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Macarena Larrain Jory

Preservice Primary Teachers' Diagnostic Competences in Mathematics

Assessment and Development

Springer Spektrum

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Preface

This thesis explores the initial education of primary teachers in the mathematics area. In this vast field, the author focuses on the diagnostic competence of preservice teachers in error situations and finds very interesting results regarding this competence and its relations with variables associated with the background and disposition of preservice teachers. Moreover, the author designs a short seminar sequence to improve the diagnostic competence of preservice teachers in error situations and reports changes in this competence and studies the relations of these changes with the same variables mentioned above.

The analysis and conclusions of this study are drawn on research data from Chilean preservice teachers studying in various universities in the capital Santiago. However, the results shed light on general issues regarding the diagnostic competence in error situations, its relation with other variables and the possibility of developing it during teachers' initial education. In addition, this thesis opens very interesting research questions to further studying this crucial competence in mathematics education.

Rather than comment on the range of validity of the thesis' conclusions and its value in terms of the advance of research in the area, which is the role of thesis readers and future peer reviewers, I would like to discuss the value of this thesis regarding ideas, clues and consequences for the initial education of teachers in Chile. By doing so, readers from other countries may possibly identify some of the ideas that can be applied to their own reality.

Around the globe, changes and reforms in education and mathematics education have been taking place aiming to adjust goals and methods to the new realities imposed by technological changes. Chile is no exception, and national curricular changes have been taking place for about ten years. However, changes are slow at the school level and, paradoxically, even slower at the university level.

This situation can be observed in various ways, but data from this thesis may provide a different angle. As a result of the study, more than one-third of the sample did show instructivist preferences for dealing with student errors and another one third didn't show substantial answer. Thus, only one-third of preservice teachers in the sample were able to provide a constructivist approach to students to deal with the observed error, that is a response aligned with reform ideas.

As a conclusion of the thesis, a higher competence to hypothesize about causes of student errors is related to stronger mathematical knowledge for teaching, to practical experience in teaching and to constructivist beliefs about the nature of mathematics and about mathematics teaching and learning. Even more, constructivist beliefs are associated with the group of preservice teachers showing constructivist preferences for dealing with student errors, and teacher-directed beliefs about mathematics learning and views of mathematics as a set of rules and procedures are associated with the group of preservice teachers showing instructivist preferences for dealing with student errors. Considering this, we can suggest that initial teacher education programs should include a strong emphasis on mathematics for teaching and practice-related activities, i.e. real and/or simulated teaching. These conclusions allow us to be optimistic about the future since these activities are strongly encouraged in the initial education of teachers in Chile.

Interestingly enough, this thesis additionally provides a short seminar sequence that showed to have positive effects in improving the competence to hypothesize about causes of student errors and a slight increase of the number of preservice teachers showing constructivist preferences for dealing with those errors. Here again, the positive changes were related to mathematical knowledge for teaching and to practical experience. These results show the suitability of this short seminar setting, with a practical approach together with discussion and reflection activities, for improving the diagnostic competence during initial education, and possibly other teaching competences.

As a result of the analysis, the conception of mathematics and mathematics teaching and learning appears as a very important variable related to preservice teachers' diagnostic competence in error situations and to the changes after the short seminar sequence in which they participated. On the one hand, we have constructivist beliefs about the nature of mathematics, in which mathematics are viewed as a flexible construction process, were rules appear as necessary, and helps to understand and solve problems. On the other hand, the conception of mathematics as a set of fixed rules and procedures that have to be learned with related conceptions of teaching and learning mathematics. Can these conceptions be changed during teachers' initial education? Certainly, it is not something

that a short seminar will achieve, but an active and constructive methodology in mathematics and mathematics teaching courses, aligned with reform ideas, should undoubtedly help in this direction. In this way, this thesis' results provide more reasons to be optimistic in the future of mathematics education.

Besides the practical consequences and ideas that this thesis provides for the development of teachers' initial education in Chile, it opens a great number of interesting questions to pursue research in diagnostic competence in error situations. Errors are definitely a rich source of teaching and learning opportunities in classrooms, as it is well documented in this thesis. I wonder why correct steps, answers or arguments are not considered as a source of teaching and learning opportunities for preservice teachers, allowing them to hypothesize about the reasons for the correct answers and about how to deal with them in order to improve students' understanding of mathematics.

I would like to end these words by congratulating Macarena Larrain for this well-grounded thesis, where she could answer several questions and open another set of interesting ones. I enjoyed and learned reading the thesis and it brought to me ideas and practical consequences for education in Chile.

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Introduction

Results from international assessments such as the Programme for International Student Assessment (PISA) or the Trends in International Mathematics and Science Study (TIMSS) have made evident the need to actively work towards improving the quality of school education in many countries. Additionally, 'concerns have been raised about whether preservice and in-service training succeeds in equipping teachers with the professional knowledge they need to deliver consistently high-quality instruction' (Baumert et al., 2010, p. 133). In other words, advancing school effectiveness requires examining and improving teachers' competences and the learning opportunities provided by initial and further teacher education programs.

Most educational reforms across the globe demand that mathematics teaching shifts from a traditional teacher-directed instructional approach to the formation of competence-oriented teaching environments, in which student thinking is highlighted and taken as a starting point for building further mathematical knowledge (e.g., National Council of Teachers of Mathematics, 2000). Moreover, the increasing heterogeneity in classrooms sets new challenges for teachers, who have to be able to support students individually. Such an educational paradigm requires teachers equipped with a series of complex professional competencies that allow them to plan and carry out teaching strategies that consider all students' needs and provide sufficient and targeted learning opportunities. In particular, understanding students' thinking has been regarded as crucial to successfully differentiate learning experiences and use individual learner's reasoning as the basis on which to build further mathematical knowledge (Cooper, 2009; Empson, 2003; Jacobs, Lamb & Philipp, 2010).

In the field of mathematics education, the professional competencies needed by teachers have been extensively described, researched, and debated (see, for

instance, Ball, Thames & Phelps, 2008; Blömeke, Gustafsson & Shavelson, 2015; Depaepe, Verschaffel & Kelchtermans, 2013; Kaiser, Blömeke, König, Busse, Döhrmann & Hoth, 2017; Shulman, 1986). It has been argued for the relevance of teachers' abilities to identify and understand what students know, what they still need to learn, and what they have incorrectly understood, to conduct ongoing analyses of students' learning and to make instructional decisions that support and cognitively activate student learning. Particularly significant have been research and discussions about teachers' noticing skills (Sherin, Jacobs & Philipp, 2011). This line of research has highlighted teachers' abilities to identify noteworthy classroom events and children's mathematical thinking and to interpret these observations by connecting them with prior knowledge and experiences (Jacobs et al., 2010; van Es & Sherin, 2002). A third component has been included, namely, teachers' skills to decide how to provide an instructional response based on what they have attended to and interpreted (Jacobs et al., 2010).

From another perspective, teachers' diagnostic competence has been regarded as essential for the evaluation and understanding of student thinking and, thus, for adapting and individualizing teaching strategies in a way that fosters further learning. According to Weinert, Schrader and Helmke (1990), diagnostic competence is one of the four key components of teacher expertise, together with classroom management competence, knowledge of instructional techniques, and subject-matter knowledge. Helmke (2017) justifies the particular importance that teachers' diagnostic competence has for the teaching and learning process because of its essential role in designing effective teaching strategies and aligning instructional responses to students' learning requirements. Prediger (2010) emphasizes the connection of teachers' diagnostic competence to a student-centered teaching style and argues that it is necessary to understand student thinking and take it as a starting point to build further learning and provide sensitive support to individual students and the whole class.

Additionally, it has been widely recognized that learning situations in which errors arise are very often a rich source of information for teachers (Ashlock, 2010; Prediger & Wittmann, 2009; Rach, Ufer & Heinze, 2013; Radatz, 1979; Scherer & Moser Opitz, 2012). Analysis of students' errors can uncover their erroneous conceptualizations or misconceptions and provide teachers with valuable insights into individual students' understanding of mathematical concepts and procedures. Based on this information, teaching strategies can be targeted to students' particular needs and, thus, better promote further mathematical learning.

Despite the agreement on the relevance of teachers' diagnostic competence for promoting student mathematical learning, more evidence is needed about the development of this competence and how it can be productively fostered in teacher

education. In particular, preservice primary school teachers' diagnostic competence in error situations requires further attention in order to better understand key characteristics and the structure of the competence and to uncover critical aspects influencing its development. It has been suggested that teaching experience is a relevant factor for the development of this competence. However, it has also been argued that preservice teachers can benefit from guided experiences and knowledge that may constitute the basis for further developing this competence (Cooper, 2009; Heinrichs, 2015). Hence, it is the interest of this study to examine the characteristics of preservice primary school teachers' diagnostic competence in error situations and how it can be fostered. Consequently, the central research question of the present study is

> *To what extent is it possible to promote preservice primary school teachers' diagnostic competence in error situations within initial teacher education?*

Towards that goal, a brief university seminar sequence was developed to be included in initial teacher education programs at Chilean universities. The seminar sequence introduces preservice teachers into the value of analyzing students' errors for improving teaching and aims at building the foundations for the development of their diagnostic competence. To investigate the effect of the seminar sequence, a computer-based pre- and post-test was developed to assess preservice teachers' diagnostic competence in error situations and evaluate their changes after taking part in the seminar sequence. The design of both the seminar sequence and the diagnostic-competence test was grounded on a study carried out in German universities with preservice secondary mathematics teachers (Heinrichs, 2015) and used its model of the diagnostic process in error situations, which consists of three facets: perceiving the error, hypothesizing about causes for the error and dealing with the error. The model's facets were also used in the present study to structure the characterization of preservice primary school teachers' diagnostic competence in error situations and describe its changes after participation in the seminar sequence.

To specify the construct of diagnostic competence in error situations and frame the interpretation and discussion of the results of the present study, the first chapter provides an overview of the current state of research on the field. The concept of teachers' professional competencies, as well as various models specifying it for mathematics teachers, are described and explained. This is followed by a review of different conceptualizations and models of the construct of teachers' diagnostic competence and its connection with teachers' professional competencies. Moreover, the relevance of teachers' diagnostic competence for the teaching and learning

process and how it can be fostered within teacher education are described. Additionally, the role of errors in mathematics teaching and learning is revised and connected to the specification of the concept of diagnostic competence for learning situations in which errors arise. Finally, the three facets of the diagnostic process in error situations are described in detail as used in this study.

After presenting the guiding research question and hypotheses in the second chapter, the third chapter specifies the study's methodological context. It begins by displaying the study's design and detailed descriptions of the four sessions of the university seminar sequence and the pre- and post-test developed to assess preservice teachers' diagnostic competence in error situations. Next, a brief overview of data collection in Chilean universities is provided. Lastly, both the qualitative and quantitative methods used in the present study are presented. Thereupon, the method of qualitative text analysis is described, and, in particular, the procedures used to conduct evaluative qualitative text analysis in the present study are specified. In a similar way, the quantitative methods used in the study are described, namely Item-Response-Theory, Latent-Class-Analysis, and several statistical tests for hypotheses testing.

The fourth chapter presents the results of the conducted analyses. First, the results of the cross-sectional analyses of the data are reported, which allows describing preservice teachers' diagnostic competence in error situations and relating these characteristics with other features of preservice teachers' background. Similarly, the results of the longitudinal analyses of the data are presented and discussed. This allows examining the changes observed in preservice teachers' diagnostic competence in error situations after their participation in the university seminar sequence. Moreover, the relations of these changes with features of preservice teachers' backgrounds provide valuable indications about relevant aspects for promoting the development of preservice teachers' diagnostic competence within initial teacher education.

In the closing chapter, the main results of the study are summarized and discussed. Then, limitations of the present study, as well as arising questions, are explicated. The chapter ends by considering and discussing implications of the results for teacher education and further research opportunities.

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1 State of the Art and Theoretical Framework

1.1 Teachers' Professional Competencies

In the light of unsatisfactory students' results in large-scale international assessments, great concern about the effectiveness of teachers' work has been raised, particularly in many Western countries. Closely together, criticism of teacher education in general has notably increased, which has, in turn, directed the attention and efforts of many researchers towards the study of teachers' professional competences, their nature and development. In particular, the professional competencies of mathematics teachers have gained much attention in the last decades.

1.1.1 The Construct of Professional Competence

Before going into details about the nature and characteristics of teachers' competencies, the concept of competence itself needs to be clarified. In educational research, the approach proposed by Weinert (2001) has strongly influenced the understanding of the concept of competence. He stated that it "refers to the necessary prerequisites available to an individual or a group of individuals for successfully meeting complex demands" and argued that these prerequisites "are comprised of cognitive and (in many cases) motivational, ethical, volitional, and/or social components" (Weinert, 2001, p. 62). Moreover, he pointed out that these prerequisites are acquired through learning. Hence, according to this approach, competencies are composed of a set of cognitive abilities, knowledge, skills and associated attitudes, motivational, volitional and social variables that

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are learned by individuals and then available for successfully solving complex demands at specific situations. Furthermore, Koeppen, Hartig, Klieme and Leutner (2008) highlight the context-specificity of competencies. They suggest that competencies are connected to the specific domains in which tasks are to be solved and, therefore, they are to be developed through learning opportunities in situations relevant to the domain.

1.1.2 Initial Models of Teachers' Professional Competencies

Weinert's (2001) conceptualization of competence distinguishes between cognitive and affective or motivational aspects that are needed to meet the demands of complex tasks. Discussions about the competencies needed by mathematics teachers have largely focused on cognitive aspects. Various researchers have studied the nature of the knowledge needed by teachers to be effective in their occupation, many of them starting from the work of Shulman (1986), who introduced the concept of 'pedagogical content knowledge' (PCK) and largely influenced the field.

1.1.2.1 The Seminal Contribution of Lee Shulman

In his groundbreaking work, Shulman (1987) outlined seven categories into which the knowledge needed by teachers to promote learning can be structured. These included content knowledge, general pedagogical knowledge, curricular knowledge, pedagogical content knowledge, knowledge of students and their characteristics, knowledge of the educational context and knowledge about the purposes and values of education. He stressed the relevance of pedagogical content knowledge (PCK) because it is distinctive of the work of teaching, a "special form of professional understanding" (p. 8). He characterized it as "the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction" (p. 8).

Shulman (1986) detailed three of the seven categories, namely on those related to content knowledge: subject matter content knowledge, pedagogical content knowledge and curricular knowledge. Regarding subject matter content knowledge, he emphasized that teachers need not only to know the facts and contents but also to understand the structures of the subject. Besides understanding concepts and knowing facts, teachers "must also be able to explain why a particular proposition is deemed warranted, why it is worth knowing, and how it relates to other propositions, both within the discipline and without, both in theory and