education at primary school are closely linked to the ideas of FREUDENTHAL (1971), who called for a geometry curriculum in which "geometry should be related to physical space […] nobody can deny that we live in space, that we move in space, that we analyze space, to be better adapted to it" (p. 418). Freudenthal's vision has been at the core of modern geometry classes in primary school and cited within the conceptual framework of international comparative studies (e. g., OECD 2004).

Spatial abilities have been generally acknowledged as important cognitive abilities contributing to geometry learning (P. BRYANT 2009; D. H. CLEMENTS & BATTISTA 1992; D. H. CLEMENTS 1999; HATTERMANN, KADUNZ, & REZAT 2015; HERSHKOWITZ ET AL. 1996; K. JONES & TZEKAKI 2016; PINKERNELL 2003; SOURY-LAVERGNE & MASCHIETTO 2015). As a matter of fact, they have been systematically incorporated into conceptions of geometry education and development of geometry curricula since the early 80s (SINCLAIR & BRUCE 2015). TAHTA (1980), for example, emphasized the role of spatial abilities by proposing that visual images are fundamental in mathematics education and that geometry in particular is all about becoming aware of imagery that "arises from a dynamic process of the mind" (p. 6). In the German mathematics education literature, the role of spatial abilities was emphasized by BESUDEN (1999a) who stated that "understanding geometry is primarily about understanding spatial relations and their interdependencies which is impossible without involving spatial abilities" (BESUDEN 1999a, p. 3, translation by the author)[1]. This idea was further supported in the study by WITTMANN (1999) and MAIER (1999b).

The idea that geometry education should engage spatial abilities has been embedded in the curricula of different countries worldwide. For example, The National Council of Teachers of Mathematics (NCTM 2000) has considered spatial abilities by emphasizing practices related to two core domains: first, using "visualization, spatial reasoning, and geometric modeling to solve problems" in real space while "building and manipulating mental representations of two- and three-dimensional objects and perceiving an object from different perspectives" (p. 41) and second specifying locations and describing relationships of spatial configurations, that is, being aware of changing relative positions in space (p. 42). Similar ideas, which partly specify the use of maps and construction plans, can be found in mathematics curricula for primary school in Australia (NEW SOUTH WALES BOARD OF STUDIES 2012), England (DEPARTMENT FOR EDUCATION 2014), Canada (MINISTÈRE DE L'ÉDUCATION ET DU DÉVELOPPEMENT DE LA PETITE ENFANCE 2016), or Germany (KMK 2004).

Despite this general consensus regarding the importance of spatial abilities in geometry education, the relationship between these abilities and geometry learning is less well understood (MULLIGAN 2015). Addressing this epistemological question about the cognitive nature of spatial and geometrical abilities and their relation in the context of geometry education, PERRIN-GLORIAN, MATHÉ, AND LECLERCQ (2013) proposed a basic model of spatial and geometrical thinking in the context of geometry education (Figure 1.1). They enlarged the theoretical framework of BERTHELOT AND SALIN (1993) who distinguished between spatial and geometrical knowledge and conceptualized teaching and learning of geometry as a problem-solving and modeling process that takes place in three types of spaces: the physical, geometrical and graphical one (PERRIN-GLORIAN ET AL. 2013). According to their model, spatial and geometrical abilities are closely related but not interchangeable and activities in geometry education involve the three spaces mentioned above.

According to PERRIN-GLORIAN ET AL. (2013), the physical space is the three-dimensional space of concrete action with objects and the space in which real problems and activities are defined. The geometrical space is the space of Euclidean axioms and formal deductions that serve as abstract-cognitive tools to solve a

[1] German original: „Geometrieverständnis ist vor allem die Einsicht in Beziehungen und Zusammenhänge, was ohne Raumvorstellung nicht denkbar ist."