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Hayato Hirai

Towards Infrared Finite S-matrix in Quantum Field Theory

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Towards Infrared Finite S-matrix in Quantum Field Theory

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Supervisor's Foreword

It is my great pleasure to introduce the work of Dr. Hayato Hirai, now published in this Springer Thesis series. His work is on the infrared problems in the quantum field theory.

The quantum field theory, which describes the physics of elementary particles, is a very successful theory. The Standard Model is formulated by a quantum field theory and describes matters and forces other than gravity very well. A good example of the success of this theory is that the anomalous magnetic moment of the electron agrees between theory and experiment to more than 8 significant digits. This is one of the best agreements between theory and experiment in physics. However, there are still some conceptually poorly understood aspects of the quantum field theory. One of them is the infrared problem. In a quantum field theory that includes a massless field, there is a divergence from the infrared region, and the S-matrix elements, which quantify the scattering, are not well defined. This is also a problem in quantum electrodynamics, which appears in the early stage of the history of the quantum field theory, and is one of the most successful examples.

A solution to this problem in standard textbooks is called inclusive formalism, which brings up the "resolution of the detector". In this formalism, although we cannot define the S-matrix elements, we can define the inclusive scattering cross section, which is an observable quantity. Although this has convinced many people, some people still think it is important to define the S-matrix elements in some way, and this has been studied for a long time.

Dr. Hirai recognizes the importance of an infrared finite S-matrix and has done these works. In this thesis, he makes two important contributions towards the construction of an infrared finite S-matrix.

One is to show that what is called "asymptotic symmetry" is a physical symmetry, and furthermore to show that the conservation law for asymptotic symmetry is equivalent to a theorem called "the soft photon theorem". Asymptotic symmetry is a subtle "symmetry" that at first glance appears to be unphysical, i.e., a symmetry in which the corresponding conserved quantity is always zero regardless of the state. Therefore, its treatment has been controversial. In this work, using a standard formulation called the BRST formalism, he and collaborators have fully clarified that the asymptotic symmetry is a physical symmetry. Furthermore, they have developed the relation between the sub-leading soft photon theorem and this symmetry. These results have had a great impact on the field.

The other one is that the asymptotic state of electrons proposed by Faddeev and Kulish is shown to be a physical state, using the BRST formalism. Chung, Faddeev, Kulish, and other people were not completely satisfied with inclusive formalism. They have tried to change the concept of asymptotic states and obtain an S-matrix without infrared divergence. They have constructed a class of states, now called "Faddeev-Kulish states". However, since the BRST formalism was not well developed at that time, the Faddeev-Kulish states were not considered to be "physical states". Therefore, people were lost in trying to make them physical. Dr. Hirai and collaborators have fully clarified that Faddeev-Kulish states are physical states using the now-standard method, the BRST formalism. This is a major step in constructing the S-matrix theory without infrared divergence. It will lead to future comparisons with inclusive formalism and experimental verification, which will have a significant impact on the field.

Osaka, Japan
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Prof. Satoshi Yamaguchi

Parts of this thesis have been published in the following journal articles:

HS1. H. Hirai and S. Sugishita, "Conservation Laws from Asymptotic Symmetry and Subleading Charges in QED", Journal of High Energy Physics **07** (2018) 122.
HS2. H. Hirai and S. Sugishita, "Dressed states from gauge invariance", Journal of High Energy Physics **06** (2019) 023.

Parts of Chaps. 2, 4 and Appendices A, B, C, E, G are reprinted from [HS1], and parts of Chaps. 5, 6 are reprinted from [HS2], with revisions and with permission.

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Contents

Chapter 1
Introduction and Summary

This thesis is devoted to a better understanding of infrared structures of scattering theory in gauge theories, in particular, *quantum electrodynamics (QED)* in four-dimensional Minkowski spacetime. The investigation of scattering phenomena in the gauge theories describing our world at low energy, such as QED, has a long history since the early nineteenth century. The comparison between theoretical predictions and experimental data of scattering cross-sections has been one of the main sources of ideas in developing the models explaining our world. Nowadays the Standard Model of particle physics has demonstrated tremendous successes in explaining the data provided by collider experiments with great accuracy, at least with the current experimental resolution.

However, the infrared dynamics in the gauge theories involving long-range forces has recently turned out to be worth reinvestigating, which was first triggered by the discovery of the *asymptotic symmetry* in Maxwell and Yang–Mills theory coupled to massless charged matters [1]. Moreover, the discovery of these new symmetries led to the discoveries of a triangle equivalence of seemingly unrelated following three subjects that concern the infrared dynamics in QED, QCD, gravity: *asymptotic symmetry, soft theorem*, and *memory effect*. This equivalence, called *infrared triangle*, is one of the main subjects in this thesis.

Asymptotic symmetry transformations, which I will define precisely later, refers to a gauge transformation which changes boundary conditions on asymptotic spacetime regions while keeping physically reasonable fall-offs behaviors of gauge fields. Although asymptotic symmetry is local symmetry, this is *physical symmetry*. The history of the asymptotic symmetry analysis goes back to the seminal work in gravity by Bondi, van der Burg, Metzner and Sachs (BMS) [2, 3] in 1962. They found the infinite-dimensional subgroup of diffeomorphisms of asymptotically flat spacetime that act non-trivially on the boundary data, which is now called the BMS group. On the other hand, as already mentioned, the asymptotic symmetries in gauge theories in asymptotically flat spacetime were revealed recently [1, 4–6].

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The *soft theorems* describe the universal property of scattering amplitudes with external *soft* particles *i.e.* massless particles whose energies are much less than the energies of external charged matters. For example, the soft photon theorem [7–14] in QED simply says that the scattering amplitude of the process $\alpha \to \beta$, say $\mathcal{M}_{\beta\alpha}$, and the one with an additional external soft photon of momentum $k^\mu = (\omega, \mathbf{k})$, say $\mathcal{M}_{\beta\alpha}(k)$, are proportional to each other as

$$\mathcal{M}_{\beta\alpha}(k) = \mathcal{M}_{\beta\alpha} \sum_{n}^{N} \frac{\eta_n e_n \epsilon(k) \cdot p_n}{p_n \cdot k} + \mathcal{O}(\omega^0), \tag{1.1}$$

where e_n and p_n are the charge and the four-momentum of n-th particle in the initial state α and the final state β, $\epsilon_\mu(k)$ is the polarization of the soft photon, and η_n is a sign factor which takes $+1$ for particles in β and -1 for particles in α. The factor of proportionality, called the (leading) soft factor, diverges as the energy of soft photon tends to zero, since it is order $\mathcal{O}(\omega^{-1})$. This fact reflects one of the important property of infrared dynamics; the slight acceleration of a charged particle results in the radiation of infinite number of low energy photons.

The *memory effect* is concerned with the detection of waves coming from faraway sources. It has been investigated mainly as a mechanism for the observation of gravitational waves, called the gravitational memory effect. It originates in a proposal in 1974 by Zel'dovich and Polnarev [15], and developed by many others [16–26]. The gravitational memory effect claims that the passage of gravitational waves coming from faraway sources produces a permanent displacement of the metric and results in a permanent displacement of relative position of freely falling particles (viewed as detectors).[1] Although the gravitational memory effect has a long history, the electromagnetic analog of the memory effect was first studied recently [23, 29, 30]. The electromagnetic memory effect is a phenomenon: the total net charge that has passed through at an local angle is given by a permanent displacement of the gauge field at the corresponding angle, instead of the metric in gravity.

The infrared (IR) triangle reveals the surprising fact that the above three subjects, describing seemingly different aspects in infrared (long distance) physics, are just different perspective of one subject. The equivalence of those was first discussed in the Yang–Mills theory [1], and extended to gravity [31–33] and also to QED [4–6]. The equivalence has also been extended to higher orders of the soft expansion (e.g. [34–37]), to higher spacetime dimensions (e.g. [38, 39]), and also to other theories (e.g. [40–42]) by many others.[2] The IR triangles are not just about the mathematical equivalence among the three things already known because the IR triangles are universal equivalences valid for many theories including massless particles and there were a few theories in which all corners of the triangle were fully understood. In fact, the insight from the viewpoint of the infrared triangle has led to the many discoveries of new corners in many different theories in the last several years (for example,

[1]Detection of the memory effect at LISA [27] and at LIGO [28] has been proposed recently.

[2]We have just referred to relatively old references here because there are too many. We will refer to the references directly related to our works in the later chapters.